T R A C T S:

CONTAINING

I, MECHÀNICS, or the DOCTRINE of MOTION.

II. The PROJECTION of the SPHERE.

III. The LAWS of CENTRIPETAL and CENTRIFUGAL FORCE.

By WILLIAM EMERSON.

A NEW EDITION.

TO WHICH IS PREFIXED,

Some ACCOUNT of the LIFE and WRITINGS of the AUTHOR,

By the Rev. W. B O W E,

Of *Scorton*, near *Catterick, Yorkſhire*.

LONDON:

Printed for, F. WINGRAVE, Succeſſor to Mr. NOURSE, in the *Strand.*

MDCCXCIII.

SOME ACCOUNT

OF THE

L I F E

OF

Mr. WILLIAM EMERSON.

T HE Writer of the following Memoirs judges it proper to premife from what fources he has derived his information. He had the good fortune to be perfonally acquainted with Mr. EMERSON during the laft three or four years of his life; and enjoyed frequent opportunities of learning, from his own mouth, accounts of circumftances which had taken place at former periods of his life. He has alfo had frequent converfations refpecting him with feveral perfons of veracity, who knew him many years before. He has not, however, accumulated an indifcriminate mafs of anecdotes, as they were offered to him, but has rejected fuch as did not appear to him to be authentic, and adopted fuch only as upon mature examination and enquiry, he had reafon to think true. In fhort, he has endeavoured to form and exhibit as juft a picture of Mr. EMERSON's Life as he could, under fome peculiar difficulties; and his motive for publifhing it was a defire to gratify, in fome meafure, that curiofity, which mankind muft always entertain, to be informed of whatever has relation to thofe men, who have drawn the

attention of their fellow-creatures by the superior
splendor of their talents, and who have enlarged
the boundaries of human knowledge.*

Mr. DUDLEY EMERSON, of *Hurworth*,
near *Darlington*, in the County of *Durham*, had
two Sons, WILLIAM, the elder, and DUDLEY,
who died whilst he was young. WILLIAM, who
afterwards lived to become so eminent a Mathe-
matician, was born at *Hurworth*, in the year
1701, and, as appears by the Parish Register, was
baptized there on the 10th of June in that year.
" In a vacant leaf of an old prayer book, in
" which DUDLEY EMERSON, the father of
" WILLIAM, had registered his marriage, and
" the births of his several children, it is written,
" William Emerson was born Wednesday,
" May 14, at ®ne o'clock in the morning, and
" 40 minutes, and baptized June 10, 1701."
His father DUDLEY who was possessed of but
a small estate, at that time taught a school, and
seems to have thought himself of some little con-
sequence in the world, for I have seen a paper,
written by himself, containing what he calls an
account of the principal transactions or events of
his life. Amongst these *memorabilia*, relating, I
think, chiefly to his movements from one place
to another, I observed nothing respecting the
birth or education of his son *William*; which he
did not foresee would be the only circumstances
or events of any importance in his life, that
might possibly rescue his name from oblivion.

<div align="right">*William*</div>

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* " Several circumstances and anecdotes have been inter-
" mixed with Mr. BOWE's narrative, or subjoined to it as
" notes. These were partly taken from other accounts of
" Mr. EMERSON, and partly communicated by gentlemen
" who had a personal acquaintance with him."

William was taught, and principally by his father, reading, writing and arithmetic, and a little Latin, perhaps as far as *Cordery*, or *Beza's Latin Teſtament.** It does not appear, however, that he was much attached to his books whilſt a boy, or exhibited any ſymptoms of thoſe ſuperior faculties, which he afterwards exerted with ſo much energy. Indeed ſo carelefs and inattentive to learning was he, at this period, that I have heard him ſay, till he was near twenty years of age, his principal and favourite employment, for one ſeaſon of the year, was that of ſeeking birds neſts. But his attachment to childiſh amuſements was now to paſs away; and his mind begaſ to be ſenfible of the charms and beauties of ſcience. He went firſt to Newcaſtle, and afterwards to York, where he applied himſelf, with confiderable attention and diligence, to the ſtudy of the Mathematics, under the direction of ſchoolmaſters, whoſe names I do not remember, but of whom he uſed to ſpeak, in the latter part of his life, with much reſpect. " He " uſed to ſay too, that his father was a tolerable " Mathematician; and, without his books and " inſtructions, perhaps his own genius (moſt " eminently fitted for mathematical difquiſitions) " would never have been unfolded."

After his return from the ſchool at York, he refided principally at Hurworth, where he continued to purſue his ſtudies and amuſements, at intervals, until the time of his marriage. In what year of his life this happened I do not exactly know, but I think it was about the thirty-ſecond or thirty-third: and from this

<center>a 3</center> <div align="right">period</div>

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* " He afterwards received ſome affiſtance in the learned " languages from a young gentleman, then curate of " Hurworth, who was boarded at his father's houſe."

period we muft date the commencement of his mathematical *labors*; or perhaps rather thé communication of them to the public. What he had done before in this line, was merely an occafional application, for his own amufement, or for the exercife and improvement of his leifure hours. But one of thofe accidents, which, as Dr. Johnfon obferves in the Life of Cowley, " produce that particular defignation of mind and propenfity for fome certain fcience, commonly called genius," took place upon this occafion, and added a powerful ftimulus to his native thirft for knowledge and for fame. His wife was the Niece of a Doctor Johnfon, Rector of Hurworth, Vicar of Manfield in the county of York, and a Prebendary of Durham, a man very eminent in his time for his fkill in furgery, and who, by a very extenfive and fuccefsful practice in this profeffion, together with the emoluments arifing from his livings, had accumulated a confiderable fortune. Dr. Johnfon had promifed to give his niece, who lived with him, five hundred pounds for her marriage portion. Some time after the marriage, Mr. EMERSON took an opportunity to mention this matter to the Doctor, and to remind him of his promife. The Doctor, however, did not recollect, or did not chufe to recollect, any thing of it, but treated our young Mathematician with fome contempt, as a perfon of no confequence, and beneath his notice. The pecuniary difappointment, EMERSON (who had as independent a fpirit as any man, and whofe patrimony, though not large, was equal to all his wants) would eafily have furmounted, but this contemptuous treatment ftung him to the very foul. He immediately went home, packed up all his wife's cloaths,

and

and fent them off to the Doctor, faying that he would fcorn to be beholden to fuch a fellow for a fingle rag; and fwearing at the fame time that he would be revenged, and prove himfelf to be the better man of the two, His plan of revenge was truly noble and laudable.—He was refolved to demonftrate to his uncourteous uncle and to the world, that he was not to be rated as an infignificant or ignorant perfon; and that the contempt and indignity, with which he had been treated, were much mifplaced, and very un-merited : and, in order to demonftrate this, he determined to labour till he became one of the firft Mathematicians of the age.

He had received from nature a ftrong and vigorous mind, and had acquired a juft relifh for the beauties of Mathematical Science, and an ardent love of Truth; he was at the fame time, ftimulated with an eager defire of diftinguifhing himfelf from the illiterate crowd of mortals: the effects of his labor, influenced by fuch motives, and directed by fuch abilities, could not therefore but be great. He made himfelf a perfect mafter of the whole circle of the Mathematics; and after having carefully planned and digefted, revifed and compleated the work to his own fatisfaction, he publifhed, in the forty-fecond year of his age, his book of *Fluxions*; and at his firft appearance in the world as an author, ftepped forth like a giant in all his might, and juftly claimed a place amongft Mathematicians of the very firft rank. By the ftrictly fcientific manner in which he eftablifh-ed the principles, and demonftrated the truth of the method of Fluxions in this work, he added another firm and durable fupport to the noble edifice of the Newtonian Philofophy, which, by fome lefs accurate and penetrating obfervers, was

fuppofed

fuppofed to have received a violent and dangerous concuffion from the metaphyfical artillery of the *analyft*, and the cavils and objections which had been advanced againft the truth of the fluxionary method.

Having thus fecured his mathematical fame upon a firm and folid Bafis, he continued from time to time to favour and inftruct the publick with other moft valuable publications upon the feveral branches of the Mathematics. Thefe appeared in the order in which they ftand arranged below: I have added to each the date of the Author's life.*

Years. Ætat. An.

1743 . 42 Fluxions, 8vo.
1749 . 48 Projections of the Sphere, and Elements of Trigonometry, 8vo.
1754 . 53 Mechanics, 4to.
1755 . 54 Navigation, 12mo.
1763 . 62 Arithmetic, Geometry, 8vo. Method of Increments, 4to
1764 . 63 Algebra, 8vo.
1767 . 66 Arithmetic of Infinites and Conic Sections, 8vo.
1768 . 67 Elements of Optics and Perspective, 8vo.
1769 . 68 Astronomy, Mechanics, Centripetal and Centrifugal Forces, 8vo.
1770 . 69 Mathematical Principles of Geography, Navigation and Dialling, Comment on the Principia, with the Defence of Newton. Tracts, 8vo.
1776 . 75 Mifcellanies, 8vo. which was his laft work.

The above Works, many of them allowed to be the beft extent upon the fubjects of which they treat, will remain a lafting monument of Mr. EMERSON's genius, penetration and induftry

to

* New editions of feveral of the above works have fince been publifhed, with improvements, communicated from the Author, by F. WINGRAVE, Succeffor to Mr. NOURSE, Bookfeller in the Strand.

to the lateft times; and render any further eulo-
gium of their Author, as a Man of Science,
totally unneceffary.

" His firft publications however did not meet
with immediate encouragement; fo that it is
probable the reft would never have appeared, or,
at leaft not in the Author's life time, had he not
about the year 1763, been recommended, by
his great admirer and friend the late EDWARD
MONTAGU, Efqr. to Mr. JOHN NOURSE, Book-
feller in London, who was himfelf an eminent
Mathematician, and well fkilled in the Newtonian
Philofophy, having had an univerfity education,
and been an early affociate with the learned
Doctors Pemberton and Wilfon, the one the
companion of Newton, and Editor of the beft
edition of the Principia, the other of Mr. Robins's
Mathematical Tracts. Mr. NOURSE was fo
highly fenfible of Mr. EMERSON's fuperior abili-
ties, that he engaged him on very liberal terms,
to furnifh a regular courfe of the Mathematics
for the ufe of young ftudents. Mr. EMERSON
made a journey to London in the fummer of the
year 1763, to fettle and fulfil this agreement. Even
in London he could not be idle : befides correcting
his fheets for the prefs,* he took lodgings at a
Watchmaker's, near Smithfield, that he might
improve himfelf, in that branch of knowledge
during his ftay there."

" Befides the above regular works, publifhed
in Mr. EMERSON's own name, he wrote feveral
other fugitive pieces, in the Ladies Diaries and
other periodical and mifcellaneous works. In the
Ladies Diaries he propofed and anfwered feveral
new queftions under the fignature *Merones*, an
anagram

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* " He always made the revifal himfelf; to truft no eyes
" but his own, being a favourite maxim with him."

anagram of his own name, containing all the letters of it tranfpofed; the queftions refolved by him were as follows, viz. prize, 1736; queft. 193, 195, 197; prize, 1737; queft. 205, 206, 207, 209, 210, 215, 217, 221, 223; prize, 1741; queft. 226, 229; prize, 1742; queft, 238, 240: and he propofed the following new queftions; No.193, 206, and 220*. Mr. EMERSON alfo took a part in the *Mifcellanea Curiofa Mathematica*, a work publifhed in quarterly numbers, by Mr. FRANCIS HOLLIDAY, (his friend and correfpondent) from the year 1745 till 1755, in 4to. In this work he refolved many queftions, as before in the Diaries, fometimes under the fignature of *Merones*, and fometimes under the ftill more whimfical one of *Philofluentimechanalgegeomaftrolongo*; and probably under feveral others." We fhall now take a view of Mr. EMERSON in his private life, as a man, and a member of fociety.

Mr. EMERSON was, in perfon, fomething below the common fize, but firm, compact, well-made, very active and ftrong. He had a good, open, expreffive countenance, with a ruddy complexion, a keen and penetrating eye, and an ardor and eagernefs of look that was very expreffive of the texture of his mind. His drefs was very fimple and plain, or, what, by the generality of people, perhaps would have been called grotefque and fhabby. A very few hats ferved him through the whole courfe of his life, and when he purchafed one (or indeed any other article of drefs) it was a matter of perfect indifference

* " See the *Diarian Mifcellany*; which is a republication " of all the ufeful parts of the Ladies Diaries, by Dr. " HUTTON In the Diaries, &c. Mr. EMERSON had alfo " fome warm controverfies with Wadfon (Mr. Dawfon of " Sedbergh) and other eminent Mathematicians."

ference to him, whether the form and fashion of it was that of the day, or of half a century before. One of these hats, of immense superficies, had in length of time lost its elasticity, and the brim of it began to droop in such a manner as to prevent his being able to view the objects before him in a direct line. This was not to be endured by an optician: he therefore took a pair of sheers, and cut it round close by the body of the hat, leaving a little to the front, which he dexterously rounded into the resemblance of the nib of a jockey's cap. His wigs were made of brown or a dirty flaxen coloured hair, which at first appeared bushy and tortuous behind, but which grew pendulous through age, till at length it became quite straight, having probably never undergone the operation of the comb: and either through the original mal-conformation of the wig, or from a custom he had of frequently inserting his hand beneath it, his hind-head and wig never came in very close contact. His coat, or more properly jacket, or waistcoat with sleeves to it, which he commonly wore without any other waistcoat, was of a drab colour. His linen came not from *Holland* or *Hibernia*, but was spun, and bleached by his wife, and woven at *Hurworth*, being calculated more for warmth and duration than for shew. He had a singular custom of frequently wearing, especially in cold weather, his shirt with the wrong side before, and buttoned behind the neck. But this was not an *affectation* of singularity (for Emerson had no *affectation* about him, though his customs and manners were singular:) he had a *reason* for it;—he seldom buttoned more than two or three of the buttons of his waistcoat, one or two at the bottom, and sometimes one at the top; leaving all the rest
open.

open. In wind, rain or fnow, therefore, he muſt have found the aperture at the breaſt inconvenient, if his ſhirt had been put on in the uſual manner. His breeches had an antique appearance, the lappet before not being ſupported by two buttons, placed in a line *parallel* to the plane of the horizon, but by buttons deſcending in a line *perpendicular* to it. In cold weather he uſed to wear, when he grew old, what he called *ſhin-covers*. Now theſe *ſhin-covers* were made of old ſacking, tied with a ſtring above the knee, and depending before the ſhins down to the ſhoe; they were uſeful in pre-ſerving his legs from being burnt, when he ſate too near the fire (which old people are apt to do;) and if they had their *uſe,* he was not ſolicitous about the figure or appearance they might make.

This ſingularity of dreſs and figure, together with his character for profound learning, and know-ledge more than human, cauſed him to be confi-dered, by ignorant and illiterate people in the neighbourhood, as a *wiſe* or *cunning man,* or *conjuror*: many of them are ſtill perſuaded that he was ſuch, and will tell you wonderful ſtories of the feats he performed, and particularly how by virtue of a magic ſpell, he pinned a fellow in the top of his pear or cherry tree, who had got up with a deſign to ſteal his fruit; and compelled him to ſit there a whole Sunday's forenoon, in full view of the congregation going to and returning from church. That he did compel a man to ſit for ſome time in the tree, I believe was a fact, not how-ever by virtue of any magic ſpell, but by ſtanding at the bottom of the tree, with a hatchet in his hand, and ſwearing that if he came down he would *hag* (i.e. hew) *his legs off*. This opinion of his ſkill in the *Black-art,* was of ſervice in defending his property from ſuch depredations, and therefore

it

it would have been impolitic to difcourage it:
but he was apt to lofe his patience very much,
when he was applied to for the recovery of ftolen
goods, or to inveftigate the fecrets of futurity. A
woman came one day to him to enquire about her
hufband, who had gone fix years before to the Weft
Indies or America, and had not been heard of,
fince.   She requefted therefore to be informed
whether he was dead or living, as a man, in her
neighbourhood, had made propofals of marriage
to her.   It was with much difficulty the fuppofed
prophet reprelfed the rifing *furor* till the conclufion
of the tale; when rifing haftily from the tripod, or
three footed ftool, on which he ufually fate, in terms
more energetic than ever iffued from the fhrine
at Delphi; he gave this plain and unequivocal
refponfe; " D—n thee, for a bitch! thy hufband's
gone to hell; and thou may go after him."   The
woman went away well pleafed and fatisfied with
the anfwer fhe had received, thinking fhe might
now liften to the propofals of her lover with a fafe
confcience.   Another young damfel, on a fimilar
errand, met with a milder reception.   Her miftrefs
had loft fome caps, or linen, and fhe wanted to
know whether her fellow fervant (of whom fhe en-
tertained fufpicions) had purloined them or not.
" *Thou's a canny young lafs*, replied the fmiling
conjurer, *but thou's over late o'coming, I can do
nought for the.*"   The poor girl went away grieved
that fhe had not made her application fooner,
fuppofing he meant that the myfterious moment
of divination was paft.   He was by many people
alfo looked upon as an atheift, but he was as
much an atheift as he was a magician.   He firmly
believed in the being of a God; he did not
believe it, as he fometimes faid, he *knew* it, he
was *certain* of it to a demonftration.   But it muft
be

be acknowledged, that he did not always fpeak of revealed religion, the Church of England or the clergy, in terms of refpect. It has often been obferved and lamented, that minds merely mathematical, are apt to tend towards fcepticifm and irreligion. The man who is always accuftomed to demonftrative proofs, and wholly engaged in a fcience which admits of them at every ftep, will not fo readily acquiefce in a feries of probabilities, where inveftigations of another kind are prefented to him; and perhaps will not have patience to examine circumftances deeply enough to afcertain, on which fide there is a preponderancy of evidence amounting nearly to demonftration. Befides, EMERSON's refentment at Dr. Johnfon's treatment of him, which I have mentioned before, might produce a bias on his mind unfavourable to religion. A man of his natural impetuofity of temper would be too apt to affociate the idea of the *profeffion* with that of the *profeffor*; and becaufe he had quarrelled with the *prieft*, would alfo quarrel with his *doctrine*. Under the influence of this prepoffeffion, he fet himfelf to work to examine the Scriptures of the Old and New Teftament, and collected two fmall 4to. vols. of what he conceived to be contradictory paffages; and arranged them, like hoftile troops confronting each other, on the oppofite pages of his book.

His diet was as fimple and plain as his drefs, and his meals gave little interruption to his ftudies, employments or amufements. During his days of clofe application, he feldom fat down to eat, but would take a piece of cold pye or meat of any kind in his hand, and retiring with it to his place of ftudy, could fatisfy his appetite for knowledge and food at the fame time. He catered for himfelf, and pretty conftantly

made

made his own market. When his ftock of groceries, or other neceffaries in the article of houfekeeping, grew low, on the Monday morning he took his *wallet*, which he flung obliquely acrofs his fhoulders, and fet forward for the market at Darlington, three miles diftant from Hurworth; whither he always walked on foot, for he feldom or never kept a horfe, and had an averfion to riding. " He would frequently lead " the horfe, when he had one, from market, by " the halter, bearing the wallet ftuffed with the " provifions he had bought at market." After having provided all the neceffary articles he did not always make directly home again, but, if he found good fair ale, and company to his mind, he would fit himfelf down contentedly in fome public houfe, for the remainder of the day, and frequently during the night too; fome times he did not reach home till late on Tuefday or even Wednefday; he remained talking, or difputing on various topics, mechanics, politics or religion, juft as his company might be, varying the fcene fometimes with a beef-ftake, mutton-chòp, or a pan of cockles; for it is remarkable that his ale did not injure, but rather improve his appetite; and that he never felt the head-ach, or any immediate ill effects afterwards. In thefe *durable* potations, he would fometimes indulge, not only at Darlington, but in Hurworth, or fome neighbouring village, and always in an ale-houfe, for he kept no ftock of ale at home; and he was upon all occafions rigidly exact in apportioning each man's quota of the reckoning. The laft time he made an excurfion to Darlington with his wallet, our Philofopher made a figure truly confpicuous: this was the only time, I believe, he ever rode thithèr,

and

and he was then mounted on a quadruped, whofe intrinfic value, independent of the fkin, might be fairly eftimated at half a crown. Being preceded and led by a boy hired for that purpofe, he crawled in flow and folemn ftate, at the rate of a mile and a half in an hour, till in due time he arrived at Darlington, and was conducted in the fame ftate, to the great entertainment of the fpectators through the ftreets to the inn, where he wifhed to refrefh himfelf and his beaft. What idea EMERSON himfelf entertained of the velocity with which the animal could move, appears from this, that when a neighbour of his from Hurworth afked him towards the evening, if he was going home? *D—n thee* (fays he) *what does thou want with my going home?* Only, fays the man, becaufe I fhould be glad of your company.—*Thou fool, thou!* (rejoined the other) *Thou'lt be home long enough before me, man! thou walks and I ride.* We muft obferve, that fuch expreffions as *D—n thee,* and *thou fool,* were mere expletives often with him, expreffive neither of indignation nor contempt.

His ftyle in converfation was generally very abrupt and blunt, abounding in fuch expletives as the above, and often vulgar and ungrammatical. And this was the reafon that led many people to fuppofe, he could not be able to write any thing like grammar, or tolerable Englifh, and that his prefaces therefore, fome of which are not ill written, could not be his own compofition, but muft have been made, or at leaft tranflated into Englifh for him by fome other hand. But we cannot, with certainty, judge how a man will write from his language in common converfation. Though EMERSON was no accurate grammarian, his reading was extenfive enough to fupply him with

a fufficient

a sufficient stock of proper words and expressions on such subjects as he had occasion to treat of, and the vigour and energy of his mind would give force, weight and perspicuity to his sentences. He was one day told, that this opinion respecting his prefaces was entertained by many ; and the disparity of his conversation and writing was pointed out as the reason of it. After a momentary pause he exclaimed, with some indignation : *A pack of fools! who could write my prefaces but my self?* They do in fact carry with them every mark of legitimacy; they could have no other father. Indeed the original prefaces, as well as the original MSS. of most of the Author's works, in his own hand writing, with a great number of his original letters, are now in the possession of the publisher, all of which are well written, and in a good stile. There are also, in some of them, quotations from the Greek Authors, elegantly written by himself. In the earlier part of his life, principally by the aid of his Dictionary, he had acquired a stock of latin, sufficient to enable him to read and translate mathematical works in that language: and he once had a design to give a translation of the Jesuits comment, on the Principia. It appears also from his *mottos* to his several books, that he sometimes *dipped* into the Classic Authors, for we must not suspect him of having been *much conversant* with such gentry as *Lucretius, Horace* and *Virgil.* He had in his library *Homer* and *Virgil travestied,* and these he read with more pleasure than *Pope's* or *Dryden's* translations. In arguments or disputes, whether in conversation or writing, the fire and impetuosity of his temper was too apt to break forth, and betray him into the use of such terms, as should always be avoided, especially by Mathematicians and Philosophers, whose employment

is, or ought to be, the cool and difpaffionate
fearch after truth. His zeal in fupport of Sir
Ifaac Newton is outrageous, and he has treated the
oppugners of his Philofophy as if he had a perfonal
quarrel againft them. "*J. Bernoulli* (according
to him) is a low critic, laborious and tedious to
the laft degree: a blunderbufs, a perfon of
eternal contradiction, blind, bigotted, prejudiced,
mad; and a whole fection of 100 pages is nothing
but a heap of abfurd, inconfiftent ftuff." He
therefore leaves him (in his great mercy) to be
drowned in a gurges of his own contriving.
Euler receives no milder treatment: he is
vortex-mad too, and is one of thofe bigotted
Philofophers, whofe brains are turned in a vortex,
or they never would prefer fuch complex vortical
fchemes before the fimple doctrine of projectile
and centripetal forces. Such Philofophers! ex-
claims our indignant defender. Leibnitz's at-
tempt to rob Sir Ifaac Newton of the honour of
the invention of Fluxions, is, as we might expect,
a caufe fufficient to make him rave with rage,
and to call down all the thunder of his indignation.
To tell us flatly " that not he that firft *found*,
but he that firft *publifhed*, deferves the praife,"
is an affertion that might move the bile in a
ftomach lefs apt to boil than EMERSON's. It
is robbing, as he truly fays, the inventor of his
due praife to give it to the thief that ftole it. But
however much Sir Ifaac might have approved of
Mr. EMERSON as the *champion* or *defender* of his
principles, he would not have admired the *weapons*
he fometimes made ufe of.

In fo difficult a problem as EMERSON himfelf
allows that of finding the preceffion of the equi-
noxes to be, one would have thought that he
would have had fome patience with thofe who
                                        differ

differ from Sir Ifaac and himfelf. But thefe are a *clan*, and Mr T. Simpfon has the misfortune to be one of the *clan*. There feems, unfortunately, to have fubfifted fome kind of animofity. or jea-loufy between thefe two great contemporary Ma-thematicians, excited and fomented perhaps by fome, who were mean enough to carry tales be-tween them, in order to curry favor. One of EMERSON's correfpondents, in London, informed him that Simpfon had faid, in company with fome mathematical friends, " that he took EMERSON (referring to his book of Fluxions) to be an in-duftrious, plodding writer, but a man of no genius." This was quite enough to irritate EMERSON to acts of hoftility.

We meet with frequent complaints in his books of the little encouragement given to ma-thematical learning in his time, and efpecially in England. How far this complaint may be well or ill founded, I know not, but this is pretty cer-tain, that if any reward or recompenfe had been offered to him for his mathematical labors, and had not come to him in his own way, he would not have accepted it. He did not wifh to be admitted a fellow of the Royal Society, " *becaufe* (he faid) *it was a d—n'd hard thing that a man fhould burn fo many farthing candles as he had done, and then have to pay fo much a year for the honor of F. R. S. after his name. D—n them and their F. R. S. too.*" He reaped one reward of his toils in queft of fcience, which was very acceptable and grateful to him; and that was the acquaintance and friendfhip of *Mr. Montagu*, who having an eftate at Eryholme, near Hurworth, fometimes vifited that neighbour-hood, and fpent a good part of his time in company and converfation with EMERSON, and at his deceafe bequeathed to him a legacy of 100l. or guineas.

When

When Mrs. Montagu's agent called upon him to discharge this legacy, EMERSON told him he would much rather have seen Mr. Montagu himself than his money. And he spoke, doubtless, from his heart, for he never mentioned Mr. Montagu's name but in terms that strongly expressed the sincerity and ardor of his affection and esteem for him.

" Mr. Montagu, in his visits to EMERSON, would often go to him in the fields, when he happened to be at work there, and would accompany him home, but could never persuade him to get into a carriage. On these occasions he would sometimes exclaim, " D--n your whim-wham! I had rather walk."

' Inheriting a small patrimonial estate, of about 60 or 70 pounds a year, he was as independent and happy as if he had enjoyed so many thousands. He was never known to ask a favour, or seek the acquaintance of a rich man, unless he possessed some eminent qualities of the mind.' " Notwithstanding his imperfect and desultory course of education, he acquired a general knowledge of most sciences. He had even paid attention to medicine, at least so far as it has been combined with mathematical principles, according to the plan of Keil, Morgan, &c. He esteemed Morgan above all others as a physician: and held Keil to be the best of anatomists.

" Mr. EMERSON often tried in practice the effect of his mathematical speculations, by constructing a variety of instruments, mathematical, mechanical, or musical, on a small scale. He made a spinning wheel for his wife, which is represented by a drawing in his book of Mechanics. He was well skilled in the science of music, the theory of sounds, and the various " scales,

" fcales, both ancient and modern ; but he was
" a very poor performer, though he fometimes
" obliged his friends in the country by tuning
" and repairing their mufical inftruments.   He
" carried that fingularity which marked all his
" actions even into this fcience.   He had, if I
" may be allowed the expreffion, two firft ftrings
" to his violin, which he faid made the E more
" melodious when they were drawn up to a perfect
" unifon.   His virginal, which is a fpecies of in-
" ftrument like the modern fpinnet, he had cut
" and twifted into various fhapes in the keys, by
" adding fome occafional half-tones, to regulate
" the prefent fcales, and to rectify fome fraction
" or difcord that will always remain in the tuning.
" This he never could get regulated to his mind,
" and generally concluded in a paffion by faying,
" It is a d—mn'd inftrument, and a foolifh thing
" to be vexed with."

" Mr. EMERSON was very fond of angling, and
" while he thus amufed himfelf, would ftand up
" to his middle in water for feveral hours toge-
" ther.—When he was building a houfe upon the
" fmall farm which he poffeffed by the fide of
" the Tees, he never hefitated to plunge into the
" water,   for the purpofe of collecting ftones
" from the bed of the river.   He was affected,
" about that time,   with fome flight gouty
" fymptoms, and faid, that *wading* was fervice-
" able to him, becaufe the water fucked the gout
" out of his legs ; a theory for which he was
" probably not indebted to his favourite Keil or
" Morgan.   When he wrote his treatife on
" Navigation, he muft needs make and fit up a
" fmall veffel ; with this he and fome young
" friends embarked on the river Tees, that ran
" by his door at Hurworth ; but the whole crew
                                        " got

" got fwampt frequently; when EMERSON,
" fmiling and alluding to his book, faid, They
" muft not do as I do, but as I fay."

" In the earlier part of his life he attempted
" to teach a few fcholars; but whether from
" his concife method, for he was not happy in
" explaining his ideas, or the natural warmth of
" his temper, he made little progrefs in his fchool,
" which he therefore foon dropped. He never
" had a fcholar that did him any credit except
" Mr. Richardfon of Darlington, who was always
" a great favourite with him, and of whom he
" ufed to fay, that he was the only boy who had
" a head in his fchool."

" Being requefted once, by letter, to commu-
" nicate fome particulars of his life, by a friend
" who wanted to put them together, he wrote for
" anfwer, I never knew you were commenced
" biographer before; they will have little to do,
" I think, that fet about writing my life: I am
" fure of this, half of it will be lies: therefore
" I chufe to die in the fame obfcurity I have
" lived."

During the greateft part of his life he had
enjoyed ftrong and uninterrupted health; but as
he advanced in years he was afflicted with the
ftone and gravel to an excruciating degree. In
the agony of thefe fits he would crawl round the
floor, on his hands and knees, fometimes praying
and fometimes fwearing, and devoutly wifhing
that the mechanifm of the human frame had been
fo contrived as to go to wreck without all that
clitter-my-clatter, as he called it; as he grew
weaker the violence of the diforder abated, and
he died at laft, apparently without much pain, on
the 21ft of May 1782, and in the 81ft year of
his age.

Mr.

" Mr. EMERSON, with much perfuafion, about
" a year before his deceafe, was prevailed on by
" his friend Dr. Cloudefley, of Darlington, to
" fit for his picture, which was taken by Sikes,
" and is now in the Doctor's poffeffion.   Towards
" the clofe of the year 1781, being fenfible of
" his approaching diffolution, he difpofed of the
" whole of his mathematical library to a book-
" feller at York: his inftruments he had for
" many years been in the habit of difpofing of,
" at times, to different perfons, for a mere trifle.
" When afked, why he chofe to fell his books,
" he faid, he had none but a pack of fools to
" leave them to, and money would be of more
" fervice to them than books.   He valued his
" library when it was fent to York at 40 or 50l."

EMERSON, like other great men, had his foibles
and defects.—He was fingular and uncouth in his
drefs and manners, and hafty and impetuous in
his temper; but whatever failings he had, they
were overbalanced by his virtues.   He had a
great, firm, and independent mind, that could not
be brought to fubmit to any thing mean, bafe or
difingenuous, by any power on earth: a pure,
genuine, ardent love of truth, and deteftation of
falfhood of whatever fpecies.   His honefty and
intregrity were fuch, that all who knew any thing
of him, repofed in him the moft implicit confi-
dence; and no man could ever juftly complain
that EMERSON had deceived him.   He had great
pleafure in doing a good and friendly fervice to
any deferving perfon, whenever he had it in his
power; and under a rough and forbidding exterior
he concealed a humane heart, that wifhed to pro-
mote the welfare and happinefs of his fellow
creatures.

He

He lies buried in the church-yard at Hurworth, at the weſt end of the church; againſt which is erected to his memory a ſtone with the following inſcription. His wife ſurvived him near two years; they had no children.

Quod ſub Pedibus ſepultum
Et neglectum jacet
Aliquando fuit
GULIELMUS EMERSON;
Vir
Priſcæ Simplicitatis,
Summæ integritatis,
Rariſſimi ingenii.
Quantus fuerit Mathematicus
Si ſcripta ejus perlegeris
Quorſum narraret ſaxum?
Si non perlegeris,
Perlege, et Scies.
Obiit 21? Mäij, 1782.
Ætat. An? 81.

Juxta ſepulta jacet
ELIZABETHA UXOR.
Quæ obiit 27? Martii, 1784,
Ætat. An. 76.

MECHANICS;

OR, THE

DOCTRINE

OF

MOTION.

COMPREHENDING,

I. The general LAWS of MOTION.

II. The DESCENT of BODIES perpendicularly, and down inclined PLANES, and alfo in CURVE SURFACES. The MOTION of PENDULUMS.

III. CENTERS of GRAVITY. The EQUILIBRIUM of BEAMS of TIMBER, and their FORCES and DIRECTIONS.

IV. The MECHANICAL POWERS.

V. The COMPARATIVE STRENGTH of TIMBER, and its STRESS. The POWERS of ENGINES, their MOTION and FRICTION.

VI. HYDROSTATICS and PNEUMATICS.

Da veniam fcriptis, quorum non gloria nobis caufa, fed utilitas officiumque, fuit.

OVID DE PONTO III.

PREFACE.

HAVING some years since written a large book of Mechanics, I have here given a short abstract of that work, as it falls properly into this course; and especially as that branch of science, is of such extensive use in the affairs of human life. I do not in the least, design this to interfere with the other book, being rather an introduction to it, as it explains several things in it more at large; particularly in the first section, as being of universal extent and use; and likewise in several other parts of the book, especially such as have been objected to by ignorant writers. I have also added several things not mentioned in the other book, which are more simple and easy, and more proper for learners. So that this short treatise may be looked upon as an introduction to the other book, and will doubtless facilitate the reading of it. As to the higher and more difficult matters, as few care to trouble their heads about them, I have said little of them here, being not so proper for an introduction. To mention one or two things; I had taken a great deal of pains to find out the true form of a bridge, that shall be the strongest, and of a ship that shall sail the fastest; both upon principles that I know to be as certain and demonstrative as the Elements of Euclid; both these you have in the other book. But, as we have no occasion in England, for the strongest bridges, or the swiftest ships; Mathematicians, for the future, may find something else to do, than run into such perplext and useless disquisitions. For indeed when any

of

PREFACE.

of these grand things are to be performed, they generally fall into the hands of such people, as know little of the nature of them. They perhaps know how to lay down upon paper, the plan of a design, by rule and compass, and to do several problems in practical geometry; and not much more. As if this was an adequate qualification, for conducting such magnificent works, as cost many thousand pounds to execute. So that these things, instead of being constructed by the rules of art; they are too often done by fancy, without any true rules.

But in this there is no great wonder, considering how few people study this art; and among those that do, how few are competent judges. For even among those that presume to write about it, it is surprizing to see what mistakes they daily run into. One denies the composition aud resolution of forces; another cannot be satisfied with some of the mechanical powers, not even so simple a thing as the wedge; all owing to the wrong notions they have imbibed. And in consequence of this, will be either condemning the art itself, or criticising on other writers, whom they do not understand.

As to what I have written on this subject; I have all along given the demonstrations of the several things I have handled; and I expect that to be my test, as to the truth or falshood thereof. And by this test I leave them to be tried by any judicious, honest reader; who is a lover of truth, and a promoter of science.

W. Emerson.

MECHANICS.

DEFINITIONS.

1. *MECHANICS* is a fcience, which treats of the forces, motions, velocities, and in general, of the actions of bodies upon one another. It teaches how to move any given weight with any given power; how to contrive engines to raife great weights, or to perform any kind of motion.

2. *Body* is the mafs or quantity of matter; an *elaftic body* is that which yields to a ftroke, and recovers its figure again. But if not, it is called an *unelaftic body*.

3. *Denfity* is the proportion of the quantity of matter in any body, to the quantity of matter in another body of the fame bignefs.

4. *Force* is a power exerted on a body to move it. If it act inftantaneoufly, it is called *percuffion*, or *impulfe*. If conftantly, it is an *accelerative force*.

5. *Velocity* is a property of motion, by which a body paffes over a certain fpace in a certain time. And is greater or leffer, as it paffes over a greater or leffer fpace in a certain time, as fuppofe a fecond.

6. *Motion* is a continual and fucceffive change of place. If the body moves equally, it is called *equable* or *uniform motion*. If it increafes, or decreafes it is called *accelerated* or *retarded motion*. When it is compared with fome body at reft, it is called

abfolute

abfolute motion. But when compared with others in motion, it is called *relative motion.*

7. *Direction* of motion is the courfe or way the body tends, or the line it moves in.

8. *Quantity* of motion, is the motion a body has, confidered both in regard to its velocity and quantity of matter. This is alfo called the *Momentum* of a body.

9. *Vis inertiæ,* is the innate force of matter, by which it refifts any change, ftriving to preferve its prefent ftate of reft or motion.

10. *Gravity* is that force wherewith a body endeavours to fall downwards. It is called *abfolute gravity* in empty fpace; and *relative gravity* when immerfed in a fluid.

11. *Specific gravity,* is the greater or leffer weight of bodies of the fame magnitude, or the proportion between their weights. This proceeds from the natural denfity of bodies.

12. *Center of gravity,* is a certain point of a body; upon which, the body, when fufpended, will reft in any pofition.

13. *Center of motion,* is a fixed point about which a body moves. And the *axis of motion* is a fixed line it moves about.

14. *Power* and *weight,* when oppofed to one another, fignify the body that moves another, and the other which is moved. The body which begins and communicates motion is the *power*; and that which receives the motion, is the *weight.*

15. *Equilibrium* is the balance of two or more forces, fo as to remain at reft.

16. *Machine* or *engine,* is any inftrument to move bodies, made of levers, wheels, pullies, &c.

17. *Mechanic powers,* are the balance, lever, wheel, pulley, fcrew, and wedge.

18. *Strefs* is the effect any force has to break a beam, or any other body; and *ftrength* is the refiftance

refiftance it is able to make againft any ftraining force.

19. *Friction* is the refiftance which a machine fuffers, by the parts rubbing againft one another.

POSTULATA.

1. That a fmall part of the furface of the earth may be looked upon as a plane. For though the earth be round, yet fuch a fmall part of it as we have any occafion to confider, does not fenfibly differ from a plane.

2. That heavy bodies defcend in lines parallel to one another. For though they all tend to a point which is the center of the earth, yet that center is at fuch a diftance that thefe lines differ infenfibly from parallel lines.

3. The fame body is of the fame weight in all places on or near the earth's furface. For the difference is not fenfible in the feveral places we can go to.

4. Though all matter is rough, and all engines imperfect; yet, for the eafe of calculation, we muft fuppofe all planes perfectly even; all bodies perfectly fmooth; and all bodies and machines to move without friction or refiftance; all lines ftraight and inflexible, without weight or thicknefs; cords extremely pliable, and fo on.

AXIOMS.

1. Every body endeavours to remain in its prefent ftate, whether it be at reft, or moving uniformly in a right line.

2. The alteration of motion by any external force is always proportional to that force, and in direction of the right line in which the force acts.

3. Action and re-action, between any two bodies, are equal and contrary.

4. The

4. The motion of any body is made up of the sum of the motions of all the parts.

5. The weights of all bodies in the same place, are proportional to the quantities of matter they contain, without any regard to their figure.

6. The *vis inertiæ* of any body, is proportional to the quantity of matter.

7. Every body will descend to the lowest place it can get to.

8. Whatever sustains a heavy body, bears all the weight of it.

9. Two equal forces acting against one another, in contrary directions, destroy one anothers effects. And unequal forces act only with the difference of them.

10. When a body is kept in equilibrio; the contrary forces in any line of direction are equal.

11. If a certain force generate any motion; an equal force acting in a contrary direction, will destroy as much motion in the same time.

12. If a body be acted on by any power in a given direction; it is all one in what point of that line of direction, the power is applied.

13. If a body is drawn by a rope, all the parts of the rope are equally stretched. And the force in any part acts in direction of that part. And it is the same thing whether the rope is drawn out at length, or goes over several pullies.

14. If several forces at one end of a lever, act against several forces at the other end; the lever acts and is acted on in direction of its length.

SECT. I.

The General Laws of MOTION.

PROP. I.

THE quantities of matter in all bodies, are in the compound ratio of their magnitudes and denfities.

For (Def. 3.) in bodies of the fame magnitude, the quantities of matter will be as the denfities. Increafe the magnitude in any ratio, and the quantity of matter is increafed in the fame ratio. Confequently the quantity of matter is in the compound ratio of the denfity and magnitude.

Cor. 1. *In two fimilar bodies, the quantities of matter are as the denfities, aud cubes of the diameters.*

For the magnitudes of bodies are as the cubes of the diameters.

Cor. 2. *The quantities of matter are as the magnitudes and fpecific gravities.*

For (by Def 3. and 11.) the denfities of bodies are as their fpecific gravities.

PROP. II.

The quantities of motion in all moving bodies, are in the compound ratio of the quantities of matter and the velocities.

For if the velocities be equal, the quantities of motion will manifeftly be as the quantities of matter. Increafe the velocity in any ratio, and the quantity of motion will be increafed in the fame ratio.

ratio. Therefore it follows univerſally, that the quantities of motion are in the compound ratio of the velocities and quantities of matter.

Cor. *Hence if the body be the ſame, the motion is as the velocity. And if the velocity be the ſame, the motion is as the body or quantity of matter.*

PROP. III.

In all bodies moving uniformly, the ſpaces deſcribed, are in the compound ratio of the velocities and the times of their deſcription.

For in any moving body, the greater the velocity, the greater is the ſpace deſcribed ; that is, the ſpace will be as the velocity. And in twice or thrice the time, &c. the ſpace will be twice or thrice as great ; that is, the ſpace will be increaſed in proportion to the time. Therefore univerſally the ſpace is in the compound ratio of the velocity, and the time of deſcription.

Cor. 1. *The time of deſcribing any ſpace, is as the ſpace directly and velocity reciprocally; or as the ſpace divided by the velocity. And if the velocity be the ſame, the time is as the ſpace. And if the ſpace be the ſame, the time is reciprocally as the velocity.*

Cor. 2. *The velocity of a moving body, is as the ſpace directly, and time reciprocally; or as the ſpace divided by the time. And if the time be the ſame, the velocity is as the ſpace deſcribed. And if the ſpace be the ſame, the velocity is reciprocally as the time of deſcription.*

PROP. IV.

The motion generated by any momentary force, or by a single impulse, is as the force that generates it.

For if any force generates any quantity of motion; double the force will produce double the motion; and treble the force, treble the motion, and so on. If a body striking another, gives it any motion, twice that body striking the same, with the same velocity, will give it twice the motion, and so the motion generated in the other will be as the force of percuffion.

Cor 1. *Hence the forces are in the compound ratio of the velocities and quantities of matter.*

For (Prop. II.) the motions are as the quantities of matter multiplied by the velocities.

Cor. 2. *The velocity generated, is as the force directly and quantity of matter reciprocally. Therefore if the bodies are equal, the velocities are as the forces. And if the forces are equal, the velocities are reciprocally as the bodies.*

Cor. 3. *The quantity of matter, is as the force directly, and velocity reciprocally. And therefore if the velocities be equal, the bodies are as the forces. And if the forces be equal, the bodies are reciprocally as the velocities.*

PROP. V.

The quantity of motion generated, by a constant and uniform force, is in the compound ratio of the force and time of acting.

For the motion generated in any given time will be proportional to the force that generates it ; and

and in twice that time, the motion will be double by the same force; and in thrice that time it will be treble; and so every part of time adds a new quantity of motion equal to the first; and therefore the whole motion, will be as the force, and the whole time of acting.

Cor. 1. *The motion lost in any time, is in the compound ratio of the force and time.*

Cor. 2. *The velocity generated (or destroyed) in any time, is as the force and time directly, and quantity of matter reciprocally. The same is true of the increase or decrease of velocity.*

For the motion, that is (Prop. II.) the body multiplied by the velocity, is as the force and time. And therefore the velocity is as the force and time directly, and the body reciprocally.

Cor. 3. *Hence if the force be as the quantity of matter, the velocity is as the time. Or if the force and quantity of matter be given, the velocity is as the time.*

And if the time and quantity of matter be given, the velocity is as the force.

And if the force and time be given, the velocity is reciprocally as the matter.

Cor. 4. *The time is as the quantity of matter and velocity directly, and the force reciprocally. Therefore,*

If the force and velocity be given, the time is as the quantity of matter.

If the quantity of matter and velocity be given, the time is reciprocally as the force.

Cor. 5. *The force is as the quantity of matter and velocity directly, and the time reciprocally. Whence,*

If the velocity is as the time, or if the velocity and time be given, then the force is as the quantity of matter.

And

And if the velocity and quantity of matter be given, the force is reciprocally as the time.

Cor. 6. *The quantity of matter is as the force and time directly, and the velocity reciprocally. Therefore, if the force and time be given, the quantity of matter is reciprocally as the velocity.*

Cor. 7. *Hence also if the body be given, the velocity is in the compound ratio of the force and time.*

And if the force be given, the time is in the compound ratio of the matter and velocity, or as the quantity of motion.

PROP. VI.

If a given body is urged by a constant and uniform force; the space described by the body from the beginning of the motion, will be as the force and square of the time.

Suppose the time divided into an infinite number of equal parts or moments. Then in each of these moments of time, the space described (Prop. III. Cor. 2.) will be as the velocity gained; that is, (by Cor. 7. Prop. V.) as the force and time from the beginning. And the sum of all the spaces, or the whole space described, will be as the force and the sum of all the moments of time from the beginning. Therefore put $t =$ the whole time, and the whole space described, will be as the force and sum of the times 1, 2, 3, 4, &c. to t. But the sum of the arithmetic progression $1 + 2 + 3 + 4$ ... to $t = \dfrac{t+1}{2} \times t = \frac{1}{2}tt$, because t is infinite or consists of an infinite number of moments. Therefore the whole space described will be as the force and $\frac{1}{2}tt$; that is, (because $\frac{1}{2}$ is a given quantity), as the force and the square of the time of description.

Cor.

Cor. 1. *If a body is impelled by a conſtant and uniform force; the ſpace deſcribed from the beginning of the motion, is as the velocity gained, and the time of moving.*

For the ſpace is as the force and ſquare of the time, or as the force × time × time. But (Cor. 7. Prop. V.) the force × time is as the velocity; therefore the ſpace, which is as the force × time × time, is as the velocity × time.

Cor. 2. *If a body urged by any conſtant and uniform force, deſcribes any ſpace; it will deſcribe twice that ſpace in the ſame time, by the velocity acquired.*

For the ſum of all the ſpaces deſcribed by that force, $1+2+3$ &c. to t, was ſhewn to be $\frac{1}{2}tt$. But the ſum of all the ſpaces deſcribed by the laſt velocity, will be $t+t+t$ &c. to t terms, whoſe ſum is tt. But tt is double to $\frac{1}{2}tt$; that is, the ſpace deſcribed by the laſt velocity, is double the ſpace deſcribed by the accelerating force.

Cor. 3. *Univerſally, in all bodies urged by any conſtant and uniform forces, the ſpace deſcribed is as the force and ſquare of the time directly, and the quantity of matter reciprocally.*

For (Cor. 1.) the ſpace is as the time and velocity. But (Prop. V. Cor. 2.) the velocity is univerſally as the force and time directly, and quantity of matter reciprocally. Therefore the ſpace is as the ſquare of the time and the force directly, and matter reciprocally; whence,

Cor. 4. *The product of the force and ſquare of the time, is as the product of the body and ſpace deſcribed.*

Cor. 5. *The product of the force and time, is as the product of the quantity of matter and velocity.*

For (Prop. V.) the product of the force and.
time,

time, is as the motion; that is, as the body and Fig.
velocity.

Cor. 6. *The product of the body, and square of
the velocity, is as the product of the force and the
space described.*

For (Cor. 5.) the product of the body and ve-
locity, is as the force and time. Therefore, the
body × velocity square, is as the force × time ×
velocity; but time × velocity is as the space (by
Prop. III.); therefore body × velocity square is as
the force × space.

SCHOLIUM.

If any quantity or quantities are given, they must
be left out. And such quantities as are propor-
tional to each other must be left out. For exam-
ple, if the quantity of matter be always the same;
then (Cor. 3.) the space described is as the force
and square of the time. And if the matter be
proportional to the force, as all bodies are in res-
pect to their gravity; then (Cor. 6.) the space des-
cribed is as the square of the velocity. Or if the
space described be always proportional to the body;
then (Cor. 6.) the force is as the square of the
velocity. Again, if the body be given, then (Cor.
4.) the space is as the force and square of the time.
And if both the quantity of matter and the force
be given, the space described is as the square of
the time. And so of others.

PROP. VII.

If ABCD *be a parallelogram; and if a body at* 1.
A, *be acted upon seperately by two forces, in the di-
rections* AB *and* AC, *which would cause the body to be
carried through the spaces* AB, AC, *in the same time.
Then both forces acting at once, will cause the body to be
carried through the diagonal* AD *of the parallellogram.*

Let the line AC be supposed to move parallel to
B   itself;

Fig.
1.

itfelf; whilft the body at the fame time moves from A, along the line AC or *bg*, and comes to *d* at the fame inftant, that AC comes to *bg*. Then fince the lines AB, AC, are defcribed in the fame time; and A*b*, A*d*, are alfo defcribed in the fame time. Therefore, as the motions are uniform, it will be, A*b* : *bd* : : AB : BD; and therefore A*d*D is a ftraight line, coinciding with the diagonal of the parallelogram.

Cor. 1. *The three forces in the directions* AB, AC, AD, are refpectively as the lines AB, AC, AD.

Cor. 2. *Any fingle force* AD *denoted by the diagonal of a parallelogram, is equivalent to two forces denoted by the fides* AB, AC.

Cor. 3. *And therefore any fingle force* AD *may be refolved into two forces, an infinite number of ways, by drawing any two lines* AB, BD, *for their quantities and directions.*

Scholium.

This practice of finding two forces equivalent to one, or dividing one force into two; is called the *compofition* and *refolution* of forces.

PROP. VIII.

2.

If three forces, A, B, C, *keep one another in equilibrio; they will be proportional to three fides of a triangle, drawn parallel to their feveral directions,* DI, CI, CD.

Produce AD to I, and BD to H, and compleat the parallelogram DICH; then (Prop. VII.) the force in direction DC, is equal to the forces DH, DI, in the directions DH, DI. Take away the force DC and putting the forces DH, DI equal thereto;

thereto; and the equilibrium will still remain. Fig.
Therefore (Ax. 10.) DI is equal to the force A 2.
oppofite to it; and DH or CI equal to its oppofite
force B. And as CD reprefents the force C, the
three forces A, B, and C, will be to one another as
DI, CI, and CD.

Cor. 1. *Hence if three forces acting against one
another, keep each other in equilibrio; thefe forces
will be refpectively as the three fides of a triangle
drawn perpendicular to their lines of direction; or
making any given angle with them, on the fame fide.*
For this triangle will be fimilar to a triangle
whofe fides are parallel to the lines of direction.

Cor. 2. *If three active forces A, B, C, keep one
another in equilibrio; they will be refpectively as the
fines of the angles, which their lines of direction pafs
through.*
For A, B, C are as DI, CI, CD; that is, as
S.DCI, S.CDI, and S.DIC. But S.DCI=S.CDH
=S.CDB. And S.CDI=S.CDA. Alfo S.DIC
=S.BDI=SBDA.

Cor. 3. *If ever fo many forces acting against one
another, are kept in equilibrio, by thefe actions; they
may be all reduced to two equal and oppofite ones.*
For any two forces may, by compofition, be re-
duced to one force acting in the fame plane. And
this laft force, and any other may likewife be re-
duced to one force acting in the plane of thefe;
and fo on, till they all be reduced at laft to the
action of two equal and oppofite ones.

PROP. IX.

*If a body impinges or acts against any plain furface; 3.
it exerts its force in a line perpendicular to that furface.*

Let the body A moving in direction AB, with a
given velocity, impinge on the fmooth plain FG at

Fig. the point B. Draw AC parallel, and BC perpen-
3. dicular to FG; and let AB reprefent the force of
the moving body. The force AB is, by the refo-
lution of forces, equivalent to AC and CB. The
force AC is parallel to the plain, and therefore has
no effect upon it; and therefore the furface 'FG
is only acted upon by the force CB, in a direction
perpendicular to the furface FG.

Cor. 1. *If a body impinges upon another body with
a given velocity; the quantity of the ftroke is as the
fine of the angle of incidence.*
For the abfolute force is AB, and the force
acting on the furface FG is CB. But AB : CB : :
rad. : S.CAB or ABF.

Cor 2. *If an elaftic body A impinges upon a hard
or elaftic plane FG; the angle of reflection will be
equal to the angle of incidence.*
For if AD be parallel to FG; the motion of A
in direction AD parallel to the plane, is not at all
changed by the ftroke. And by the elafticity of
one or both bodies, the body A is reflected back
to AD in the fame time it moved from A to B;
let it pafs to D; then will AC=CD, being def-
cribed in equal times; confequently the angle
ABC = angle CBD; and therefore the angle
DBG = angle ABF.

Cor. 3. *If a non-elaftic body ftrikes another non-
elaftic body, it lofes but half the motion, that it would
lofe, if the bodies were elaftic.*
For non-elaftic bodies only ftop, without re-
cedeing from one another; but elaftic bodies
recede with the fame velocity.

PROP.

PROP. X.

*The sum of the motions of two or more bodies,
in any direction towards the same part, cannot be
changed by any action of the bodies upon each other.*

Here I reckon progreffive motions affirmative;
and regreffive ones negative, and to be deducted
out of the reft to get the fum.

1. 'If two bodies move the fame way; fince ac-
tion and re-action are equal and contrary, what one
body gains the other lofes; and the fum remains
the fame as before. And the cafe is the fame, if
there were more bodies.

2. If bodies ftrike one another obliquely; they
will act on one another in a line perpendicular to
the furface acted on. And therefore by the law of
action and re-action there is no change made in
that direction.

3. And in a direction parallel to the ftriking
furface, there is no action of the bodies, therefore
the motion remains the fame in that direction.
Whence the motions will remain the fame in any
one line of direction.

Cor. 1. *Motion can neither be increafed nor de-
creafed, confidered in any one direction; but muft
remain invariably the fame for ever.*

This follows plainly from this Prop. for what
motion is gained to one (by addition) is loft to
another body (by fubtraction); and fo the total
fum remains the fame as before.

Scholium.

This Prop. does not include or meddle with fuch
motions as are eftimated in all directions. For
upon this fuppofition, motion may be increafed or

.Fig. decreafed an infinite number of ways. For example, if two equal and non-elaftic bodies, with equal velocities, meet one another; both their motions are deſtroyed by the ſtroke. Here at the beginning of the motion, they had both of them a certain quantity of motion, but to be taken in contrary directions; but after the ſtroke they had none.

· PROP. XI.

The motion of bodies included in a given ſpace, is the ſame, whether that ſpace ſtands ſtill; or moves uniformly in a right line.

For if a body be moving in a right line, and any force be equally impreſſed both upon the body and the line it moves in; this will make no alteration in the motion of the body along the right line. And for the ſame reaſon, the motions of any number of bodies in their ſeveral directions will ſtill remain the ſame; and their motions among themſelves will continue the ſame, whether the including ſpace is at reſt, or moves uniformly forward. And ſince the motions of the bodies among themſelves; that is, their relative motions remain the ſame, whether the ſpace including them be at reſt, or has any uniform motion. Therefore their mutual actions upon one another, muſt alſo remain the ſame in both caſes.

SECT. II.

*The perpendicular defcent of heavy bo-
dies, their defcent upon inclined Planes,
and in Curve Surfaces. The Motion
of Pendulums.*

PROP. XII.

*THE velocities of bodies, falling freely by their own
weight, are as the times of their falling from reft.*

For fince the force of gravity is the fame in all
places near the earth's furface (by Poft. 3.) and
this is the force by which bodies defcend. There-
fore the falling body is urged by a force which acts
conftantly and equally; and therefore (by Cor. 3,
Prop. V.) the velocity generated in the falling
body in any time, is as the time of falling.

Cor. 1. *If a body be thrown directly upwards with
the fame velocity it falls with; it will lofe all its
motion in the fame time.*

For the fame active force will deftroy as much
motion in any time, as it can generate in the fame
time.

Cor. 2. *Bodies defcending or afcending gain or lofe
equal velocities in equal times.*

PROP. XIII.

*In bodies falling freely by their own weight; the
fpaces defcribed in falling from reft, are as the fquares
of the times of falling.*

For fince gravity is fuppofed to be the fame in
all places near the earth. Therefore the falling

Fig. body will be acted on by a force which is conſtantly the ſame; and therefore (by Prop. VI.) the ſpaces deſcribed, from the beginning of the motion, or ſince their falling from reſt, will be as the ſquares of the times of falling.

Cor. 1. *The ſpaces deſcribed by falling bodies, are as the ſquares of the velocities.*

For (Prop. XII.) the velocities are as the times of falling.

Cor. 2. *The ſpaces deſcribed by falling bodies, are in the compound ratio of the times, and the velocities acquired by falling.*

Cor. 3. *If a body falls through any ſpace, and move afterwards with the velocity gained in falling; it will deſcribe twice that ſpace in the time of its falling.*

Cor. 4. *A body projected upward with the velocity it gained in falling, will aſcend to the ſame height it fell from.*

Scholium.

All theſe things would be true if it was not for the reſiſtance of the air, which will retard their motion a little. In very ſwift motions, the reſiſtance of the air has a very great effect in deſtroying the motions of bodies.

PROP. XIV.

4. *If a heavy body* W *be ſuſtained upon an inclined plane* AC, *by a power* F *acting in direction* WF *parallel to the plane; and if* AB *be parallel to the horizon and* BC *perp. to it. Then if the length* AC *denotes the weight of the body, the height* CB *will denote the power at* F *which ſuſtains it, and the baſe* AB *the preſſure againſt the plane.*

Draw BD perpendicular to AC, then CB will be the direction of gravity, DC parallel to WF will
<div align="right">be</div>

be the direction of the force at F, and DB the di-
rection of the preſſure (by Prop. IX.) Therefore
(by Prop. VIII.) the weight of the body, the
power at F, and the preſſure; will be reſpectively
as BC, CD and DB. But the triangles ABC, BDC
are ſimilar, and therefore BC, CD and DB are
reſpectively as AC, CB and BA. Therefore the
weight, power and preſſure, are as the lines AC,
CB and AB.

Cor. 1. *The weight of the body, the power F that
ſuſtains it on the plane, and the preſſure againſt the
plane; are reſpectively as radius, the ſine and coſine
of the plane's elevation above the horizon.*
For AC, CB and AB are to one another, as
radius, ſine of CAB, and ſine of ACB.

Cor. 2. *The power that urges a body* W *down the
inclined plane is* $=\dfrac{CB}{AC} \times$ *weight of* W. *Hence,*

Cor. 3. *If a priſmatic body whoſe length is* AC
lie upon the inclined plane AC; *it is urged down the
plane with a force equal to the weight of the priſm
of the length* CB.

Cor. 4. *If there be two planes of the ſame height,
and two bodies be laid on them proportional to the
lengths of theſe planes; the tendency down the planes
will be equal in both bodies.*

PROP. XV.

If AC *be an inclined plane,* AB *the horizon,* BC
perp. to AB. *And if* W *be a heavy body upon the
plane, which is kept there by the power* P *acting in
direction* WP. *Draw* BDE *perp. to* WP. *Then
the weight* W, *the power* P, *the preſſure againſt the
plane; will be reſpectively as* AB, DB *and* AD.

For AB being a horizontal plane is perpendi-
cular to the action of gravity; and BD is perp. to
the

Fig.
5.
the direction of the power P; and AD is the plane, which is perp. to the direction of the pref-fure againft that plane. Therefore (Cor. 1. Prop. VIII.) the weight of the body, the power P, and the preffure; are as AB, BD and AD. And if the direction of the power WP be under the plane, the proportion will be the fame, as long as BD is perpendicular to WP.

Cor. 1. *The weight of the body* W, *is to the power* P *that fuftains it* : : *as cofine of the angle of traction* CWP : *to the fine of the plane's elevation* CAB.

For the weight : power : : AB : BD : : S.ADB or WDE : S.BAD : : cof. DWE or CWP : S.BAC, where the angle CWP made by the plane and di-rection of the power is called the angle of traction.

Cor. 2. *Hence it is the fame thing as to the power and weight, whether the line of direction is above or below the plane, provided the angle of traction be the fame. For an equal power will fuftain the weight in both cafes.*

' Cor. 3. *The weight of the body : is to the preffure againft the plane* : : *as the cofine of the angle of trac-tion* CWP : *to the cofine of* BNP, *the direction of the power above the horizon.*

For the weight W : preffure : : AB : AD : : S.ADB : S.ABD : : S.ADE : : SNBE : : cof. EWD or PWC : cof. BNP.

Cor. 4. *Hence the preffure againft the plane is greater when the direction of the power is below the plane, the weight remaining the fame.*

SCHOLIUM.

Although the power has the fame proportion to the weight, when the angle of traction is the fame; whether the direction of the power be above or
below

below the plane. Yet, since the pressure upon the Fig.
plane is greater, when the line of direction is 5.
below the plane. Therefore in practice, when a
weight is to be drawn up hill, if it is to be done
by a power whose direction is below the plane,
the greater pressure in this case will make the car-
riage sink deeper into the earth, &c. and for that
reason will require a greater power to draw it up,
than when the line of direction is above the plane..

PROP. XVI.

If a weight W *upon an inclined plane* AC, *be in* 6.
equilibrio with another weight P *hanging freely; then*
if they be set a moving, their perpendicular velocities
in that place, will be reciprocally as their quantities of
matter.

Take WA a very small line upon the plane AC;
draw AB parallel to the horizon, and BC perp. to
it. Draw AF, and WR, BE perpendicular to it;
and WT, DV perp. to AB. Let W descend through
the small line WA upon the plane, then P will
ascend a height equal to AR perpendicularly; and
WT will be the perpendicular descent of W. The
triangles AWR and ADE are similar; and like-
wise the triangles AWT and ADV. Therefore
WT : DV :: AW :: AD :: WR : DE. And
alternately, WT : WR :: DV : DE; and WR :
AR :: DE : AE; therefore WT : AR :: DV :
AE :: (by the similar triangles DBV and AEB)
DB : AB :: (Prop. XV.) power P : weight W.

Cor. 1. *If any two bodies be in equilibrio upon two*
inclined planes; their perpendicular velocities will be
reciprocally as the bodies.

Cor. 2. *If two bodies sustain each other in equili-*
brio, on any planes; the product of one body × by its
                                                    perp.

Fig. *perp. velocity, is equal to the product of the other*
6. *body × by its perp. velocity.*

PROP. XVII.

7. *If a heavy body runs down an inclined plane* CA ;
*the velocity it acquires in any time, moving from rest ;
is to the velocity acquired by a body falling perpendi-
cularly in the same time ; as the height of the plane* CB,
to its length CA.

The force by which a body endeavours to descend
on an inclined plane, is to its weight or the force
of gravity; as CB to CA (by Prop. XIV.)   And
as these forces always remain the same, therefore
(Cor. 2. Prop. V.) the velocities generated will be
as these forces, and the times of acting, directly ;
and the bodies reciprocally.   And since the times
of acting, and the bodies are the same in both
cases, the velocities generated will be as these
forces; that is, as the height of the plane CB to
its length CA.

Cor. 1. *The velocity acquired by a body running
down an inclined plane, is as the time of its moving
from rest.*

Cor. 2. *If a body is thrown up an inclined plane,
with the velocity it acquired in descending ; it will
lose all its motion in the same time.*

PROP. XVIII.

7. *If a heavy body descends down an inclined plane*
CA : *the space it describes from the beginning of the
motion, is to the space described by a body falling per-
pendicularly in the same time ; as the height of the
plane* CB, *to its length* CA.

For the force urging the body down the plane is
to the force of its gravity, as CB to CA (by Prop.
XIV.)

XIV.) which forces remain conſtantly the ſame. Fig.
And ſince (Prop. XVII.) the velocities generated 7.
in equal times on the plane, and in the perpendi-
cular, are conſtantly as CB to CA; the ſmall par-
ticles of ſpace deſcribed with theſe velocities in
all the infinitely ſmall portions of time, will ſtill
be in the ſame ratio; and therefore the ſums of
all theſe ſmall ſpaces, or the whole ſpaces deſcribed
from the beginning, will be in the ſame conſtant
ratio of CB to CA.

Cor. 1. *The ſpace deſcribed by a body falling down
an inclined plane, in a given time, is as the ſine of
the plane's elevation.*
For if CB be given, and alſo the perp. deſcent;
that ſpace will be reciprocally as CA, or directly
as S.CAB.

Cor. 2. *The ſpaces deſcribed by a body deſcending
from reſt, down an inclined plane, are as the ſquares
of the times.*

Cor. 3. *If BD be drawn perp. to the plane CA;
then in the time a body falls perpendicularly through
the height CB, another body will deſcend through the
ſpace CD upon the plane.*
For by ſimilar triangles CA : CB : : CB : CD.

PROP. XIX.

If AC is an inclined plane, the time of a body's 7.
*deſcending through the plane CA, is to the time of fall-
ing perpendicularly through the height of the plane CB,
as the length of the plane CA to the height CB.*

For if BD is perp. to CA, then (Cor. 2. Prop.
XVIII.) ſpace CD : ſpace CA : : ſquare of the
time in CD : ſquare of the time in CA : : that is,
as the ſquare of the time of deſcending perpendi-
cularly

Fig.
7. cularly in CB (Cor. 3. laft) : fquare of the time in CA.   But CD : CA : : CB² : CA². Therefore CB² : CA² : fquare of the time in CB : fquare of the time in CA.   And CB : CA : : time in CB : time in CA.

Cor. 1. *If a body be thrown upwards on the plane with the velocity acquired in defcending; it will in an equal time afcend to the fame height.*

Cor. 2. *The times wherein different planes, of the fame height, are paffed over; are as the lengths of the planes.*
Let the planes be CA, CF.   Then time in CA : time in CB : : CA : CB.   And the time in CB : time in CF : : CB : CF. Therefore, *ex equo*, time in AC : time in CF : : CA : CF.

PROP. XX.

8.   *If a body falls down an inclined plane, it acquires the fame velocity as a body falling perpendicularly through the height of the plane.*

Let the body run down the plane CA whofe height is CB.   Draw DF parallel to AB, and infinitely near it.   Then the velocities in DA and FB, may be looked upon as uniform.   Now (Prop. XIX.) the times of defcribing CA and CB, will be as CA and CB.   Likewife the times of defcribing CD and CF, will be as CD and CF; that is, as CA and CB.   And by divifion, the difference of the times, or the times of defcribing DA and FB, will alfo be as CA and CB; that is, as DA and FB.   But (from Prop. III.) the velocities are equal when the fpaces are as the times of defcription. Therefore velocity at A is equal to the velocity at B.

Cor.

Cor. 1. *The velocities acquired, by bodies descending* Fig.
on any planes, from the same height to \the same 8.
horizontal line,• are equal.

Cor. 2. *If the velocities be equal at any two equal*
altitudes D, F ; *they will be equal at all other equal*
altitudes A, B.

Cor. 3. *Hence also, if several bodies be moving in*
different directions, through any space contained between
two parallel planes; and be acted on by any force,
which is equal·at equal distances from either plane.
Then if their velocities be equal at entering that
space; they will also be equal at emerging out of it.
For dividing that space into infinitely small
parts by parallel planes. Then the force between
any two planes may be supposed uniform; and
supposing DF, AB to represent two of these planes,
then (by Cor. 2.) the velocities at D and F being
equal; the velocities at A and B will be equal;
that is, the velocities at entering the first part of
space being equal, the velocities at emerging out
of it, or at entering the second space will be equal.
And for the same reason the velocities at entering
the second space being equal, those at emerging
out of it into the third, will be equal. And con-
sequently the velocities at entering into, and
emerging out of the third, fourth, fifth, &c. to
the last will be equal respectively.

PROP. XXI,

If a body falls from the same hight, through any 9.
number· of contiguous planes .AB, BC, CD; it will
at last gain the same velocity as a body falling per-
pendicularly from the same height.

Let FH be a horizontal line, FD perp. to it.
Produce the planes BC, DC to G and H. Then
(Cor.

Fig.
9.

(Cor. 1. Prop. XX.) the velocity at B is the fame whether the body defcend through AB or GB. And therefore the velocity at C will be the fame, whether the body defcends through ABC or through GC, and this is the fame as if it had defcended through HC. And confequently it will have the fame velocity at D, in defcending through the planes ABCD, as in defcending through the plane HD; that is, (Prop. XX.) as it has in defcending through the perpendicular FD.

Cor. 1. *Hence a body defcending along any curve furface, will acquire the fame velocity, as if it fell perpendicularly through the fame height.*

For let the number of planes be increafed, and their length diminifhed *ad infinitum,* and then ABCD will become a curve. And the velocity acquired by defcending through thefe infinite planes; that is, through the curve ABCD, will be the fame as in falling perpendicularly through FD.

Cor. 2. *If a body defcends in a curve, and another defcends perpendicularly from the fame height. Their velocities will be equal at all equal altitudes.*

Cor. 3. *If a body, after its defcent in a curve, fhould be directed upwards with the velocity it had gained; it will afcend to the fame height from which it fell.*

For fince gravity acts with the fame force whether the body afcends or defcends, it will deftroy the velocity in the afcent, in the fame time it did generate it in the defcent.

Cor. 4. *The velocity of a body defcending in any curve, is as the fquare root of the height fallen from.*

For it is the fame as in falling perpendicularly; and in falling perpendicularly, it is as the fquare root of the height.

Cor.

Cor. 5. *If a body, in moving through any space* Fig.
ED, *be acted on uniformly by any force ; its velocity* 9.
at emerging out of it at D, *will be equal to the square
root of the sum of the squares of the velocity at* E
*in entering of it, and of the velocity acquired in falling
from rest through that space* ED.   *And this holds,
whether the body moves perpendicularly or obliquely.*

For let the body enter the space ED at E, with
the velocity acquired in falling through FE. Then
(Prop. XIII. Cor. 1.) the square of the velocity at
E will be as FE ; and the square of the velocity at
D, as FD; and the square of the velocity at D
falling from E, will be as ED.   But FD = FE
+ ED; therefore the square of the velocity at
D (falling through FD) = square of the velocity at
E + square of the velocity at D (falling through
ED.)   And (Cor. 1. of this Prop.) the velocity
will be the same whether the body descends per-
pendicularly or obliquely.

PROP. XXII.

The times of bodies descending through two similar 10.
*parts of similar curves, placed alike, are as the square
roots of their lengths.*

Let ABCD and *abcd* be two similar curves, and
suppose BC and *bc* to be infinitely small, and simi-
lar to the whole; that is, so that BC : *bc* : : AB :
ab.   Draw FA parallel to the horizon, and HB,
hb perp. to it.   Then if two bodies descend from
A and *a* (Cor. 4. Prop. XXI.) the velocities at B
and *b* will be as $\sqrt{HB}$ and $\sqrt{hb}$; that is, as $\sqrt{AB}$
and $\sqrt{ab}$, because AB, *ab* are similar parts.
Therefore (Prop. III. Cor. 1.) the times of des-
cribing BC and *bc*, are as $\dfrac{BC}{\sqrt{AB}}$ and $\dfrac{bc}{\sqrt{ab}}$; that is,

C   as

Fig.
10. as $\dfrac{AB}{\sqrt{AB}}$ and $\dfrac{ab}{\sqrt{ab}}$, or as $\sqrt{AB}$ and $\sqrt{ab}$; that is,
as $\sqrt{AD}$ and $\sqrt{ad}$, becaufe the curves are fimi-
larly divided in B and b.   After the fame manner
the times of defcribing any other two fimilar parts
as BC, bc, will be as $\sqrt{AD}$ and $\sqrt{ad}$.   Therefore
by compofition the times of defcribing all the BC's,
and all the bc's will be as $\sqrt{AD}$ and $\sqrt{ad}$.   That
is, the time of defcribing the curve AD to the
time of defcribing the curve ad, is as $\sqrt{AD}$ to
$\sqrt{ad}$.

Cor. *If two bodies defcend through two fimilar
curves* ABD, *and* abd; *the axes of the curves* FD, Fd
are as the fquares of the times of their defcending.

For $\sqrt{FD} : \sqrt{Fd} :: \sqrt{AD} : \sqrt{ad} ::$ time of
defcending through ABD : time of defcending
through *abd*.   And FD, Fd, are as the fquares of
the times.

PROP.   XXIII.

11.   *A body will defcend through any chord of a circle,
in the fame time that another defcends perpendicularly
through the diameter.*

Draw the diameter AB perpendicular to the
horizon, and the chords CA, CB.   Then fince BC
is perpendicular to AC, therefore (Prop. XVIII.
Cor. 3.) the time of defcending through the chord
AC is equal to the time of falling through AB.
Draw CD parallel to AB, and DB parallel to
CA, then is CD equal to AB.   And by reafon of
the parallels, the angle DBC = angle BCA = a
right angle.   Then fince DB is perp. to CB, there-
fore (Cor. 3. Prop. XVIII.) a body will defcend
through the inclined plane CB, in the fame time
that another falls through CD, or, which is the
fame thing, through its equal AB.

<div align="right">Cor.</div>

Cor. 1. *Hence the times of defcending through all* Fig.
the chords of a circle drawn from A *or* B, *are equal* 11.
among themfelves.

Cor. 2. *The velocity gained by falling through the*
chord CB, *is as its length* CB.

For the velocity gained in falling through CB
is the fame as is gained by falling through EB;
and that velocity is to the velocity gained by
falling through AB, as √BE to √AB (by Cor. 1.
Prop. XIII.); that is, as BC to BA. Therefore
if the given velocity in falling down AB be repre-
fented by AB, the velocity gained in falling down
CB will be reprefented by CB; and fo that in any
other chord, by its length.

PROP. XXIV.

If a pendulum vibrates in the fmall arch of a circle; 12.
the time of one vibration, is to the time of a body's
falling perpendicularly through half the length of
the pendulum; as the circumference of a circle, to the
diameter.

If a pendulum fufpended by a thread, &c. be
made to vibrate in any curve; it is the fame thing
as if it defcended down a fmooth polifhed body
made in the form of that curve. For the motions,
velocities, and times of moving will be the fame
in both.

Let OD or OE be the pendulum vibrating in the
arch ADC, whofe radius is OD. Let OD be perp.
to the horizon, and take the arch E*e* infinitely
fmall, and draw ABC, EFG, *efg*, perp. to OD;
and draw the chord AD. About BD defcribe the
femicircle BGD. Draw *er* and G*s* perp. to EG.

Put *t* = time of defcending through the diameter
2OD, or through the chord AD. Then the velocities

gained

Fig.
12.
gained by falling through 2OD, and by the pendulum's defcending through the arch AE, will be as $\sqrt{2OD}$ and $\sqrt{BF}$. And the fpace defcribed in the time t, after the fall through 2OD, is 4OD. But the times are as the fpaces, divided by the velocities. Therefore, $\dfrac{4OD}{\sqrt{2OD}}$ or $2\sqrt{2OD} : t$ (time of

its defcription) $::\dfrac{Ee}{\sqrt{BF}}$: time of defcribing Ee

$$=\frac{t \times Ee}{2\sqrt{2OD \times BF}}.$$

But by the fimilar triangles OEF, Eer; and KGF, Ggs; we fhall have $\dfrac{EF}{OD} \times Ee = er = Ff$

$= Gs = \dfrac{FG}{KD} \times Gg$. Whence $Ee = \dfrac{OD \times FG}{KD \times EF} \times$

Gg. Therefore the time of defcribing $Ee =$

$$\frac{t \times OD \times FG \times Gg}{2KD \times EF\sqrt{BF \times 2OD}} =$$

$$\frac{t \times OD \times \sqrt{BF \times FD} \times Gg}{2KD\sqrt{BF} \times \sqrt{DO+OF} \times FD \times \sqrt{2OD}} =$$

$$\frac{t \times \sqrt{OD} \times Gg}{2KD \times \sqrt{DO+OF} \times \sqrt{2}} = \frac{t \times \sqrt{2OD}}{4KD \times \sqrt{DO+OF}}$$

$\times\, Gg = \dfrac{t \times \sqrt{2OD}}{2BD \times \sqrt{2OD-DF}} \times Gg$. But DF,

in its mean quantity for all the arches Gg, is nearly equal to DK. Therefore the time of defcribing

$Ee = \dfrac{t \times \sqrt{2OD}}{2BD\sqrt{2OD-DK}} \times Gg$. Whence the time

of defcribing the arch $AED = \dfrac{t \times \sqrt{2OD}}{2BD\sqrt{2OD-DK}}$

$\times$ BGD. And the time of defcribing the whole arch ADC, or the time of one ofcillation is $=$

$t \times$

Fig.
12.

$$\frac{t \times \sqrt{2\,OD}}{2\,BD\sqrt{2\,OD - DK}} \times 2\,BGD.$$ But when the arch ADC is exceeding ſmall, DK vaniſhes, and then the time of oſcillation in a very ſmall arch is $=$

$$\frac{t \times \sqrt{2\,OD}}{2\,BD\sqrt{2\,OD}} \times 2\,BGD = \tfrac{1}{2} t \times \frac{2\,BGD}{BD}.$$ But if t be the time of deſcending through $2\,OD$, $\tfrac{1}{4} t$ is the time of deſcending through $\tfrac{1}{2}\,OD$. And therefore BD the diameter, is to $2\,BGD$ the circumference ; as the time of falling through half the length of the pendulum, to the time of one vibration.

Cor. 1. *In a ſmall arch* AED, *the time of deſcending through the chord* AD, *is to the time of deſcending through the arch* AED; *as the diameter* BD, *to* $\tfrac{1}{4}$ *the circumference.*

For the time of deſcending through the arch $AED = t \times \dfrac{BGD}{2\,BD}$; therefore $BD : \tfrac{1}{4} BGD : : t :$ time in AED.

Cor. 2. *All the vibrations of the ſame pendulum, in arches not very large, are performed nearly in the ſame time.*

Cor 3. *If* KD *be biſected in* L, *and* T *be* $=$ *time of vibration in a very ſmall arch. Then* T $+$

$\dfrac{KL}{DO + OK} \times$ T *will be the time of vibration in any arch* ADC, *nearly.*

For we found the time of vibration in $ADC =$

$$\frac{t \times BGD}{BD} \times \sqrt{\frac{2\,OD}{2\,OD - DK}} = T \times \sqrt{\frac{2\,OD}{OD + OK}};$$

and the three lines DO $+$ OK, DO $+$ OL, and DO $+$ OD are in arithmetical progreſſion ; but ſince KD is very ſmall, they are nearly in geometri-

　　　　cal

Fig.
12.
cal progreſſion; whence $\sqrt{\dfrac{2OD}{DO+OK}}=\dfrac{DO+OL}{DO+OK}$.

Therefore the time of vibration $=T\times\dfrac{DO+OL}{DO+OK}$

$$=T\times\dfrac{DO+OK+KL}{DO+OK}=T+T\times\dfrac{KL}{DO+OK}.$$

Cor. 4. *Hence a falling body will deſcend through a ſpace of 16 feet and 1 inch, in a ſecond of time.*

For by obſervation, a pendulum 39.13 inches long will ſwing ſeconds. And $t\times\dfrac{BGD}{BD}=1$ ſecond,

and $\dfrac{BD}{BGD}=t$, or $\dfrac{2}{3.1416}=$ time of falling through

2×39.13. Whence (Prop. XIII.) $\dfrac{4}{3.1416^2}:2\times$

$39.13::1^2:\dfrac{39.13}{2}\times3.1416^2=193.096$ inches

$=16.09$ feet.

Cor. 5. *The ſpace deſcribed by a falling body in the time of one vibration : is to half the length of the pendulum : : as the ſquare of the circumference : to the ſquare of the diameter.*

For the ſpaces being as the ſquares of the times, the ſpace fallen : $\frac{1}{4}$ length of the pendulum : : as the ſquare of the time of vibration : to the ſquare of the time of falling through $\frac{1}{4}$ length of the pendulum : : (by this Prop.) as the ſquare of the circumference : to the ſquare of the diameter.

PROP. XXV.

The lengths of two pendulums, deſcribing ſimilar arches, are as the ſquares of the times of vibration.

For (Prop. XXII.) the times of deſcending through two ſimilar curves, are as the ſquare roots of the lengths of the curves; that is, as the ſquare

roots

roots of the lengths of the pendulum, their centers Fig.
being alike fituated.   Therefore the lengths of
the pendulums are as the fquares of the times of
vibrating.

Cor. 1. *The times of vibration of pendulums in
fmall arches of circles, are as the fquare roots of the
lengths of the pendulums.*
For if the arches are fimilar, the times of vi-
bration are in that proportion. And (Prop. XXIV.
Cor. 2.) if the arches are fmall, though not
fimilar, the vibration will be the fame as before.

Cor. 2. *The velocity of a pendulum at its loweft
point, is as the chord of the arch it defcends through.*
For the velocity at the loweft point is equal to
the velocity gained in defcending through the
chord; for they are both of them the fame as a
body acquires by falling through their common
altitude. And (Prop. XXIII, Cor. 2.) the velocity
gained in falling through the chord, is as the length
of the chord.   Therefore the velocities of a pen-
dulum in different arches, are in the fame ratio.

PROP.   XXVI.

*Pendulums of the fame length vibrate in the fame
time, whether they be heavier or lighter.*

For let the two pendulums P, p, be of the
fame length; they will each of them fall through
half the length of the pendulum in the fame time.
For (Cor. 2. Prop. V.) the velocity generated in
any given time, is as the force directly and matter
reciprocally. But in the two pendulums, the forces
that generate their motions, are their weights,
which are as their quantities of matter.   Whence
we have the velocity of P, to the velocity of p; as
$\frac{P}{P}$ to $\frac{p}{p}$ or as 1 to 1; and therefore equal veloci-
ties are generated in the fame time. Confequently,

Fig. equal fpaces will be defcribed in the fame time, and therefore they will fall through half the length of one of them in an equal time. And therefore (Prop. XXIV.) their vibrations will be performed in the fame time.

Cor. *Hence all bodies, whether greater or leffer, heavier or lighter, near the earth's furface, will fall through equal fpaces in equal times; abating the refiftance of the air.*

Becaufe they are as much retarded by their matter, as accelerated by their weight. The weight and the matter being exactly proportional to one another.

PROP. XXVII.

The lengths of pendulums vibrating in the fame time, in different places of the world, will be as the forces of gravity..

For (by Prop. V. Cor. 2.) the volocity generated in any time is as the force of gravity directly, and the quantity of matter reciprocally. And the matter being fuppofed the fame in both pendulums, the velocity is as the force of gravity; and the fpace paffed through in a given time, will be as the velocity; that is, as the gravity. Therefore, if any two fpaces be defcended through in any time, and two pendulums be made, whofe lengths are double thefe fpaces; thefe pendulums (by Prop. XXIV.) will vibrate in equal times; therefore the lengths of rhe pendulums, being as the fpaces fallen through in equal times, will be as the forces of gravity.

Cor. 1. *The times wherein pendulums of the fame length will vibrate, by different forces of gravity, are reciprocally as the fquare roots of the forces.*

For (Cor. 2. Prop. V.) when the matter is given, the velocity generated is as the force × by the time.

And

And (Prop. VI.) the fpace defcended through by Fig.
any force, is as the force and fquare of the time.
Let thefe fpaces be the lengths of the pendulums.
Then the lengths of the pendulums are as the forces
and the fquares of the times of falling through
them.   But (Prop. XXIV.) the times of falling
through them are in a given ratio to the times of
vibration; whence the lengths of pendulums are
as the forces and the fquares of the times of vi-
bration; therefore when the lengths are given,
the forces will be reciprocally as the fquares of
the times; and the times of vibration reciprocally
as the fquare roots of the forces.

Cor. 2. *The lengths of pendulums in different
places, are as the forces of gravity, and the fquares
of the times of vibration.*
This is proved under Cor. 1.   Hence,

Cor. 3. *The times wherein pendulums, of any length,
perform their ofcillations; are as the fquare roots of
their lengths directly, and the fquare roots of the gra-
vitating forces reciprocally.*

Cor. 4. *The forces of gravity in different places,
are as the lengths of pendulums directly, and the
fquares of the times of vibration reciprocally.*

PROP.   XXVIII.   *Prob.*

*To find the length of a pendulum, that fhall make
any number of vibrations in a given time.*

Reduce the given time into feconds, then fay,
as the fquare of the number of vibrations given :
to the fquare of this number of feconds : : fo is
39.13 : to the length of the pendulum fought, in
inches.
Ex. Suppofe it makes 50 vibrations in a minute;
here a minute is = 60 feconds; then,
As 2500 (the fquare of 50) : 3600 (the fquare
of

Fig. of 60) : : 39.13 : to the length $=\dfrac{3600 \times 39.13}{2500}$

$=\dfrac{140868}{2500}= 56.34$ inches, the length required.

If it be required to find a pendulum that shall vibrate such a number of times in a minute; you need only divide 140868, by the square of the number of vibrations given, and the quotient will be the length of the pendulum.

This practice is deduced from Prop. XXV. for let p be the length of the pendulum, n the number of vibrations, t the time they are to be performed in. Then $39.13 : 1^2 :: p : \dfrac{p}{39.13} =$ square of the time of one vibration, and $\sqrt{\dfrac{p}{39.13}} =$ time of one vibration; then if t be divided by $\sqrt{\dfrac{p}{39.13}}$ it will give n; that is, $t\sqrt{\dfrac{39.13}{p}} = n$, whence $tt \times 39.13 = nnp$, and $nn : tt :: 39.13 : p$.

If the pendulum is a thread with a little ball at it, then the distance between the point of suspension and the center of the ball is esteemed the length of the pendulum. But if the ball be large, say as the distance between the point of suspension, and the center of the ball, is to the radius of the ball; so the radius of the ball to a third proportional. Set ⅔ of this from the center of the ball downward, gives the center of oscillation. Then the whole distance from the point of suspension to this center of oscillation, is the true length of the pendulum.

13. If the bob of the pendulum be not a whole sphere, but a thin segment of a sphere, as AB, as in most clocks; then to find the center of oscillation, say as the distance between the point of suspension, and the middle of the bob, is to half the
breadth

breadth of the bob; fo half the breadth of the Fig. bob, to a third proportional. Set one third of this length from the middle of the bob downwards, gives the center of ofcillation. Then the diftance between the centers of fufpenfion and ofcillation, is the exact length of the pendulum.

PROP. XXIX. *Prob.*

Having the length of a pendulum given; to find how many vibrations it fhall make in any given time.

Reduce the time given into feconds, and the pendulum's length into inches; then fay, as the given length of the pendulum : to 39.13 : : fo is the fquare of the time given : to the fquare of the number of vibrations, whofe fquare root is the number fought.

Example. Suppofe the length of the pendulum is 56.34 inches, to find how often it will vibrate in a minute.

1 minute = 60 feconds. Then 56.34 (the length of the pendulum) : 39.13 : : 3600 (the fquare of 60) : to the fquare of the number of vibrations

$$\text{brations} = \frac{3600 \times 39.13}{56.34} = \frac{140868}{56.34} = 2500, \text{ and}$$

$\sqrt{2500} = 50 =$ the number of vibrations fought.

If the time given be a minute, you need only divide 140868 by the length, and extract the root of the quotient for the number of vibrations.

This is the reverfe of the laft problem, therefore, fuppofing as before in that problem, we have $tt \times 39.13 = nnp$; therefore $p : 39.13 : : tt : nn.$

They that would fee a further account of the motions of bodies upon inclined planes, the vibrations of pendulums, and the motion of projectiles; may confult my large book of Mechanics, where they will meet with full fatisfaction.

·SECT.

Fig.

(44)

SECT. III.

Of the Center of Gravity; the equili-
brium of beams of timber; the di-
rections and quantities of the forces
neceſſary to ſuſtain them.

PROP. XXX.

A BODY cannot deſcend or fall downwards, except
only when it is in ſuch a poſition, that by its
motion, its center of gravity deſcends.

14. Let the body A ſtand upon the horizontal plane
BK, and let C be its center of gravity; draw CD
perpendicular to the plane BK. And let the body
be ſuspended at the point C, upon the perpendicu-
lar line CD. Then (def. 12) it will remain un-
moved upon the line CD. And as CD is perp.
to the horizon, it has no inclination to move one
way more than another, therefore it will move no
way but remain at reſt. Take away the line CD,
and let the body be ſupported by the line BC; if
BC be fixed, the body will remain at reſt on the
line BC. But if BC be movable about B, the
body ſuspended at C, will endeavour to move
with its center of gravity downwards along the
arch CE, about B as a center, towards N. And
for the ſame reaſon the body will endeavour to
fall the contrary way, moving about the point
N; I ſay, this will be the caſe when CD is
ſituated between B and N. But theſe two motions
being contrary to one another, will hinder each
other's effects; and the body will be ſuſtained
without falling.

Again,

Again, let the body F be fufpended with its cen- Fig.
ter of gravity I upon the perpendicular IH.   As 14.
this line has no inclination to move to any fide, it
will therefore remain at reft.   Take away the line
IH, and let the center of gravity I be fufpended
on the line IG, then the body will endeavour to
defcend along the arch IK, for the higheft point
of the arch is in the perpendicular erected at G.
For the fame reafon if the body be fufpended on
the line OI, it will endeavour to defcend towards
K, about the center O; now as both thefe motions
tend the fame way, and there is nothing to oppofe
them; the body muft fall towards K.   In both
thefe cafes it is plain, that when the center of
gravity by its motion, defcends, the body will fall;
but if not, the body will be fupported without falling.

Cor. 1. *If a body ftands upon a plane, and a line be
drawn from the center of gravity perpendicular to the
horizon; if this line fall within the bafe on which the
body ftands, it will be fupported without falling.   But
if the perpendicular falls without the bafe, the body
will fall.*

For when the perpendicular falls within the bafe,
the body can be moved no manner of way, but the
center of gravity will rife.   And when the per-
pendicular falls without the bafe, towards any
fide; if the body be moved towards that fide, the
center of gravity defcends; and therefore the
body will fall that way.

Cor. 2. *If a perpendicular drawn from the center
of gravity perp. to the horizon, fall upon the extremity
of the bafe, the body may ftand; but the leaft force
whatever, will caufe it to fall that way.   And the
nearer the perp. is to any fide, the eafier it will be
made to fall, or is fooneft thruft over.   And the
nearer the perp. is to the middle of the bafe, the firmer
the body ftands.*

<div align="right">Cor.</div>

Fig.   Cor. 3. *Hence if the center of gravity of a body be*
14. *supported, the whole body is supported. And the place*
of the center of gravity must be deemed the place of
the body; and is always in a line drawn perpendicular
to the horizon, through the center of gravity.

    Cor. 4. *Hence all the natural actions of animals*
may be accounted for from the properties of the center
of gravity.

    When a man endeavours to walk, he ſtretches
out his hind leg, and bends the knee of his fore
leg, by which means his body is thruſt forward,
and the center of gravity of his body is moved
forward beyond his feet; then to prevent his
falling, he immediately takes up his hind foot,
and places it forward beyond the center of gravity;
then he thruſts himſelf forward, by his leg which
now is the hindmoſt, till his center of gravity be
beyond his fore foot, and then he ſets his hind
foot forward again; and thus he continues walking
as long as he pleaſes.

    In ſtanding, a man having his feet cloſe toge-
ther cannot ſtand ſo firmly, as when they are at
ſome diſtance; for the greater the baſe, the firmer
the body will ſtand; therefore a globe is eaſily
moved upon a plane, and a needle cannot ſtand
upon its point, any otherwiſe than by ſticking
it into the plane.

    When a man is ſeated in a chair, he cannot
riſe till he thruſts his body forward, and draws
his feet backward, till the center of gravity of
his body be before his feet; or at leaſt upon
them; and to prevent falling forward, he ſets
one of his feet forward, and then he can ſtand,
or ſtep forward as he pleaſes.

    All other animals walk by the ſame rules; firſt
ſetting one foot forward, that way the center of
gravity leans, and then another.

                               In

In walking up a hill, a man bends his body Fig.
forward, that the center of gravity may lie forward
of his feet; and by that means he prevents his
falling backwards.

In carrying a burthen, a man always leans the
contrary way that the burthen lies; so that the
center of gravity of the whole of his body and
the burthen, may fall upon his feet.

A fowl going over an obstacle, thrusts his head
forward, by that means moving the center of gra-
vity of his whole body forwards; so that by
setting one foot upon the obstacle, he can the
more easily get over it.

PROP. XXXI.

In any two bodies A, B, *the common center of* 15.
gravity C, *divides the line joining their centers, into
two parts, which are reciprocally as the bodies;* AC :
BC : : B : A.

Let the line ACB be suppoſed an inflexible
lever; and let the lever and bodies be suſpended
on the point C.   Then let the bodies be made to
vibrate about the immovable point C; then will
A and B deſcribe two arches of circles about the
center C, and theſe arches will be as the velocities
of the bodies, and theſe arches are alſo as the radii
of the circles AC and BC.   Therefore their velo-
cities are as the radii.   Whence, velocity of A :
velocity of B : : AC : CB : : (by ſuppoſition) B
: A.   Therefore A × velocity of A = B × velocity
of B.   Whence (Prop. II.) the quantities of mo-
tion of the bodies A and B are equal, and (Ax. 9.)
therefore they cannot move one another, but muſt
remain at reſt; and conſequently (def. 12.) C is
the center of gravity of A and B.

Cor.

Fig.   Cor. 1. *The produĉts of each body multiplied by its*
15.   *diflance from the common center of gravity, are equal.*
$CA \times A = CB \times B$.

Cor. 2. *If a weight be laid upon* C, *a point of the
inflexible lever* AB, *which is fupported at* A *and* B;
the preffure at A *to the preffure at* B, *will be as* CB
to CA.

For let the bodies A, B, be both placed in C;
then (Cor. 3. Prop. XXX.) fince it is the fame
thing whether the bodies be at A and B, or both
of them at C, their center of gravity; therefore the
preffures at A and B will be the fame in both cafes.
But when they are at A and B, upon the lever
ACB, their preffures are A and B, being the fame
with the weights; therefore when they are both at
C, the preffures at A and B will ftill be A and B.
Therefore (Cor. 1.) fince it is $CA \times A = CB \times B$; therefore $CA : CB :: B : A ::$ preffure at B
: preffure at A.

PROP.   XXXII.

15.   *If there be three or more bodies, and if a line be
drawn from one body* E *to the center of gravity of
the reft* C. *Then the center of gravity of all the
bodies divides the line* CE, *in two parts in* D, *which
are reciprocally as the body* E *to the fum of all the
other bodies.* $CD : DE :: E : A + B$ &c.

For fuppofe AB and CE to be two inflexible
lines; and let the body $W = A + B$ &c. and let
W be placed in the center of gravity C.   Then
by the laft Prop. $CD : DE :: E : W$ or $A + B$ &c.

Cor. *The body* $E \times DE$ *the diflance from the com-
mon center of gravity, is equal to the fum of the
bodies* $A + B$ &c. × *by* DC *the diflance of their
center from the common center of gravity.*

PROP.

PROP. XXXIII.

If A, B, *be two bodies,* C *their center of gravity.* 16.
F *any point in the line* AB.   *Then will* $\overline{FA} \times A +$
$FB \times B = FC \times \overline{A + B}$.

For (Cor. 1. Prop. XXXI.) $CA \times A = CB \times B$;
that is, $\overline{FA - FC} \times A = \overline{FC - FB} \times B$; whence,
by tranſpoſition $FA \times A + FB \times B = FC \times \overline{A + B}$.

Cor. *Hence the bodies* A *and* B *have the ſame force
to turn the lever* AF *about the point* F, *as if they
were both placed in* C, *their center of gravity.*

PROP. XXXIV.

If ſeveral bodies, A, B, E &c. *be placed on an* 17.
inflexible ſtraight lever; and if D *be their common
center of gravity; and if* F *be any point in the line*
AE, *then* $FA \times A + FB \times B + FE \times E$ &c. $= FD$
$\times \overline{A + B + E}$ &c.

For if $A + B = W$, then $FA \times A + FB \times$
$B + FE \times E = FC \times \overline{A + B} + FE \times E = FC$
$\times W + FE \times E = $ (Prop. XXXIII.) $FD \times$
$\overline{W + E} = FD \times \overline{A + B + E}$, in the three bodies
A, B, E. And after the ſame manner, if there
be four bodies, put $W = A + B + E$, and it will
be proved the ſame way, that the ſum of all the
products, $FA \times A + FB \times B$ &c. $=$ diſtance of
the common center of gravity. $\times$ by all the four.
And ſo on for more bodies.

Cor. *The ſame Prop. will hold good, when the
bodies are not in the line* AF, *but any where in the
perpendiculars paſſing through the points* A, B, E &c.

PROP. XXXV.

17. *If there be any number of bodies* **A**, **B**, **E**, *&c.*
either placed in the line AF, *or any way in the per-*
pendiculars passing through A, B, E. *And if* D *be*
the center of gravity of all the bodies; and F *be any*
point in the line AF. *Then the distance of the center*
of gravity $FD = \dfrac{FA \times A + FB \times B + FE \times E}{A + B + E}$.

For whether the bodies be in the points A, B,
E, or in the perpendiculars, it will be (by Prop.
XXXIV. and Cor.) that $FA \times A + FB \times B + FE \times E = FD \times \overline{A + B + E}$. Whence $FD = \dfrac{A \times FA + B \times FB + E \times FE}{A + B + E} = $ sum of all the pro-
ducts of each body multiplied by its distance,
divided by the sum of the bodies.

Cor. 1. *If a single body only was placed on the*
lever AF; *then the distance of the center of gravity*
of that body is equal to the sum of the products of all
the particles of the body, each multiplied by its dis-
tance from a given point F, *and divided by the body.*
For if A, B, E &c. are several particles of the
body, then A + B + E &c. = the body; and
$FD = \dfrac{A \times FA + B \times FB + E \times FE}{body}$.

Cor. 2. *If there be several bodies* A, B, E, *&c.*
placed upon the lever AF. *They act with the same*
force in turning the lever about any given point F, *as*
if they were all placed in D *the common center of*
gravity of all the bodies.

SCHOLIUM.

If any of the bodies be placed on the contrary side
of F, *their respective products will be negative.*

For

For they act the contrary way in turning the Fig.
lever about,   17.

PROP. XXXVI.

If several bodies A, B, E, G, H, *be placed on* 18.
the lever AH, *and* F *be the center of gravity of all
their weights.*   *Then* $FA \times A + FB \times B + FE \times E$
$= FG \times G + FH \times H$.

For let the lever be suspended on the point F,
then the two ends will be in equilibrio, as F is the
center of gravity.   Let D be the center of gravity
of A, B, E; and I the center of gravity of G,
H.   Then (Cor. Prop. XXXIII.) it is the same
thing whether the bodies on one side be placed at
A, B, E, or all of them in the point D.   And
whether those at the other end be placed at G, H;
or all of them at I.   But since F is the center of
gravity, $DF \times \overline{A + B + E} = FI \times \overline{G + H}$, and
therefore $A \times AF + B \times BF + E \times EF = G \times GF$
$+ H \times HF$ (by Prop. XXXIV.)

Cor. 1. *If several bodies* A, B, E, G, H, *be
placed on an inflexible lever, and if* $A \times FA + B \times$
$FB + E \times FE = G \times FG + H \times FH$.   *Then* F *is
the center of gravity of all the bodies.*
For no other point will answer the equation.

Cor. 2. *If several bodies* A, B, E, G, H, *be
placed upon a lever* AH, *or suspended at these points
by ropes; and if* $A \times FA + B \times FB + E \times FE =$
$G \times FG + H \times FH$; *they will be in equilibrio upon
the point* F.
This appears by Def. 12, and F is center of
gravity.

PROP. XXXVII.

19.   *If a heavy body* AB, *suspended by two ropes* AC, BD, *remains at rest; a right line perpendicular to the horizon, passing through the intersection* F *of the ropes; will also pass through the center of gravity* G, *of the body.*

If AC and BD be produced to F where they intersect; then (Ax. 12) it is the same thing whether the force acting in direction AC be applied to C or F; and whether the force acting in direction BD be applied to the point D or F. Suppose then that they both act at F, and then it is the same thing, as if the body was suspended at F by the two strings AF, BF. And since the body is at rest, therefore (Ax. 7.) the body, that is, the center of gravity G, is at the lowest place it can get; and therefore is in the plumb line FG. For if the body be made to vibrate, the center of gravity G will describe an arch of a circle, of which G (being in the perp. FG) is the lowest point.

Cor. 1. *Hence if* GN *be drawn parallel to* AC; *then the weight of the body, the forces acting at* C *and* D, *are respectively as* FG, GN, *and* FN.
This is evident by Prop. VIII.

Cor. 2. *If a heavy body* AB, *be supported by two planes,* IKL, *and* EHG, *at* H *and* K; *and* HF *and* KF *be drawn perpendicular to these planes; and if* FG *be drawn from the intersection* F, *perp. to the horizon, it will pass through the center of gravity* G, *of the body.*
For since the body is sustained by these planes, therefore the planes re-act against the body (by Prop. IX.) in the directions HF, KF perpendicular to these planes. Therefore it is the same thing

as
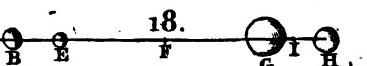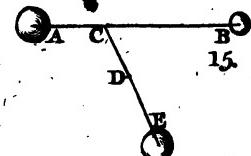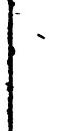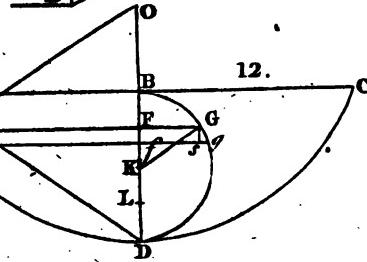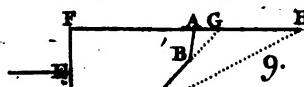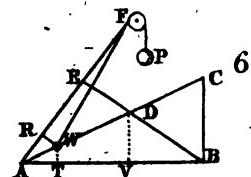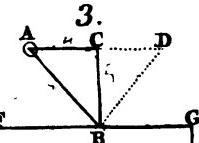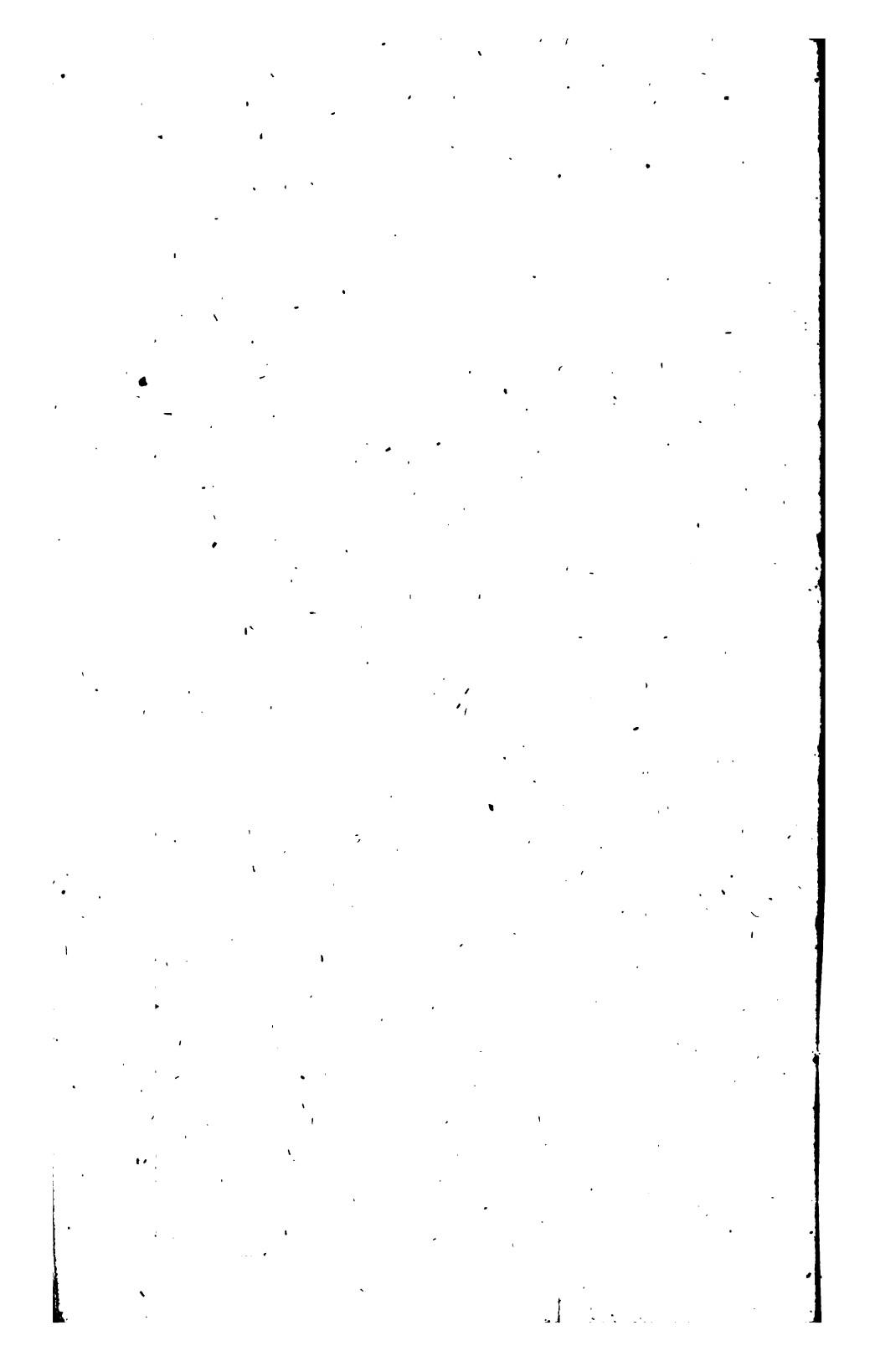
as if the body was fuftained by the two ropes HF, Fig.
KF. For the direction and quantities of the 19.
forces, acting at H and K are the fame in both
cafes. And further, if the body be made to vi-
brate round F, the points H, K will defcribe two
arches of circles, coinciding with the touching
planes at H, K; therefore moving the body up
and down the planes, will be juft the fame thing,
as making it vibrate in the ropes, HF and KF;
and confequently, the body can reft in neither cafe,
but when the center of gravity G is in the perpen-
dicular FG.

SCHOLIUM.

If any body fhould deny the truth of this Prop.
or its corollaries, againft the cleareft force of de-
monftration. It lies upon them to fhew where the
demonftration fails, or what ftep or fteps thereof do
not hold good, or are not truly deduced from the
foregoing. If they cannot do this, what other rea-
fons they may affign, can fignify nothing at all to
the purpofe. And if fuch perfon, ignorant of the
laws of nature, and the refolution of forces, would
object againft this practice, of fubftituting planes
perpendicular to the lines or cords fuftaining any
weights, inftead of thefe cords, let him firft read
Sir *I. Newton*'s *Principia*, Cor. 2. to the laws of
nature, where he will fee this practice exemplified,
and then make his objections.

And for the fake of fuch perfons as underftand
not how to apply the method of compofition and
refolution of forces, I fhall add a few problems to
prevent their being mifled by the rafh judgment of
fome people, who having brought out falfe folu-
tions to fome problems by their own ill manage-
ment, condemn the method as erroneous; when
the fault really lies in their own ignorance, and not
at all in the method itfelf.

D 3   PROP.

CENTER OF GRAVITY.

PROP. XXXVIII. *Prob.*

To determine the position of a beam CD, *movable about the end* C, *and sustained by a given weight* Q, *hanging at a rope* QAD, *which goes over a pulley at* A, *and is fixed to the other end* D.

Draw AF, CK parallel to the horizon, FDE perp. to it, and KD perp. to CD; and let G be the center of gravity, w = weight of the beam. Then if the beam was to lie horizontally (Prop. XXXI. Cor. 2.) it would be GC : GD : : pressure at D : pressure at C; and GC : CD : : pressure at D : w; whence the pressure at D $= \frac{GC}{CD} w$, horizontally. And (Prop. XIV.) CD : CE : : $\frac{GC}{CD} w : \frac{CE \times CG}{CD^2} w$ = pressure in direction DK. Produce AD to O, and draw OI parallel to CD. Then the beam is sustained by three forces in the directions OD, DI and IO; and DI : DO : : S.IOD or ODC or ADC : rad; whence S.ADC : rad. : : $\frac{CE \times CG}{CD^2} \times w$: force DO or Q $= \frac{\text{rad.} \times CE \times CG}{S.ADC \times CD^2}$ w. Therefore w : Q : : S.ADC : rad. $\times \frac{CE \times CG}{CD^2}$: : S.ADC : $\frac{CG}{CD} \times$ S.FDC, because $\frac{\text{rad.} \times CE}{CD}$ = S.EDC or FDC.

PROP. XXXIX. *Prob.*

Let the beam ED, *be sustained by the weights* P, Q, *by means of the ropes* DCP, EAQ, *going over the pullies* C, A, *in the horizontal line* AC. *To find the position of the beam; having the weights* P, Q, *given.*

One Way.

Let G be the center of gravity of the beam. Through D, E, draw HDS, FER perpendicular to AC.

AC.   Then (Cor. 2. Prop. VIII.) S.EDS : P the Fig.
tenſion of the thread CD : : S.CDS or CDH : 21.
$\frac{S.CDH}{S.EDS} \times$ P the tenſion of DE.   Alſo, S.DER or

EDS : Q : : S.AER or AEF : $\frac{S.AEF}{S.EDS} \times$ Q the
tenſion of DE in a contrary direction.   Then as
the beam is in equilibrio, theſe forces or tenſions
balance one another; therefore $\frac{S.CDH}{S.EDS} \times$ P =
$\frac{S.AEF}{S.EDS} \times$ Q.   Then P : Q : : S.AEF : S.CDH;
which may be otherwiſe expreſſed, for AE : rad : :
AF : S.AEF = $\frac{AF}{AE} \times$ rad; and DC : rad : : HC :
$\frac{HC}{DC} \times$ rad = S.HDC.   Whence P : Q : : $\frac{AF}{AE}$:
$\frac{CH}{CD}$: : $\frac{AF}{HC}$: $\frac{AE}{DC}$.

Second Way.

Let R, S, be the perpendicular preſſures of the
ends E, D; w = weight of the beam.   Then
(Cor. 2. Prop. XXXI.) R = $\frac{DG}{ED} w$, and S = $\frac{EG}{ED} w$.

And (Cor. 2. Prop. VIII.) S.AED : R or $\frac{DG}{ED} w$: :

S.AEF : tenſion of DE = $\frac{S.AEF \times DG}{S.AED \times ED} w$.   And

S.CDE : S or $\frac{EG}{ED} w$: : S.CDH : contrary tenſion

of DE = $\frac{S.CDH \times EG}{S.CDE \times ED} w$, and theſe two forces

of DE being equal, we have $\frac{S.AEF \times DG}{S.AED \times ED} w$ =

S.CDH

Fig. 21.

$$\frac{S.CDH \times EG}{S.CDE \times ED} w, \text{ and } \frac{S.AEF \times DG}{S.AED} = \frac{S.CDH \times EG}{S.CDE}.$$

Whence $EG : DG :: \dfrac{S.AEF}{S.AED} : \dfrac{S.CDH}{S.CDE} :: S.AEF$

$\times S.CDE : S.CDH \times S.AED.$

Third Way.

$$S.CDE : S.EDS :: S : P = \frac{S.EDS}{S.CDE} \times S, \text{ and}$$

$$S.AED : S.RED :: R : \frac{S.RED}{S.AED} R = Q. \text{ Then } P :$$

$$Q :: \frac{S.EDS}{S.CDE} \times S : \frac{S.RED}{S.AED} \times R :: S.AED \times S :$$

$S.CDE \times R ::$ (laſt method) $S.AED \times \dfrac{EG}{ED} w :$

$S.CDE \times \dfrac{DG}{ED} w :: S.AED \times EG : S.CDE \times DG.$

And $S.AED : S.CDE :: P \times DG : Q \times EG.$

Fourth Way.

Draw Cm, Fn parallel to DE, and FE, HD perp. to the horizon. Then by the reſolution of forces, $CD : Dm :: P : \dfrac{Dm}{CD} P = $ perpendicular

force at D; and $nE : EF :: Q : \dfrac{EF}{nE} Q = $ perpen-

dicular force at E. Therefore $EG : GD :: \dfrac{Dm}{CD} P :$

$\dfrac{EF}{nE} Q :: \dfrac{S.CDE}{S.CmD} \times P : \dfrac{S.FnE}{S.nFE} \times Q :: S.CDE \times$

$P : S.AED \times Q.$ For $S.CmD = S.mDF = S.FED$
$= S.nFE.$ That is, $EG : GD :: S.CDE \times P :$
$S.AED \times Q.$ As in the third way.

Fifth Way.

Let R, S be the perpendicular weights of the ends E, D; or which is the ſame, the tenſions of the
'perpendicular

perpendicular ropes, FE, HD. By the refolution Fig.
of forces, if Cm, Fn be parallel to DE. The force
FE or R is equivalent to En, Fn; and the force
Dm or S, to DC, mC; therefore EF : Fn : : R :
$\dfrac{F n}{FE}$ R = force at E in direction Fn. And Dm :

mC : : S : $\dfrac{m C}{m D}$ S = force at D in direction mC. But
the beam being in equilibrio, thefe two oppofite
forces muft be equal; therefore $\dfrac{F n}{FE}$ R = $\dfrac{m C}{m D}$ S.
Whence R : S : : $\dfrac{m C}{m D}$: $\dfrac{F n}{FE}$: : $\dfrac{S.CDH}{S.CDE}$: $\dfrac{S.AEF}{S.AED}$
: : S.AED × S.CDH : S.AEF × S.CDE. But (Cor.
2. Prop. XXXI.) R : S : : DG : EG. Whence
DG : EG : : S.AED × S.CDH : S.AEF × S.CDE;
the fame as by the 2d method. And the fame
thing likewife follows from the 1ft and 4th method
together.

From thefe feveral ways of proceeding, it is
evident, that which ever way we take, the procefs
if rightly managed always brings us to the fame
conclufion; and it comes to the fame thing which
way we ufe, fo that we proceed in a proper manner.
And this among other things, fhews the great ufe
and extent of that noble theory of the compofition
and refolution of forces.

What is calculated above is concerning the an-
gles, or the pofition of the feveral lines to one another,
depending on the feveral forces. Then in regard to
the weight of the beam, put it = w, then DC :
Dm : : P : $\dfrac{D m}{DC}$ P = S, and En : EF : : Q : $\dfrac{EF}{E n}$ Q

= R. And $w = R + S = \dfrac{D m}{DC} P + \dfrac{EF}{E n} Q$, an
equation fhewing the relation of the weights to
one another.

Sixth

Fig.
21.

Sixth Way, by the Center of Gravity.

Produce AE, CD to B, and from B draw BGO
through the center of gravity; which (by Prop.
XXXVII.) will be perp. to AC, and therefore
parallel to EF, DH. Then the angle EBG = AEF,
and DBG = CDH. Then EB : BD : : S.BDE or
CDE : S.BED or AED, and (Trigonom. B. II.
Prop. V. Cor. 1.) EG : GD : : EB × S.EBG : BD ×
S.DBG : : EB × S.AEF : BD × S.CDH : : S.CDE ×
S.AEF : S.AED × S.CDH; the fame as by the 2d
way. Whence all the reft will be had as before.

Cor. *It will be exactly the fame thing, whether
the weights* P, Q, *remain, or the ftrings* AE, CD,
*be fixed in that pofition to two tacks, any way in
thefe lines. And if a beam* ED, *hang upon two
tacks* A, C, *by ropes fixed there; it makes no dif-
ference, if you put two pullies inftead of the tacks,
for the ropes to go over, and then hang on weights*
Q, P, *equal to the tenfions of the ftrings* AE, CD.
For in both cafes, the forces or the tenfions of
the ftrings, and their directions, remain the fame.
And there is nothing elfe to make a difference in
the fituation of the beam.

SCHOLIUM.

Every one that knows any thing of mechanical
principles will eafily underftand, that if any forces,
which keep a body at reft, be refolved into others,
to have the fame effect; the contrary forces, or
thofe directly oppofite, muft act againft a fingle
point; or elfe the equilibrium will be deftroyed.
And therefore in the prefent Prop. fuppofe any one
fhould divide the forces CD, AE, into the two HD,
DY, and FE, EX, one perpendicular, the other
parallel to the horizon. The forces HD, EF, will
indeed balance the force of gravity at D and E, to
which

which they are directly oppofite. And therefore Fig.
the beam will remain unmoved by thefe.   But the 21.
equal forces DY, EX, being parallel, never meet in
a point; but acting obliquely on the beam, one of
them drawing at D in direction DY, and the other
at E in direction EX, the effect will be, that they
will turn the beam ED about the center of gravi-
ty G.   Therefore to prevent this, the forces DY,
EX, muft be fubdivided; that is, they muft be
refolved into others, one whereof is perp. to the
horizon, the other parallel to ED.   Then gravity
will balance thefe perp. to the horizon, and the
others, being equal and oppofite, acting in the line
EGD, act equally againft any of the points D, G,
or E.   And fo the beam will remain at reft.   But
this is much better done at once at the firft, by di-
viding DC, EF, each into two forces, one perp. to
the horizon, the other parallel to the beam ED.
And then the oppofite forces will exactly balance
one another, and the beam remain unmoved.

PROP. XL. *Prob.*

To find the pofition of the beam ED, hanging by 22.
the rope EBD, whofe ends are faftened at E and D,
and goes over a pulley fixed at B.

Let G be the center of gravity of the beam, then
(Prop. XXXVII.) BG will be perp. to the horizon.
Then as the cord runs freely about the pulley B;
therefore (Ax. 13.) the tenfion of the parts of the
rope EB, BD are equal to one another, fuppofe
= T.   By the refolution of forces, the force EB
is equivalent to EG, GB; and DB to DG, GB.

Therefore $BE : EG :: T : \dfrac{EG}{EB} T =$ force in direc-

tion EG.   And $BD : DG :: T : \dfrac{DG}{BD} T =$ force in

direction

Fig.
22.
direction DG, which is equal and oppofite to that in EG; therefore $\dfrac{EG}{EB}$ T $= \dfrac{DG}{DB}$ T. Whence EG :

EB : : DG : DB. And therefore BG bifects the angle EBD.

Cor. *Hence* ED : *ftring* EBD : : EG : EB *the part* EB *of the ftring* : : *and fo* GD : DB *the part* DB *of the ftring.*

<div align="center">SCHOLIUM.</div>

If GD be lefs than GE, then the center of gravity G, will be loweft, when the beam hangs perpendicular with the end D downward. And in many cafes it will be higheft, when it hangs perpendicular, with the end E downward.

<div align="center">PROP. XLI.   *Prob.*</div>

23.   *There is a beam* BC *hanging by a pin at* C, *and lying upon the wall* BE; *to find the forces or preffures at the points* B, *and* C, *and their directions.*

Produce BC to K, fo that CK may be equal to CB. Draw BA parallel, and CL perpendicular, to the horizon; and draw BL, CN, KI perp. to BCK. Through the center of gravity G, draw GF parallel to CL. By Prop. XIV. if a body lies upon an inclined plane, as BC; its weight, its inclination down the plane, and preffure againft it, are as BC, CA and AB; that is, as CL, CB and BL. Therefore if CL reprefent the weight of the body, CB will be the force of urging it down the plane, and BL the total preffure againft the plane. And fince GF is parallel to CL, BL is divided in F, in the fame ratio, as BC is divided in G. And therefore (Cor. 2. Prop. XXXI.) BF will be the part of the preffure acting at C, and FL the part acting at B. Make CN equal to BF, and compleat

<div align="right">the</div>

the parallelogram CNIK, and draw CI. Then Fig.
since BC or CK is the force in direction CK, and 23.
CN the force in direction CN ; then by compofi-
tion, CI will be the fingle force by which C is fuf-
tained, and CI its direction. But the triangles
CKI, CBF are fimilar and equal, and CI = CF,
and in the fame right line; therefore CF is the
quantity and direction of the force acting at C to
fuftain it. Therefore the weight of the body, the
preffure at B, and the force at C; are refpectively
as CL, FL, and CF.

Cor. 1. *Produce* FG *to interfect* CN *in* H; *then
the weight of the body, the preffure at* B, *and the
force acting at* C; *are refpectively as* HF, HC, *and*
CF.
For in the parallelogram CLFH, HF = CL,
and HC = FL.

Cor. 2. *If the beam was fupported by a pin at* B,
and laid upon the wall AC ; *the like conftruction muft
be made at* B, *as has been done at* C, *and then the
forces and directions will be had.*

Cor. 3. *If a line perp. to the horizon be drawn
through* F, *where the direction of the forces* CF, *and*
BF *meet; it paffes through* G *the center of gravity of
the beam.*

Cor. 4. *It is all one whether the beam is fuftained
by the pin* C *and the wall* BE, *or by two ropes* CI,
BP *acting in the directions* FC, FB, *and with the
forces* CF, FL.

<center>SCHOLIUM.</center>

The proportions and directions of the forces here
found, are the fame as in Prop. LXIV. of my large
book of Mechanics, and obtained here by a dif-
ferent method. The principles here ufed are in-
difputable; and the principle made ufe of in that
<div align="right">LXIV.</div>

Fig. LXIV. Prop. is here demonſtrated in the third
23. Cor. ſo that the reader may depend upon the
truth of them all.

PROP. XLII. *Prob.*

24.　BC *is a heavy beam ſupported upon two poſts* KB,
LC; *to find the poſition of the poſts, that the beam
may reſt in equilibrio.*

Let G be the center of gravity; draw BA pa-
rallel to the horizon, and BF, GD, CAN perpendi-
cular to it. Then (Prop. XXXI. Cor. 2.) if BC
be the weight of the body, CG will be the part of
the weight acting at B, and BG the weight at C.
Therefore make CN = BG, and BF = GC; and
from N and F, draw NI, FK, parallel to BC; and
make NI = FK, of any length, and lying con-
trary ways. Then draw IC and KB, which will
be the poſition of the poſts required.

For BF is the weight upon B; and CN, that
upon C, which forces being in direction of the
lines BF, CN, the beam will remain at reſt by
theſe forces. And the forces NI, FK, in direction
BC, being equal and contrary, will alſo keep the
beam in equilibrio, therefore the forces KB, IC,
compounded of the others, will alſo keep the beam
in equilibrio, acting in the directions KB, IC, or
MB, LC.

Cor. 1. *Hence if DG be produced, it will paſs
through the interſection* H, *of the lines* LC, MB.

For the triangles INC, CGH are ſimilar; there-
fore IN : NC : : CG : GH, the interſection with
CL. Alſo the triangles KFB, BGH are ſimilar;
therefore KF : BG : : BF : GH the interſection
with MB, which muſt needs be the ſame as the
other, ſince the three firſt terms of the proportion
are the ſame; for KF = NI, BG = NC, and
BF = CG.

Cor.

Cor. 2. *If a line be drawn through the center of* Fig. *gravity* G, *of the beam, perpendicular to the horizon* ; 24. *and from any point* H *in that line, (above or below* G) *the lines* HBK *and* HCM *be drawn* ; *then the props* BM *and* CL *will sustain the beam in equilibrio.*

Cor. 3. *If* GO *be drawn parallel to* HC ; *then the weight of the beam, the preſſure at* C, *and thruſt or preſſure at* B ; *are reſpectively as* HG, OG, *and* HO, *and in theſe directions.*

Cor. 4. *It is all one for maintaining the equilibrium, whether the beam* BC *be ſupported by two poſts or props* MB, LC ; *or by two ropes* BH, CH ; *or by two planes perpendicular to theſe ropes at* B *and* C.

For in all theſe caſes the forces and directions are the ſame; and there is nothing elſe concerned, but the forces and directions.

<center>SCHOLIUM.</center>

It does not always happen that the center of gravity is at the loweſt place it can get, to make an equilibrium. For here if the beam BC be ſupported by the poſts MB, LC ; the center of gravity is at the higheſt it can get; and being in that poſition, it has no inclination to move one way more than another, and therefore it is as truly in equilibrio, as if it was at the loweſt point. It is true, any the leaſt force will deſtroy that equilibrium, and then if the beam and poſts be movable about the angles M, B, C, L, which is all along ſuppoſed, the beam will deſcend till it is below the points M, L, and gain ſuch a poſition as deſcribed in Prop. XXXIX. and its Cor. ſuppoſing the ropes fixed at A, C (fig. 21.); and then G will be at the loweſt point, and will come to an equilibrium again. In planes the center of gravity G may be either at its higheſt or loweſt point. And there are caſes, when
<div align="right">the</div>

Fig.
24. the center of gravity is neither at its higheſt nor loweſt, as may happen in the caſe of Prop. XL. ſo that they who who contend, that in caſe of an equilibrium, the center of gravity muſt *always* be at the loweſt place, are much miſtaken, and know little about the principles of mechanics, or the nature of things.

Thoſe that want to ſee more variety about the motion of bodies, on inclined planes; the preſſure, and direction of the preſſure of beams of timber; centers of gravity, and alſo the centers of oſcillation and percuſſion, &c. may conſult my large book of Mechanics.

SECT.

SECT. IV.

The MECHANIC POWERS; the Strength and Strefs of Timber.

PROP. XLIII.

*IN a balance, where the arms are of equal length;
if two equal weights be fufpended at the ends, they
will be in equilibrio.*

The balance is a ftraight inflexible rod or 25.
beam, turning about a fixed point or axle in the
middle of it; to be loaded at each end with
weights fufpended there.

Let AB be the beam or lever, C the middle
point or center of motion; D, E the weights
hanging at the ends A and B. Then let the beam
and the weights, or the whole machine, be fuf-
pended at C. And fuppofe the beam and the
weights be turned about upon the center C; then
the points A, B being equidiftant from C will
defcribe equal arches, and therefore the velocities
will be equal, and if the bodies D and E be
equal, then the motion of D will be equal to the
motion of E, as their quantities of matter and
velocities are equal; and confequently, if the beam
and weights are fet at reft, neither of them can
move the other, but they will remain in equilibrio.

Cor. *If one weight be greater than the other;
that weight and fcale will defcend, and raife the other.*

SCHOLIUM.

The ufe, of the balance, or a common pair of
fcales, is to compare the weights of different bo-
dies;

Fig. dies; for any body whose weight is required, be-
25. ing put into one scale, and balanced by known
weights put into the other scale, these weights will
shew the weight of the body. To have a pair of
scales perfect, they must have these properties.
1. The points of suspension of the scales, and the
center of motion of the beam, A, C, B, must be
in a right line. 2. The arms AC, BC, must be
of equal length from the center. 3. That the
center of gravity be in the center of motion C.
4. That there be as little friction as possible.
5. That they be in equilibrio, when empty.

If the center of gravity of the beam be above
the center of motion, and the scales be in equi-
librio, if they be put a little out of that position,
by putting down one end of the beam, that end
will continually descend, till the motion of the
beam is stopt by the handle H. For by that mo-
tion, the center of gravity is continually descend-
ing, according to the nature of it. But if the cen-
ter of gravity of the beam be below the center of
motion; if one end of the beam be put down a
little, to destroy the equilibrium; it will return
back and vibrate up and down. For by this motion
the center of gravity is endeavouring to descend.

PROP. XLIV. Prob.

25. *To make a false balance; or one which is in equi-*
librio when empty, and also in equilibrio, when loaded
with unequal weights.

Make such a balance as described in the last
Prop. only instead of making the center of mo-
tion in the middle at C, make it nearer one end, as
at F. And make the weight of the scales such that
·they may be in equilibrio upon the center F. Then
if two weights D, E, be made to be in equilibrio
in the two scales; these weights will be unequal,
for

for they will be reciprocally as the lengths of the Fig.
arms AF, BF. That is, AF : BF : : E : D. 25.

For (Prop. XXXI. Cor. 1.) fince F is the
center of gravity of D and E, fuppofing them to
act at A and B; therefore FA × D = FB × E. And
FA : FB : : E : D. But AF is greater than FB,
therefore E is greater than D.

*Cor. 1. Hence to difcover a falfe balance, make the
weights in the two fcales to be in equilibrio; then
change the weights to the contrary fcales. And if
they be not in equilibrio, the balance is falfe.*

*Cor. 2. Hence alfo to prove a pair of good fcales,
they muft be in equilibrio when empty, and likewife in
equilibrio with two weights. Then if the weights be
changed to the contrary fcales, the equilibrium will
ftill remain, if the fcales are good.*

*Cor. 3. From hence alfo may be known what is gained
or loft, by changing the weights, in a falfe balance.*

Take any weight, as 1 pound, to be put into
one fcale and balanced by any fort of goods in the
other. Since AF × D = BF × E; let the weight
D be 1; then $E = \dfrac{AF}{BF}$ the weight of the goods
in the fcale E. Then changing the fcales, let the
weight E be 1; then $D = \dfrac{BF}{AF}$ the weight of the
goods in the fcale D. Then $\dfrac{AF}{BF} + \dfrac{BF}{AF} = $ whole
weight of the goods, and $\dfrac{AF}{BF} + \dfrac{BF}{AF} - 2 = $ gain
to the buyer in 2lb. &c. Therefore
$$\frac{AF^2 + BF^2 - 2AF \times BF}{AF \times BF} = \text{gain in 2 lb.} =$$
$$\frac{\overline{AF - BF}\,^2}{AF \times BF}; \text{ and } \frac{\overline{AF - BF}\,^2}{2AF \times BF} = \text{gain in 1lb.}$$

E 2 Therefore

Fig. Therefore if w is any weight to be bought; the
25. gain to the buyer, for the weight w, by changing

the fcales, will be $\dfrac{\overline{AF-BF}\,|^2}{2AF \times BF}\, w.$ For example,

if AF be 16, and BF 15; then the gain will be

$$\dfrac{\overline{16-15}^2}{2 \times 16 \times 15} \cdot w = \dfrac{1}{480}\, w.$$

Scholium.

In demonſtrating the properties of the mecha-
nical powers; fince the weight is commonly fome
large body whofe weight is to be overcome or ba-
lanced; therefore the power which acts againſt it,
will be moſt fitly reprefented by another weight;
and thus the power and weight being of the fame
kind, may moſt properly and naturally be com-
pared together.   For fuch a weight may reprefent
any power, as the ſtrength of a man's hand,   the
force of water or wind, &c.   And this weight re-
prefenting the power, being fufpended by a rope,
may hang perpendicular where the power is to act
perpendicular to the horizon; or may go over a
pulley when it acts obliquely.

PROP. XLV.

26.   *If the power and weight be in equilibrio upon any*
27. *lever, and act in lines perpendicular to the lever:*
28. *then the power* P *is to the weight* W; *as the diſtance*
29. *of the weight from the fupport* C, *is to the diſtance of*
the power from the fupport.

There are four forts of levers.   1. When the
fupport is between the weight and the power.
2. When the weight is between the power and the
fupport. 3. When the power is between the weight
and the fupport.   4. When the lever is not ſtraight
but crooked.

A lever

A lever is any inflexible rod or beam, of wood Fig.
or metal, made ufe of to move bodies. · The fup- 26.
port is the prop it refts on, in moving or fuftaining 27.
any heavy body, and this is the fame as the center 28.
of motion. 29.

Let the power P act at A by means of a rope;
then as C is the prop or center of motion, if the
lever be made to move about the center C, the
arches defcribed by A and W, that is, the velo-
cities of A and W, will be as the radii CA and CW.
But the velocity of P is .the fame as that of the
point A. Therefore velocity of P : velocity of
W : : CA : CW : : (by fuppofition) W : P; there-
fore P × velocity of P = W × velocity of W.
Confequently their motions are equal, and they
cannot move one another, but muft remain in
equilibrio. And if they be in equilibrio, they
muft have this proportion affigned.

Cor. 1. *If a power* P *act obliquely againft the lever* 30.
W A ; *the power and weight will be in equilibrio,
when the power* P *is to the weight* W, *as the diftance
of the weight* CW, *to* CD *the perpendicular, drawn
from the fupport to the line of direction of the power.*

For in this cafe WCD becomes a bended lever,
and the power P acts perpendicular to CD at D;
and (Ax. 12.) it is all one whether the power acts
at D or A, in the line of direction AD. And hence

Cor. 2. *If any force be applied to a lever* ACW
at A, *its power to turn it about the center of motion*
C, *is as the fine of the angle of application* CAD.

For if CA be given, CD is as the fine of CAD.

Cor. 3. *In a ftraight lever, of thefe three, the
power, the weight, and the preffure upon the fupport;
the middlemoft is equal to the fum of the other two.*

For the middle one acts againft both the others,
and fupports them.

E 3 Cor.

Fig.　　Cor. 4. *From the foregoing properties of the lever,*
30.　*the effects of several sorts of simple machines may be*
　　explained; and likewife the manner of lifting, carrying,
　　drawing of heavy bodies, and such like.

26.　　For example, if a given weight W is to be
　　raifed by a fmall power applied at A, the end of
　　the lever AW. If we divide WA in C, fo that it
　　be as CA : CW : : as the weight W : to the power
　　P; then fixing a prop or fupport at C, or rather a
　　little nearer W; then the power P with a fmall
　　addition, will raife the weight W.

27.　　Again, if two men be to carry a weight W, upon
　　the lever CA. The weight the man at A carries,
　　is to the weight the man at C carries, as CW to
　　AW. And therefore the lever or beam CA ought
　　to be divided in that proportion at W, the place
　　where the weight is to lie, according to the ftrength
　　of the men that carry it.

31.　　Likewife if two horfes be to draw at the fwing-
　　tree AB, by the ropes AF, BG; and the fwing-
　　tree draws a carriage, &c. by the rope CD; then
　　the force acting at A will be to the force acting at
　　B, as BC to AC. And therefore BC ought to be
　　to AC, as the ftrength of the horfe at F, to the
　　ftrength of the horfe at G; the weaker horfe having
　　the longer end. But it is proper to make the
　　crofs bar AB crooked at C; that when the ftronger
　　horfe pulls his end more forward, he may pull
　　obliquely, and at a lefs diftance from the center;
　　whilft the weaker horfe pulls at right angles to
　　his end, and at a greater diftance.

　　　Again, fuch inftruments as crows and handfpikes
　　are levers of the firft kind, and are very ufeful and
　　handy for raifing a great weight to a fmall height.
　　Alfo fciffars, pinchers, fnuffers, are double levers
　　of the firft kind, where the joint is the fulcrum or
　　fupport. The oars of a boat, the rudder of a fhip,
　　cutting knives fixed at one end, are levers of the
　　　　　　　　　　　　　　　　　　　　　　　fecond

second kind. Tongs, sheers, and the bones of animals, are levers of the third kind. A ladder rais-ed upright, is a lever of the third kind. A hammer drawing out a nail is a lever of the fourth kind.

The *Steel Yard* is nothing but a lever of the first kind, whose mechanism or conftruction is this. Let AB be the beam, C the point of fufpenfion; P the power, movable along the beam CB. The beam being fufpended at D, move the power P, along towards C, till you find the point O, where P reduces the beam to an equilibrium. Then at A hang on the weight W of 1 pound; and move P to be in equilibrio with it at 1; then hang on W of 2 pound, and shift P till it be in equilibrio, at 2. Proceed thus with 3, 4, 5, &c. pounds at W, and find the divifions 3, 4, 5, &c. Or, if you will, after having found the points O, 1; make the divifions, 12, 23, 34, &c. each equal to O1. But for more exactnefs and expedition, having found the point O, where P makes the beam in equilibrio: hang on any known number of pounds, as W; and move P to the point B, where it makes an equilibrium. Then divide OB into as many equal parts as W confifts of pounds: mark thefe divifions 1, 2, 3, 4, &c. Then any weight W being fufpended at A; if P be placed to make an equilibrium therewith, then the number where P hangs will shew the pounds or weight of W.

To prove this, we muft obferve, that AC is the heavier end of the beam; therefore let Q be the *Momentum* at that end to make an equilibrium with P fufpended at O; that is, let $Q = CO \times P$. But (Cor. 2. Prop. XXXVI.) $Q + CA \times W = CF \times P = CO \times P + OF \times P$. Take away Q $= CO \times P$, and then $CA \times W = OF \times P$. Whence $AC : P :: OF : W$. But AC and P are always the fame; therefore W is as OF; that is,

if

Fig. if OF be 1, 2, 3, &c. divifions, then W is 1, 2,
32. 3, &c. pounds.

We may take notice that the divifions O1, 12,
23, &c. are all equal; but CO may be greater or
leffer, or nothing.

If you would know how much the weight P is,
take the diftance CA, and fet it from O along the
divifions O, 1, 2, 3, &c. and it will reach to the
number of pounds.. But this is of no confequence,
being only matter of curiofity.

PROP. XLVI.

33. *In the compound lever, or where feveral levers act
upon one another, as* AB, BC, CD, *whofe fupports
are* F, G, I; *the power* P : *is to the weight* W : :
as BF × CG × DI : *to* AF × BG × CI.

For the power P acting at A : force at B : : BF :
AF; and force or power at B : force at C : : CG :
GB; and force or power at C : weight W : : DI :
IC. Therefore *ex equo*, power P : weight W : :
BF × CG × DI : AF × GB × IC.

And it is the fame thing in the other kinds of
levers, taking the refpective diftances, from the
feveral props or fupports.

PROP. XLVII.

34. *In the wheel and axle; the weight and power will
be in equilibrio, when the power* P *is to the weight*
W; *as the radius of the axle* CA, *where the weight
hangs; to the radius of the wheel* CB, *where the
power acts.*

This is a wheel fixed to a cylindrical roller,
turning round upon a fmall axis; and having a
rope going round it.

Through

20.

21.

24.

26.

27.

28.

29.

31.

32.
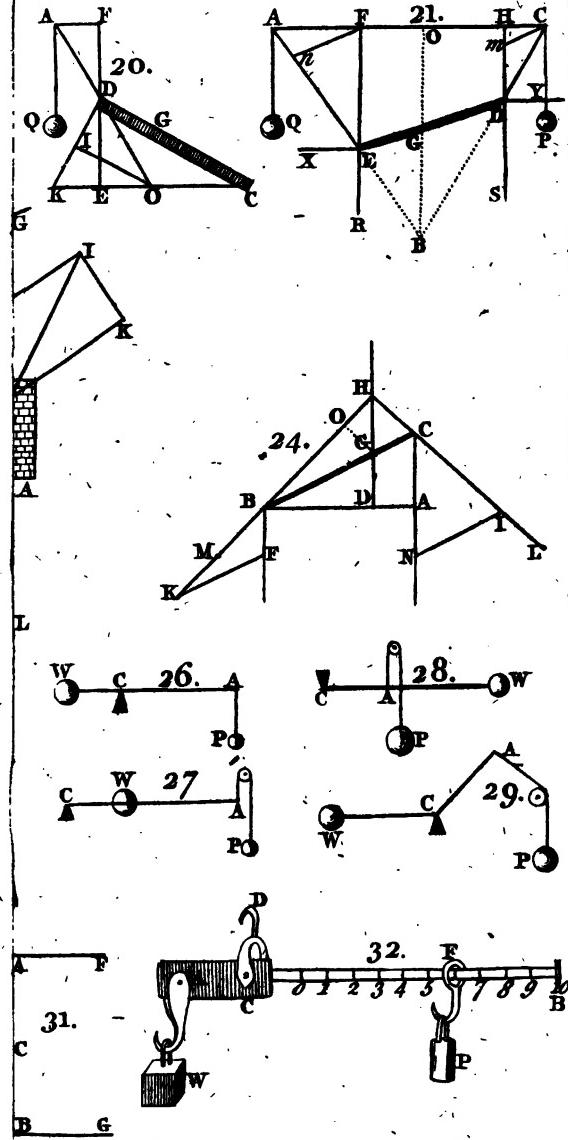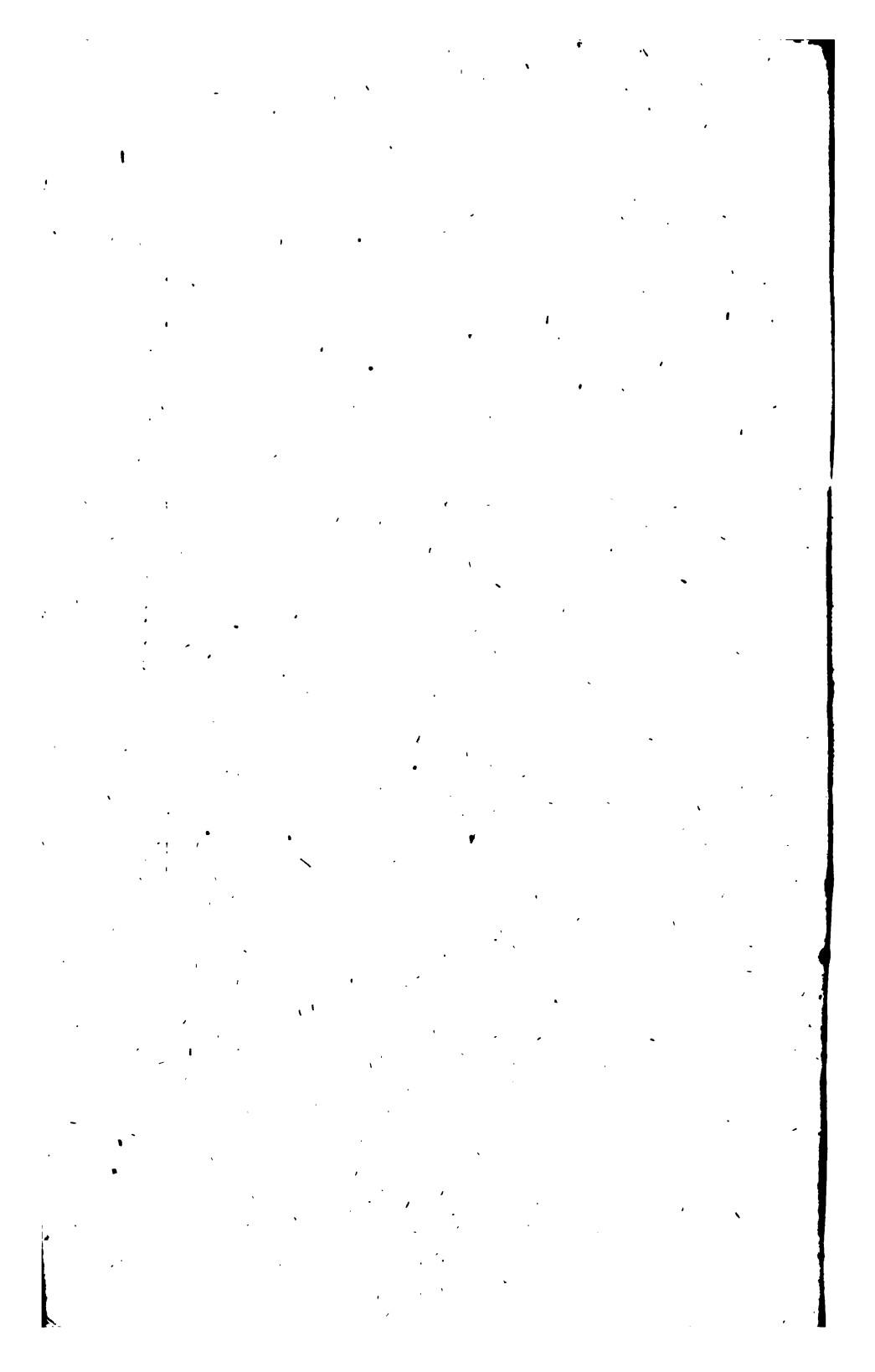
Through the center of the wheel C, draw the Fig. horizontal line BCA. Then BP and AW are per- 34. pendicular to BA; and BCA will be a lever whose support is C; and the power acts always at the distance BC, and the weight at the distance CA; which remain always the same.   Therefore the weight and power acts always upon the lever BCA. But by the property of the lever (Prop. XLV.) BC : CA : : W : P, to have an equilibrium.

Otherwise.

If the wheel be set a moving, the velocity of the point A or of W, is to that of B or P, as CA to CB; that is (by suppofition) as P : W.   Therefore W × velocity of W = P × velocity of P; therefore, the motions of P and W being equal, they cannot, when at rest, move one another.

Cor. 1. *If the power acting at the radius* CB, *act* 35. *not at right angles to it; draw* CD *perpendicular to* BP *the direction of the power ; then the power* P : *is to the weight* W : : *as the radius of the axle* CA : *to the perpendicular* CD.

For in the lever DCA, whose support is C, the power P : weight W : : CA : CD.

Cor. 2. *In a roller turning round on the axis or* 36. *spindle* FC, *by the handle* CBG; *the power applied perpendicularly to* BC *at* B, *is to the weight* W : : *as the radius of the roller* DA, *to the length of the handle* CB.

For in turning round, the point B describes the circumference of a circle; the same as if it was a wheel whose radius is CB.

Scholium.

All this is upon suppofition that the rope fuf- taining the weight is of no fenfible thicknefs. But if it is a thick rope, or if there be feveral folds of it

Fig.
36. it about the roller or barrel; you muſt meaſure to the middle of the outſide rope to get the radius of the roller. For the diſtance of the weight from the center is encreaſed ſo much, by the rope's going round.

From hence the effects of ſeveral ſorts of machines, or inſtruments may be accounted for. A roller and handle for a well or a mine, is the ſame thing as a wheel and axle, a windlaſs and a capſtain in a ſhip is the ſame; and ſo is a crane to draw up goods with. A gimblet and an augur to bore with, may be referred to the wheel and axle.

The wheel and axle has a particular advantage over the lever; for a weight can but be raiſed a very little way by the lever. But by continual turning round of the wheel and roller, the weight may be raiſed to any height required.

PROP. XLVIII.

37. *In a combination of wheels with teeth; if the power* P *be to the weight* W : : *as the product of the diameters of all the axles or pinions, to the product of the diameters of all the wheels; the power and weight will be in equilibrio.*

AC, CD are the radii of one wheel and its axle; DG, GH, the radii of another; and HI, IK are thoſe of another. Theſe act upon one another at D and H, then as the power or force P is propagated through all the wheels and axles to W; we muſt proceed to find the ſeveral forces acting upon them, by Prop. XXXVII. Thus,

$$CD : CA : : P : \frac{CA}{CD} P = \text{force acting at D.}$$

$$\text{and } GH : GD : : \frac{CA}{CD} P \text{ (force at D)} : \frac{CA \times GD}{CD \times GH} P$$
$$= \text{force acting at H.}$$

and

and IK : IH : : $\dfrac{CA \times GD}{CD \times GH}$ P (force at H) : Fig. 37.

$\dfrac{CA \times GD \times IH}{CD \times GH \times IK}$ P = force at K = W. And CA ×

GD × IH × P = CD × GH × IK × W; whence
P : W : : CD × GH × IK : : CA × GD × IH.

Cor. 1. *If the weight and power be in equilibrio, and made to move; the velocity of the weight is to the velocity of the power; as the product of the diameters of all the axles or pinions, to the product of the diameters of all the wheels. Or instead of the diameters, take the number of teeth in these axles and wheels that drive one another. And the same is true of wheels carried about by ropes.*

For the power is to the weight; as the velocity of the weight to the velocity of the power. And the number of teeth in the wheels and pinions, that drive one another, are as the diameters. And the ropes supply the place of teeth.

Cor. 2. *In a combination of wheels with teeth. The number of revolutions of the first wheel, is to the number of revolutions of the last wheel, in any time; as the product of the diameters of the pinions or axles, to the product of the diameters of the wheels: or as the product of the number of teeth in the pinions, to the product of the number of teeth in the wheels which drive them. And the same is true of wheels going by cords.*

For as often as the number of teeth in any pinion is contained in the number of teeth of the wheel that drives it; so many revolutions does that pinion make for one revolution of the wheel.

Scholium.

A pinion is nothing but a small wheel, fixed at the other end of the axis, opposite to the wheel;
and

Fig. and confifts but of a few leaves or teeth; and
37. therefore is commonly lefs than the wheel. But
in the fenfe of this propofition, a pinion may, if
we pleafe, be bigger than the wheel. As if we
put the power and weight into the contrary places,
the wheels will become the pinions, and the
pinions the wheels, according to the meaning of
this propofition.

PROP. XLIX.

If a power fuftains a weight by means of a fixed
pulley; the power and weight are equal: but if the
pulley be movable along with the weight, then the
weight is double the power.

A pulley is a fmall wheel of wood or metal,
turning round upon an axis, fixed in a block; on
the edge of the pulley is a groove for the rope to
go over.

38. Through the centers of the pullies, draw the
39. horizontal lines AB, CD; then will AB reprefent
a lever of the firft kind, and its fupport is the
center of the pulley, which is a fixed point, the
block being fixed at F. And the points A, and
B, where the power and weight act, being equally
diftant from the fupport, therefore (Prop. XLV.)
the power P = weight W.

Alfo CD reprefents a lever of the fecond kind,
whofe fupport is at C, a fixed point; the rope CG
being fixed at G. And the weight W acting at
the middle of CD, and the power acting at D,
twice the diftance from C; therefore (Prop. XLV.)
the power P is to the weight W : : as ½CD to
CD; or as 1 to 2.

Cor. Hence all fixed pullies are levers of the firft
kind, and ferve only to change the direction of the
motion; but make no addition at all to the power.
                                                    And

And therefore if a rope goes over several fixed pullies; Fig.
the power is not increased, but rather decreased, by 38.
the friction.   39.

SCHOLIUM.

The use of a fixed pulley is of great service in
raising a weight to any height, which otherwise
must be carried by strength of men, which is
often impracticable. Therefore if a rope is fixed
to the weight at W (fig. 38.) and passed over
the pulley BA; a man taking hold at P will draw
up the weight, without moving from the place.
And if the weight be large, several persons may
pull together at F, to raise the weight up; where
in many cases they cannot come to it, to raise it
by strength.

PROP. L.

In a combination of pullies, all drawn by one rope 40.
going over all the pullies; if the power P *is to the*
weight W; *as* 1 *to the number of the parts of the*
rope proceeding from the movable block and pullies.
Then the power and weight will be in equilibrio.

Let the rope go from the power about the
pullies in this order, *ntovrs*, where the last part *s*
is fixed to the lower block B. Now (Ax. 13.) all
the parts of the rope *ntovrs* are equally stretched,
and therefore each of them bears an equal weight;
but the part *n* bears the power P, which goes to
the fixed block A. All the other parts sustain the
weight and movable block B, each with a force
equal to P. Therefore P is to the sum of all the
forces, sustained by *o*, *r*, *s*, *t*, *v*, or the weight
W, as 1 to the number of these ropes immediately
communicating with the movable block B. And
all the ropes having an equal tension, none of
them

Fig.
40. them can move the reſt, but they muſt remain in equilibrio.

And if you take away the power at **P,** and apply a force at the rope *t* equal to **P,** to pull upwards in direction *t*A; this will make no alteration, for the rope *t* draws from the movable block with the ſame force as before, and therefore the weight is ſuſtained as before; for the upper pulley (by Prop. XLIX, Cor.) which the rope *nt* goes over, ſerves only to change the direction. And therefore as there are the ſame number of ropes ſtill drawing from the movable block as before; the propoſition holds good alſo in this reſpect. And it would be the ſame thing if the rope *s* was fixed to the weight **W** inſtead of the block B; but had it been fixed to the block **A,** there muſt have been a pulley more below, and a rope more, which would have increaſed the power, according to the propoſition.

Cor. 1. *Hence it appears to be a diſadvantage to the power to pull againſt the fixed block.*

For the rope *n* has no more purchaſe, or no more effect than the rope *t* has which draws againſt the movable block; and therefore when one draws by the rope *n*, there muſt be a pulley more, which will create more friction.

Cor. 2. *Hence one may explain the effects of all ſorts of machines compoſed of pullies; or find out ſuch a conſtruction or combination of them as to anſwer any purpoſe deſired. And to find its force, begin at the power and call it. 1; then all the parts of the running rope that go and return about ſeveral pullies, muſt be each numbered alike. And any rope that acts againſt ſeveral others muſt be numbered with the ſum of theſe. And ſo on to the weight.*

. *For*

For example; suppose a man wanted to draw him- Fig.
self up to the top of a house or a church. Get a 41.
pulley A fixed at the top, and place another B at
the bottom. Let a rope be fixed to the upper
block A, and brought down about the pulley B,
and then put round the upper pulley, and so
brought to the ground at H.   Then if a cross
·stick CD be fastened to the block B by a rope ; a
man may get astride of the stick, and then draw
himself up by the rope H. And the power to draw
himself up, , will be little more than ⅓ of his
weight.   For the power at H, and the two parts
of the rope going about the pulley B, sustain all
his weight ; and each of them sustains one third
of it.

If . instead of the stick CD, he takes a chair to
sit in ; then when he has drawn himself up to any
height he pleases, he may fix the rope H to the
chair, and then do any sort of business, as set up
a dial, point the walls, and such like, as is com-
monly done.

Again, several tackles are used aboard a ship, 42.
for hoisting goods and the like. Let A, B be two
blocks with pullies, the upper one being fixed,
and let a weight W be suspended at the single pul-
ley and rope, one end of the rope being fixed at
F, and the other fastened to the movable block B.
This pulley and rope BCF is called a *Runner*. Let
the power be at P, call it 1 ; then all the ropes go-
ing from B to A, must be each of them 1, and
the rope going from the block B, acting against
these four must be marked 4, and the other part
of it CF must also be 4.   Lastly, the weight act-
ing against these two , must be 8.   And then the
power P is to the weight W, as 1 to 8.

ABCD is another tackle with a runner BAD, A 43.
being a fixed pulley ; the two blocks B, C, are
both movable. The rope DAB is fixed to the
                                        weight

Fig.
43.

44.

45.

weight at D, and to the block B.   The rope PB goes and returns about the pullies BC, and at laft is faftened to the block C.   Let P be the power, mark it 1, then the other parts of the rope between the blocks, muft alfo be 1 apiece. · Then CI acting againft 3, muft be 3.   And AB is 4, as it acts againft 4; likewife AD muft be 4.   Therefore the whole force that fuftains the weight W is 3 and 4, or 7.   And the power to the weight as 1 to 7.

The following is a fort of *Spanifh burton*, A and F are two fixed pullies; C and B two movable ones.   The rope going from the power P, goes round C, B, and A, and is faftened to the block B.   Another rope is faftened to the block B, and goes over the pulley F, and is fixed to the block C.   Then marking the power P, 1.   Then each part of the rope, continued over C, B, and A to B again muft be each 1.   Then FC muft be 2, as it acts againft two parts; and likewife the other part of it FB muft be 2.   Then the whole that lifts the weight W, is $1 + 1 + 1 + 2 = 5$. And therefore the power is to the weight as 1 to 5.

The friction between the pullies and blocks is fometimes confiderable.   To remedy which, they muft be as large as they can conveniently be made, and kept oiled or greafed.

PROP. LI.

In the fcrew, if the power applied at E, *be to the weight, preffure, &c. at* B; *as the diftance of two threads of the fcrew, taken parallel to the axis of it, is to the circumference defcribed by the power at* E; *then the weight and power will be in equilibrio.*

A fcrew is an inftrument confifting of two parts AB, CD, fitting into one another.   AB is the male fcrew, called the top or fpindle; this is a long cylindrical body, having its furface cut into ridges and

34.

B C A

P W

38. F

B A

W P

G 39.

P

C D

W

43. A

4 4

B

z z z

C

3

D

W P z

44. A F

2 2

z C

z z

B

5

W

z P
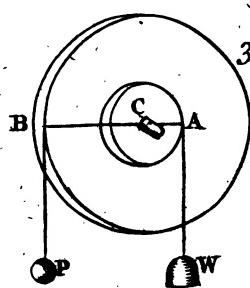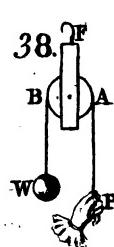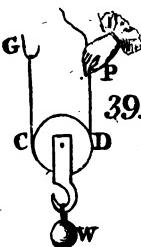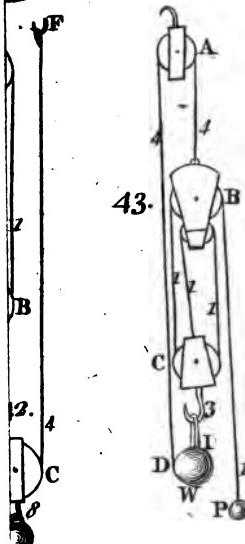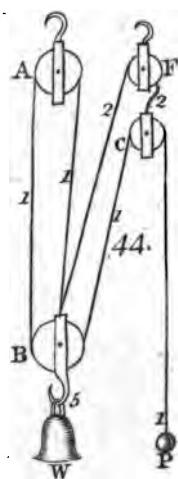
SCHOLIUM.

Screws with sharp threads have far more friction than those with square threads, and therefore move a body with more difficulty.

The screw, in moving a body acts like an inclined plane. For it is just the same as if an inclined plane was forced under a body to raise it; the body being prevented from flying back, and the base of the plane being driven parallel to the horizon.

The use of this power is very great. It is of great service for fixing several things together by help of screw nails; it is likewise very useful for squeezing or pressing things close together, or breaking them; also for raising or moving large bodies. The screw is used in presses for wine, oil, or for squeezing the juice out of any fruit. The very friction of this machine has its particular use, for when a weight is raised to any height; if the power be taken away, the screw will retain its position, and hinder the weight from descending again by its friction, without any other power to sustain it.

In the common screw, such as is here supposed; the threads are all one continued spiral from one end to the other; but where there are two or more spirals, independent of one another, as in the worm of a jack; you must measure between thread and thread of the same spiral, in computing the power.

PROP. LII.

In the endlefs fcrew, where the teeth of the worm 46.
or fpindle AB, drives the wheel CD, by acting againft
the teeth of it.   If the power applied at P, is to the
weight W, acting upon the edge of the wheel at C
: : as the diftance of two threads or teeth, between
fore fide and fore fide, taken along AB; is to the
circumference defcribed by the power P. , Then the
weight and power will be in equilibrio.

The *endlefs* or *perpetual fcrew* is one that turns
perpetually round the axis AB; and whofe teeth
fit exactly into the teeth of the wheel CD, which
are cut obliquely to anfwer them : fo that as AB
turns round, its teeth take hold of the teeth of
the wheel CD, and turns it about the axis I, and
raifes the weight W.
     For by one revolution of the power at P, the
wheel will be drawn forward one tooth; and the
weight W will be raifed the fame diftance. There-
fore the velocity of the power, will be to that of
the weight; as that circumference, to one tooth : :
that is (by fuppofition) as the weight W, to the
power P.   Therefore the power P × velocity of
P = W × velocity of W; therefore their motions
being equal, they will be in equilibrio.

Cor. *If a weight N be fufpended at E on the axle*
EF; then if the power P, is to the weight N : : as
the breadth of a tooth × EF, to the circumference
defcribed by P × CD.   They will be in equilibrio.
     Or if the power P; is to the weight N : : as
radius of the axle EI, to the radius of the handle
FP × by the number of teeth in CD; they will be in
equilibrio.

Fig. For power P : weight W :: 1 tooth : circumference.
46. and weight W : weight N : : EF : CD.

therefore P : N :: 1 tooth × EF : CD × circumference.

Or thus, whilſt EF turns roûnd once, P turns round as oft as CD has teeth; whence EI : BP × number of teeth : : velocity of N : velocity of P : : P : N.

Scholium.

47. As the teeth of the wheel CD, muſt be cut obliquely to anſwer the teeth or ſcrew on AB; ſuppoſing AB to lie in the plane of the wheel CD; and therefore the wheel will be acted on obliquely by the ſcrew AB. To remedy that, the ſcrew AB may be placed oblique to the wheel, in ſuch a poſition, that when the teeth of the wheel are cut ſtraight or perp. to its plane, the teeth of the ſcrew AB, may coincide with them, and fit them. By that means the force will be directed along the plane of the wheel CD. Fig. 47 explains my meaning.

This machine is of excellent uſe, not only in itſelf, for raiſing great weights, and other purpoſes; but in the conſtruction of ſeveral ſorts of compound engines.

PROP. LIII.

48. *In the wedge ACD, if a power acting perpendicular to the back CD, is to the force acting againſt either ſide AC, in a direction perpendicular to it; as the back CD, to either of the ſides AC; the wedge will be in equilibrio.*

A wedge is a body of iron or ſome hard ſubſtance, in fo m of a priſm, contained between two iſoceles triangles, as CAD. AB is the height, and CD the back of it; AC, AD the ſides.

Let AB be perp. to the back CD, and BE, BF, perp. to the ſides AC, AD. Draw EG, FG parallel

'rallel to BF, BE ; then all the fides of the paral- Fig.
lelogram BEGF are equal.   The triangles EGB, 48.
ADC are fimilar ; for draw EOF, which will be
perp. to AB; then the right angled triangles AEB,
AEO, are fimilar, and the angle ABE = AEO =
ACB; that is, GBE = ACD, and likewife BGE
= ADC, whence CAD = BEG.

Now let BG be the force acting at B, in direc-
tion BA, perp. to CD; then (Prop. IX.) the forces
againft the fides AC, AD, will be in the directions
EB, FB ; and therefore EB, EG will reprefent
thefe forces (by Prop. VIII.), when they keep one
another in equilibrio. Therefore force BG applied
to the back of the wedge, is to the force BE,
perp. to the fide AC; as BG to BE; that is, (by
fimilar triangles) as CD to CA.

Cor. 1. *The power acting againft the back at B, is
to that part of the force againft AC, which acts pa-
rallel to the back CD; as the back CD, is to the
height AB.*
For divide the whole force BE into the two BO,
OE ; the part EO acts parallel to CD ; therefore
the force acting at B, is to the force in direction
QE or BC ; as BG to OE ; that is, (by fimilar tri-
angles) as CD to AB.

Cor. 2. *By reafon of the great friction of the
wedge, the power at B, muft be to the refiftance
againft one fide AC ; at leaft as twice the bafe CD,
to the fide AC, taking the refiftance perp. to AC.
Or as twice the bafe CD, to the height AB, for the
refiftance parallel to the bafe CD ; to overcome the re-
fiftance.   But the power muft be doubled for the refif-
tance againft both fides.*
For fince the wedge retains any pofition it is
driven into ; therefore the friction muft be at leaft
equal to the power that drives it.

Fig.
48.
Cor. 3. *If you reckon the refiftance at both fides
of the wedge; then, if there is an equilibrium, the
power at B, is to the whole refiftance; as the back
CD, to the fum of the fides, CA, AD, reckoning the
refiftance perp. to the fides. Or as the back CD, to
twice the height AB, for the refiftance parallel to the
back CD.*

This follows directly from the Prop. and Cor. 1.

<div align="center">SCHOLIUM.</div>

The principal ufe of the wedge is for the cleav-
ing of wood, or feparating the parts of hard bo-
dies, by the blow of a mallet. The force im-
preffed by a mallet is vaftly great in comparifon of
a dead weight. For if a wedge, which is to
cleave a piece of wood, be preffed down with
never fo great a weight, or even if the other me-
chanical powers be applied to force it in ; yet the
effect of them will fcarce be fenfible ; and yet the
ftroke of a fledge or mallet will force it in. This
effect is owing very much to the quantity of mo-
tion the mallet is put into, which it communicates
in an inftant to the wedge, by the force of percuf-
fion. A great deal of the refiftance is owing to
friction, which hinders the motion of the wedge ;
but the ftroke of a mallet overcomes it ; upon
which account the force of percuffion is of excel-
lent ufe ; for a fmart ftroke puts the body into a
tremulous vibrating motion, by which the parts
are difunited and feparated ; and by this means the
friction or fticking is overcome, and the motion of
the wedge made eafy.

This mechanic power is the fimpleft of any ;
and to this, may be reduced all edge tools, as
knives, axes, chiffels, fciffars, fwords, files, faws,
fpades, fhovels, &c. which are fo many wedges
faftened to a handle. And alfo all tools or inftru-
<div align="right">ments</div>

ments with a sharp point, as nails, bodkins, nee- Fig.
dles, pins; and all instruments to cleave, cut, slit, 48.
chop, pierce, bore, and the like.   And in general
all instruments that have an edge or point.

This Prop. is the same as Prop. XXX. in my
large book of Mechanicks, but demonstrated after
a different way; and both come to the same thing,
which evinces the truth thereof.

In this Prop. I have shewn under what circum-
stance the wedge is in equilibrio; and that is,
when the power is to the force against either side;
as the back, is to that side.   Therefore it must be
very strange, that any body should understand it,
as if I had said, that the power is to the whole
resistance; as the back is to one side only.   They
that do this, must be blind or very careless.

SECT. V.

*The comparative Strength of Beams of
Timber, and the Stress they sustain.
The Powers of Engines, their Motion,
and Friction.*

PROP. LIV.

49. *IF a beam of wood* AB, *whose section is a parallel-
logram, be supported at the ends* A *and* B, *by two
props* C, D. *And the weight* E *be laid on the middle
of it, to break it; the strength of it will be as the
square of the depth* EF, *when the breadth is given.*

For divide the depth EF into an infinite num-
ber of equal parts at *n, o, p, q, r,* &c. Now the
strength of the beam consists of the strength of
all the fibres F*n, no, op,* &c. And to break these
fibres, is to break the beam. Also when the beam
is stretched by the weight, the fibres F*n, no, op,*
&c. are stretched by the power of the bended
levers AEF, AE*n,* AE*o,* &c. whose support is at
E, and power at A. For the pressure at A being
half the weight E, we must suppose that pressure
applied to A, to overcome the resistances at F, *n,
o,* &c. Put the force or pressure at A = P, then
P acts against all the fibres at F, *n, o,* &c. by
help of the bended levers AEF, AE*n,* AE*o,* &c.
But it is a known property of springs, fibres, and
such like expanding bodies; that the further they
are stretched, the greater force they exert, in pro-
portion to the length. Therefore when the beam
breaks; that is, when the tension of the fibre F*n*

is at its utmoft extent; then thofe in the middle Fig.
between F and E will have but half the tenfion, and 49.
thofe at all other diftances, will have a tenfion pro-
portional to that diftance. This being fettled, let the
utmoft tenfion of nF be $= 1$; then the tenfions at n,

o, p, &c. will be $\dfrac{E n}{EF}$, $\dfrac{E o}{EF}$, $\dfrac{E p}{EF}$, &c. and the feve-

ral forces, thefe exert againft the point A, by means
of the bended levers FEA, nEA, oEA, &c. will

be $\dfrac{FE}{EA}$, $\dfrac{E n^2}{EF \times EA}$, $\dfrac{E o^2}{EF \times EA}$, $\dfrac{E p^2}{EF \times EA}$ &c. and

the fum of all is $= \dfrac{1}{EF \times EA} \times$ into $EF^2 + E n^2$

$+ E o^2 + E p^2$ &c. to o. But (Arith. Inf. Prop.
III.) the fum of the progreffion $EF^2 + E n^2 + E o^2$
&c. to o, is $\frac{1}{3}EF^3$. Therefore the fum of all the

forces exerted at A; that is, $P = \dfrac{1}{EF \times EA} \times$

$\frac{1}{3}EF^3 = \dfrac{EF^2}{3EA}$. But $P = \frac{1}{4}$ weight E, therefore

weight $E = \frac{2}{3} \times \dfrac{EF^2}{EA}$, when the beam breaks.

In like manner for any other depth Ep, the weight

e that would break it is $= \dfrac{2 E p^2}{3 EA}$. Whence the

weight E to the weight e, is as EF^2 to $E p^2$; that

is, as the fquares of the depths; for $\dfrac{2}{3EA}$ is a

given quantity. Therefore the ftrength of the
beams, are as the fquares of the depths.

Cor. 1. *Hence the ftrength of feveral pieces of the
fame timber, are to one another as the breadths and
fquares of the depths.*

For by this Prop. they are as the fquares of the
depths when the breadth is given. And if the
breadth be increafed in any proportion, it is evi-
dent

Fig.
49.
dent the ſtrength is encreaſed in the ſame propor-
tion. So that a beam of the ſame depth being
twice as broad is twice as ſtrong, and thrice as
broad is thrice as ſtrong, &c.

50.   Cor. 2. *If ſeveral beams of timber as* AF *of the
ſame length, ſtick out of a wall; their ſtrength to
bear any weight* W *ſuſpended at the end, is as the
breadth and ſquare of the depth.*

This follows from Cor. 1. only turning the beam
upſide down, to make the weight W ſuſpended at
A act downwards inſtead of preſſing upwards.

49.   Cor. 3. *If ſeveral pieces of timber be laid under
one another, they will be no ſtronger than if they
were laid ſide by ſide.*

For not being connected together in one ſolid
piece, they can only exert each its own ſtrength,
which will be the ſame in any poſition. ·

Cor. 4. *Hence the ſame piece of timber is ſtronger
when laid edgeways or with the flat ſide up and down,
than when laid flat ways, or with the flat ſide hori-
zontal; and that in proportion of the greater breadth
to the leſſer.*

For let B be the greater breadth, or the breadth
of the flat ſide; *b* the leſſer breadth, being the
narrow ſide. Then the ſtrength edge ways is BB*b*,
and flat ways B*bb*; and they are to one another as
B to *b*.

PROP. LV.

49.   *If a beam of timber* AB *be ſupported at both ends;
and a given weight* E *laid on the middle of it; the
ſtreſs it ſuffers by the weight, will be as its length* AB.

For half the weight E is ſupported at A, by the
prop C; and the preſſure at C is equal to it. And
this preſſure is always the ſame whatever length
                    AB

AB is of.   But it was shewn in the last Prop. that Fig.
the pressure at A, breaks the fibres F*n*, *no*, *op*, &c. 49.
by means of the bended levers AEF, AE*n*, AE*o*,
&c.   But (by Prop. XLV.) when the lengths EF,
E*n*, E*o*, &c. are given, and the power at A also
given; the effect at F, *n*, *o*, &c. is so much greater,
as the arm AE is longer; that is, the stress at the
section EF, is proportional to the distance AE,
or to the length of the beam AB.

Cor. 1. *If AF be a beam sticking out of a wall,* 50.
and a weight W *hung at the end of it.   The stress it*
suffers by the weight, at any point G, *will be as the*
distance AG.

For this has the same effect, as in the case of
this Prop. only turning the beam upside down.   Or
thus, suppose AHG to be a bended lever, whose
fulcrum is H; then since GH is given, and the
weight W; therefore by the power of the lever,
the longer AH is, the more force is applied at G,
or any other points, in GH, to separate the parts
of the wood; and therefore the stress is as AG.

Cor. 2. *Therefore instead of a weight, if any*
force be applied at the end A, *of the lever* AF;
the stress at any part G, *will be as the force, and*
distance AG.

For augmenting the force, the stress is increased
in the same ratio.

Cor. 3.. *Hence also if any weight lie upon the* 49.
middle of a horizontal beam; the stress there will
be as the weight and length of the beam.

For if the weight be increased, the stress will
be increased proportionally, all other circumstances
remaining the same.

Cor. 4. *The stress of beams by their own weight,*
will be as the squares of the lengths.

For here the weight is as the length.

PROP.

PROP. LVI.

51. *If* AB *be a beam of timber whose length is given; and supported at the ends* A *and* B; *and if a given weight* W *be placed at any point of it* G. *The stress of the beam at* G, *will be as the rectangle* AGB.

Let the given weight be W. Then (Cor. 3. Prop. XLV.) the weight W is equal to the pressure at both A and B. And (Cor. 2. Prop. XXXI.) pressure at A : pressure at B : : BG : AG, and press. A : press. A + press. B : : BG : BG + AG ; that is, press. A : weight W : : BG : AB, therefore pressure at $A = \dfrac{BG}{AB}W$, and this is the force re-acting at A. But (Prop. LV. Cor. 2.) the stress at G by this force acting at the distance AG, is as the force multiplied by AG; that is, as $\dfrac{AG \times BG}{AB} \times W$. But W and AB are given, and therefore the stress at G is as $AG \times GB$.

Cor. 1. *The greatest stress of a beam is when the weight lies in the middle.*

For the greatest rectangle of the parts, is in that point.

Cor. 2. *The stress at any point* P *by a weight at* G; *is equal to the stress at* G, *by the same weight at* P.

For when the weight is at W, the stress at G is $AG \times GB$, and the stress at $P = \dfrac{BP}{BG} \times$ the stress at $G = \dfrac{BP}{BG} \times AG \times GB = BP \times AG$. Again, when the weight is at P, the stress at P is $AP \times PB$; and the stress at $G = \dfrac{AG}{AP} \times$ last stress $= \dfrac{AG}{AP} \times AP \times PB = AG \times PB$, the same as before.

PROP.

PROP. LVII.

If the distance of the walls AD *and* BC *be given,* 52.
and AB, AC *be two beams of timber of equal thick-*
ness; the one horizontal, the other inclined. And if
two equal weights P, Q, *be suspended in the middle*
of them; the stress is equal in both, and the one will
as soon break as the other, by these equal weights.

For (Prop .XIV.) AC : AB : : weight P :
$\dfrac{AB}{AC}$P = pressure against the plane, or the part of
the weight the beam AC sustains.   And (Cor. 3.
Prop. LV.) the stress upon AC is $\dfrac{AB}{AC}$P × AC or
AB × P; and the stress on AB is Q × AB, which
is equal to AB × P, because the weights P, Q are
equal.   Therefore, the stress being the same, and
the beams being of equal thickness, one will bear
as much as the other, and they will both break
together.

Cor. 1. *If the beams be loaded with weights in*
any other places in the same perpendicular line as F,
G; *they will bear equal stress, and one will as soon*
break as the other.
For they are cut into parts similar to one ano-
ther; and therefore stress at F : stress by P : :
AFC : ¼AC² : : AGB : $\dfrac{AB^2}{4}$: : stress by B :
stress by Q or stress by P. Therefore stress at F =
stress at B.

Cor. 2. *If the two beams be loaded in proportion*
to their lengths; the stress by these weights, or by
their own weights, will be as their lengths; and there-
fore the longer, that stands aslope, will sooner break.
For

Fig.
52.

For the ſtreſs upon AC was AB x P, and the ſtreſs on AB was AB x Q; but ſince P and Q are to one another as AC and AB, therefore the ſtreſs on AC and AB will be as AB x AC and AB x AB; that is, as AC to AB. And in regard to their own weights, theſe are alſo proportional to their lengths.

PROP. LVIII.

53.

Let AB, AC, be two beams of timber of equal length and thickneſs, the one horizontal, the other ſet ſloping. And if CD be perp. to AB, and they be loaded in the middle with two weights P, Q, which are to one another as AC to AD. Then the ſtreſs will be equal in both, and one will as ſoon break as the other.

For (Prop. XIV.) $AC : AD : : P : \frac{AD}{AC}P =$ preſſure of P in the middle of AC. And by ſuppoſition, $AC : AD : : P : Q;$ therefore $\frac{AD}{AC}P = Q$, the weight in the middle of AB. Therefore the forces in the middle of the two beams are the ſame; and the lengths of the beams being the ſame, therefore (Prop. LV.) the ſtreſs is equal upon both of them; and being of equal thickneſs, if one breaks, the other will break.

Cor. *If the weights P, Q, be equal upon the two equal beams AB, AC. The ſtreſs upon AB will be to the ſtreſs upon AC, as AB or AC to AD . The ſame holds in regard to their own weights.*
For the weight Q is increaſed in that proportion.

SCHOLIUM.

Many more propoſitions relating to the ſtrength of timber might be inſerted; as for example, if a
weight

weight was difpóſed equally through the length of Fig.
the beam AB (fig. 51.), fuppoſed at both ends; 53.
the ſtreſs in any point G, is as the rectangle AGB.
And the ſtreſs at any point G is but half of the
ſtreſs it would fuffer, if the whole weight was fuf-
pended at G. Alſo if AF (fig. 50.) be a beam
fixed in a wall at one end, and a weight be dif-
perſed uniformly through all the length of it. The
ſtreſs at any point G, with that weight (or with its
own weight, if it be all of a thicknefs), will be as
AG ſquare, the ſquare of the diſtance from the
end. And the ſtreſs at any point G by a weight
fuſpended at A, will be double the ſtreſs at the
ſame point G, when the ſame weight is difperſed
uniformly through the part AG. They that would
fee theſe and fuch like things demonſtrated, may
conſult my large book of Mechanics, to which I
refer the reader.

PROP. LIX.

*If feveral pieces of timber be applied to any me-
chanical uſe where ſtrength is required ; not only
the parts of the ſame piece, but the feveral pieces in
regard to one another, ought to be ſo adjuſted for big-
nefs, that the ſtrength may be always proportional to
the ſtreſs they are to endure.*

This Prop. is the foundation of all good Mecha-
- niſm, and ought to be regarded in all forts of
tools and inſtruments we work with, as well as in
the feveral parts of any engine. For who that is
wiſe, will overload himſelf with his work tools, or
make them bigger and heavier than the work re-
quires ? neither ought they to be fo flender as not to
be able to perform their office. In all engines, it muſt
be conſidered what weight every beam is to carry,
and proportion the ſtrength accordingly. All le-

Fig. vers muſt be made ſtrongeſt at the place where they are ſtrained the moſt; in levers of the firſt kind, they muſt be ſtrongeſt at the ſupport. In thoſe of the ſecond kind, at the weight. In thoſe of the third kind, at the power, and diminiſh proportionally from that point. The axles of wheels and pullies, the teeth of wheels, which bear greater weights, or act with greater force, muſt be made ſtronger. And thoſe lighter, that have light work to do. Ropes muſt be ſo much ſtronger or weaker, as they have more or leſs tenſion. And in general, all the parts of a machine muſt have ſuch a degree of ſtrength as to be able to perform its office, and no more. For an exceſs of ſtrength in any part does no good, but adds unneceſſary weight to the machine, which clogs and retards its motion, and makes it languid and dead. And on the other hand, a defect of ſtrength where it is wanted, will be a means to make the engine fail in that part, and go to ruin. So neceſſary it is to adjuſt the ſtrength to the ſtreſs, that a good mechanic will never neglect it; but will contrive all the parts in due proportion, by which means they will laſt all alike, and the whole machine will be diſpoſed to fail all at once. And this will ever diſtinguiſh a good mechanic from a bad one, who either makes ſome parts ſo defective, imperfect and feeble as to fail very ſoon; or makes others, ſo ſtrong or clumſey, as to out laſt all the reſt.

From this general rule follows

Cor. 1. *In ſeveral pieces of timber of the ſame ſort, or in different parts of the ſame piece; the breadth multiplied by the ſquare of the depth, muſt be as the length multiplied by the weight to be borne.*

For then the ſtrength will be as the ſtreſs.

Cor. 2. *The breadth multiplied by the ſquare of the depth, and divided by the product of the length and weight, muſt be the ſame in all.* Cor.

Cor. 3. *Hence may be computed the strength of* Fig. *timber, proper for several uses in building.* As,

1. To find the dimensions of joists and boards for flooring. Let b, d, l be the breadth, depth and length of a joist, $n =$ number of them, $x =$ their distance, $g =$ depth of a board, $w =$ weight; then $nbdd =$ strength of all the joists, and $wl =$ stress on them, also $nlgg =$ strength of the boards, and wx their stress; therefore $\dfrac{nbdd}{wl} = \dfrac{nlgg}{wx}$; and $x = \dfrac{llgg}{bdd}$, for the distance of the joists, or the length of a board between them. Or $b = \dfrac{llgg}{ddx}$, or $dd = \dfrac{llgg}{bx}$, and so on, according to what is wanted.

2. To find the dimensions of square timber for the roof of a house. Let r, s, l be the length of the ribs, spars and lats, so far as they bear; x, y, z their breadth or depth, n the distances of the lats, $w =$ weight upon a rib, $c =$ cosine of elevation of the roof. Then by reason of the inclined plane, $\dfrac{lw}{r} \times c =$ weight upon a spar. And $\dfrac{lnw}{rs} =$ weight upon a lat: for the ribs and lats lie horizontally. Therefore (Cor. 2.) $\dfrac{x^3}{wr} = \dfrac{y^3}{\frac{slw}{r} \times c} = \dfrac{z^3}{l \times \frac{lnw}{rs}}$.

Whence $x^3 = \dfrac{rry^3}{cls}$, and $x^3 = \dfrac{rrsz^3}{lln}$. Hence if any one x, y, or z be given, and all the rest of the quantities; the other two may be found. Or in general, any two being unknown, they may be found, from having the rest given.

For example, let $r = 9$ feet, $s = 4$ feet, $l = 15$ inches, $n = 11$ inches, $c = .707$ the cosine of $45°$, the pitch of the roof. And assume $y = 2\frac{1}{4}$

inches;

inches; then $x = 2\frac{1}{2}\sqrt[3]{\dfrac{.81}{3.535}} = 7.1$ inches. And

$z = y\sqrt[3]{\dfrac{ln}{css}} = 2\frac{1}{2}\sqrt[3]{\dfrac{55}{543}} = 1\frac{1}{8}$ inches.

54.   3. To find the curve ACB, into the form of which, if a joift be cut, on the upper or under fide ; and having the two fides parallel planes, which are perp. to the horizon. That the faid joift fhall be equally ftrong every where to bear a given weight, fufpended on it.

Let the weight be placed in the ordinate CD ; and the breadth of the beam, and the weight being given ; then (Prop. LIV.) the ftrength at C is as CD². And (Prop. LVI.) the ftrefs is as ADB. Therefore that the ftrength may be as the ftrefs, CD² is as the rectangle ADB ; and therefore the curve ACB is an ellipfis.

55.   4. To find the figure of a beam AB, fixed with one end in a wall, and having a given weight W fufpended at the other end B ; and being every where of the fame depth; it may be equally ftrong throughout.

Let CD be the breadth at C ; then (Prop. LIV.) the ftrength is as CD. And (Cor. 1. Prop. LV.) the ftrefs is as CB. Therefore CD is every where as CB, and therefore CDB is a plane triangle. And the beam is a prifm, whofe upper and under fides are parallel to the horizon.

56.   5. To find the figure of a beam AB, fticking with one end in a wall, and of a given breadth ; having a weight W fufpended at the end B ; fo that it may be equally ftrong throughout.

Let CD be the depth at C. Then fince the breadth is given, the ftrength is as CD². And the ftrefs as DB ; therefore CD² is as DB. Whence CD is a common parabola.

57.   6. To find the figure of a beam AB, of the fame breadth and depth, fticking in a wall with
                                 one

one end, and bearing a weight fufpended at the Fig.
other end B; fo that it may be equally ftrong 57.
throughout.

Let CD be the thicknefs at O. Then the ftrength
is as CD³, and the ftrefs is as BO. Therefore BO
is as CD³ or as CO³. And confequently ACB is
a cubic parabola, whofe vertex is at B.

7. In like manner, if CDB be a beam fixed with 58.
one end in a wall, and all the fides of it be cut
into the form of a concave parabola, whofe vertex
is at B.   It will be equally ftrong throughout for
fupporting its own weight.

For putting $BO = x$, $CO = y$, then by nature
of the curve, $ay = xx$. But the folidity of CBD is
$\frac{3.1416yyx}{5}$. And the center of gravity J, is diftant
$\frac{1}{6}x$ from B, therefore $OI = \frac{1}{6}x$.   Now CD³ or

$8y^3 = $ ftrength at O. And CBD × OI or $\frac{3.1416yyx}{5}$ ×
$\frac{1}{6}x = $ ftrefs. Therefore the ftrength : to the ftrefs : :
is as $8y^3$: to $\frac{3.1416y^2xx}{30}$: : $8y : \frac{3.1416xx}{30}$: : $8y$:

$\frac{3.1416ay}{30}$: : $340 : 3.1416a$, that is, in a given
ratio.   And as as this happens every where, the
folid is equally ftrong in all parts.

I muft take notice here that the 116th figure in
my large book of Mechanics, is drawn wrong. It
fhould be concave inftead of being convex.

8. Again, if AB be the fpire of a church which 59.
is a folid cone or pyramid; it will be equally ftrong
throughout for refifting the wind.   For the quan-
tity of wind falling on any part of it ACD, will
be as the fection ACD.   Therefore let $AO = x$,
$CD = y$, and $x = ay$; then the ftrength at $O = $
y^3, and if I be the center of gravity of ACD, then

Fig. $OI = \frac{1}{3}x$. And the ſtreſs at O = wind $ACD \times$
59. $OI = xy \times \frac{1}{3}x$. Therefore the ſtrength is to the
ſtreſs : : as y^3 : $\frac{1}{3}xzy$: : yy : : $\frac{1}{3}xx$ or $\frac{1}{3}aayy$: : 3 :
aa; that is, in a given ratio. Therefore the ſpire
is equally ſtrong every where.

SCHOLIUM.

It is all along ſuppoſed that the timber, &c. is
of equal goodneſs, where theſe proportions for
ſtrength are made.   But if it is otherwiſe, a pro-
per allowance muſt be made for the defeſt.

In theſe Propoſitions, I have called every thing
Strength, that contributes in a direſt proportion to
reſiſt any force aſting againſt a beam to break it;
and I call *Streſs*, whatever weakens it in a direſt
proportion.   But the whole may be referred to
the article of ſtrength; for what I have called ſtreſs
may be reckoned ſtrength in an inverſe ratio. Thus
the ſtrength of a piece of timber·may be ſaid to
be direſtly as the breadth and ſquare of the depth,
and inverſely as its length, and the weight or force
applied; and that is equivalent to taking in the
ſtreſs.   But I had rather keep them diſtinſt, and
refer to each of them their proper effeſts, as I
have all along done in the foregoing examples.

A piece of wood a foot long, and an inch ſquare,
will bear as follows; oak from 320 to 1100;· elm
from 310 to 930; fir from 280 to 770 pounds,
according to the goodneſs.

PROP.

PROP. LX.

In any machine contrived to raife great weights; if the power applied, be to the weight to be raifed ; as the velocity of the weight, to the velocity of the power ; the power will only be in equilibrio with the weight. Therefore to raife it, the power muft be fo far increafed, as to overcome all the friction and re-fiftance arifing from the engine or otherwife ; and then the power will be able to raife the weight.

A man would be much miftaken, who fhall make an engine to raife a great weight, and give his power no greater velocity, in regard to the ve-locity of the weight, than the quantity of the weight has in regard to the quantity of the power. For when he has done that, his weight and power will but have equal quantities of motion, and therefore they cannot fet one another a moving, but muft always remain at reft. It is neceffary then, that he do one of thefe two things. 1. That he apply a power greater than in that proportion, fo much as to overcome all the friction and other ac-cidental refiftance that may happen : and in fome engines thefe are very great. Or 2. He muft fo continue his engine, that the velocity of the power, which fuppofe he has given, may be fo much greater than the velocity of the weight ; as the quantity of the weight, friction, and refiftance and all together, is greater than the power. This be-ing done, the greater power will always overcome the leffer, and his engine will work.

If a man does not attend to this rule, he will be guilty of many abfurd miftakes, either in attempt-ing things that are impoffible, or in not applying means proper for the purpofe. Hence it is that engines contrived for mines and water-works fo of-

ten

Fig. ten fail ; as they muſt when either the quantity or
velocity of the power is too little ; or which is the
ſame thing, when the velocity of the weight is too
great, and therefore would require more power
than what is propoſed.   As the weight is to move
ſlow, the conſequence is, that it will be ſo much a
longer time in moving through any ſpace. But there
is no help for that.   For as much as the weight to
be raiſed is the greater, the time of raiſing it will
be ſo much greater too.

Cor. 1. *Hence in raiſing any weight, what is gain-*
ed in power is loſt in time. Or the time of riſing
through any height will be ſo much longer as the
weight is greater.

If the power be to the weight as 1 to 20; then
the ſpace through which the weight moves will be 20
times leſs, and the time will be 20 times longer in
moving through any ſpace, than that of the power.
The advantage that is gained by the ſtrength of
the motion, is loſt in the ſlowneſs of it.   So that
though they increaſe the power, they prolong the
time.   And that which one man may do in 20 days,
may be done by the ſtrength of twenty men in one
day.

Cor. 2. *The quantity of motion in the weight is not*
at all increaſed by the engine.   And if any given
quantity of power be immediately applied to a body at
liberty, it will produce as much motion in it, as it
would do by help of a machine.

PROP. LXI.

If an engine be compoſed of ſeveral of the ſimple
mechanic powers combined together ; it will produce
the ſame effect, ſetting aſide friction ; as any one ſim-
ple mechanic power would do, which has the ſame
power or force of acting.

For let any compound engine be divided into all
the ſimple powers that compoſe it.   Then the force
or

or power applied to the firft part, will caufe it to Fig.
act upon the fecond with a new power, which
would be deemed the weight, if the machine had
no more parts.   This new power acting on the fe-
cond part, will caufe it to act upon the third part :
and that upon a fourth, and fo on till you come
at the weight, which will be acted on, by all thefe
mediums, juft the fame as by a fimple machine,
whofe power is equal to them all.

    Cor. 1. *Hence a compound machine may be made,*
which fhall have the fame power, as any fingle one
propofed.
    For if a lever is propofed whofe power is 100 to
1 ; two levers acting on one another will be equi-
valent to it, where the power of the firft is as 10
to 1, and that of the fecond alfo as 10 to 1, or the
firft 20 to 1, and the fecond 5 to 1 ; or any two
numbers, whofe product is 100.
    Again, a wheel and axle whofe power is as 48
to 1, may be refolved into two or more wheels
with teeth, to have the fame power; for exam-
ple, make two wheels, fo that the firft wheel and
pinion be as 8 to 1, and the fecond as 6 to 1.
They will have the fame effect as the fingle one. Or
break it into three wheels, whofe feveral powers
may be 4 to 1, and 4 to 1, and 3 to 1.
    If a fimple combination of pullies be as 36 to
1 ; you may take three combinations to act upon
one another, whofe powers are 3 to 1, 3 to 1, and
4 to 1.
    And after the fame manner it is to be done in
machines more compounded.
    And this is generally done to fave room.   For
when an engine is to have great power, it is hard-
ly made of one wheel, it would be fo large; but
by breaking it into feveral wheels, after this man-
ner, it will go into a little room, and have the

fame

Fig. fame power as the other. All the inconvenience is, it will have more friction; for the more parts acting upon one another, the more friction is made.

Cor. 2. *Hence also it follows, that in any compound machine, its power is to the weight, in the compound ratio of the power to the weight in all the simple machines that compose it.*

Cor. 3. *Hence it will be no difficult matter to contrive an engine that shall overcome any force or resistance assigned.*

For if you have the quantity of power given, as well as of the weight or resistance; it is but taking any simple machine, as a lever, wheel, &c. so that the power may be to the weight in the ratio assigned, adding as much to the weight as you judge the friction will amount to. When this simple machine is obtained; break it or resolve it into as many other simple ones as you think proper; so that they may have the fame power.

And as to the several simple machines, it matters not what fort they are of, as to the power; whether they be levers, wheels, pullies, or screws; but some are more commodious than others for particular purposes; which a mechanic will find out beft by practice. In general, a lever is the moft ready and fimple machine to raife a weight a fmall diftance; and for further diftances, the wheel and axle, or a combination of pullies; or the perpetual fcrew. Alfo thefe may be combined with one another; as a lever with a wheel or a fcrew, the wheel and axle with pullies, pullies with pullies, and wheels with wheels, the perpetual fcrew and the wheel. But in general a machine fhould confift of as few parts as is confiftent with the purpofe it is defigned for, upon account of leffening the friction; and to make it ftill lefs, the joints muft

be

be oiled or greafed.   All parts that act on one
another muft be polifhed fmooth.   The axles or
fpindles of wheels muft not fhake in the holes, but
run true and even. Likewife the larger a machine
is, if it be well executed, the better and truer it
will work.   And large wheels and pullies, and
fmall axles or fpindles have the leaft friction.

The power applied to work the engine may be
men or horfes; or it may be a weight or a fpring;.
or wind, water, or fire; of which one muft take
that which is moft convenient and cofts the leaft.
Wind and water are beft applied to work large
engines, and fuch as muft be continually kept
going.   A man may act for a while againft a
refiftance of 50 pounds; and for a whole day againft
30 pounds. A horfe is about as ftrong as five men.

If two men work at a roller, the handles ought
to be at right angles to one another.

When a machine is to go regular and uniform,
a heavy wheel or fly muft be applied to it.

<div align="center">SCHOLIUM.</div>

Two things are required to make a good engi-
neer. 1. A good invention for the fimple and eafy
contrivance of a machine, and this is to be at-
tained by practife and experience.   2. So much
theory as to be able to compute the effect any
engine will have; and this is to be learned from
the principles of Mechanics.

<div align="center">PROP. LXII.</div>

The friction or refiftance arifing by a body moving
upon any furface, is as the roughnefs of the furface, and
nearly as the weight of the body; but is not much in-
creafed by the quantity of the furface of the moving
body, and is fomething greater with a greater velocity.

It is matter of experience that bodies meet with
a great deal of refiftance by fliding upon one ano-
ther,

Fig. ther, which cannot be entirely taken away, though the bodies be made never so smooth: yet by smoothing or polishing their surfaces, and taking off the roughnefs of them, this refiftance may be reduced to a fmall matter. But many bodies, by their natural texture, are not capable of bearing a polifh; and thefe will always have a confiderable degree of refiftance or friction. And thofe that can be polifhed, will have fome of this refiftance arifing from the cohefion of their furfaces. But in general, the fmoother or finer their furfaces, the lefs the friction will be..

As the furfaces of all bodies are in fome degree rough and uneven, and fubject to many inequalities; when one body is laid upon another, the prominent parts of one fall into the hollows of the other; fo that the body cannot be moved forward, till the prominent parts of one be raifed above the prominent parts of the other, which requires the more force to effect, as thefe parts are higher; that is, as the body is rougher. And this is fimilar to drawing a body up an inclined plane, for thefe protuberances are nothing elfe but fo many inclined planes, over which the body is to be drawn. And therefore the heavier the body, the more force is required to draw it over thefe eminences; whence the friction will be nearly as the weight of the body.

But whilft the roughnefs remains the fame, or the prominent parts remain of the fame height, there will always be required the fame force, to draw the fame weight. And the increafing of the furface, retaining the fame weight, can add nothing to the refiftance on that account; but it will make fome addition upon other accounts. For when one furface is dragged along another, fome part of the refiftance arifes from fome parts of the moving furface, taking hold of the parts of the other, and tearing them off; and this is called *wearing*. And
therefore

therefore this part of the friction is greater in a Fig.
greater surface, in proportion to that surface. There
is likewise in a greater surface, a greater force of
cohesion, which still adds something to the friction.
But the two parts of the friction, arising from the
wearing and tenacity, are not increased by the ve-
locity: but the other part, of drawing them over
inclined planes, will increase with the velocity. So
that in the whole, the friction is somewhat increased
by the quantity of the surface, and by the ve-
locity, but not much.   But more in some bodies
than others, according to their particular texture.

Cor. 1. *Hence there can be no certain rule, to esti-
mate the friction of bodies ; this is a matter that can
only be decided by experiments. But it may be observed,
that,* cæteris paribus, *hard bodies will have less
resistance than softer; and bodies oiled or greased, will
have far less.*

For the particles of hard bodies, cannot so well
take hold of one another to tear themselves off.
And when a surface is oiled, it is the same thing as
if it run upon a great number of rollers or spheres.

Cor. 2. *Hence also a method appears of measuring
the friction of a body sliding upon another body, by
help of an inclined plane.*

Take a plank CB of the same matter, raise it at 60.
one end C so high, till the body whose friction is
sought, being laid at C, shall just begin to move
down the plane CB.   Then the weight of the
body, is to the friction, as the base AB to the
height AC of the plane.   For the pressure against
the plane is the part of the weight that causes the
friction, and the tendency down the plane is equal
to the friction.   And (Prop. XIV.) that pressure
is to the tendency as AB to AC.

                                                  If

Fig.
60.

If you puſh the body from C downward, and obſerve it to keep the ſame velocity through D to B; then you will have the friction for that velocity. If it increaſes its velocity, lower the end of the plank C; if it grows ſlower, raiſe the end C till you get the body to have the ſame velocity quite through the plane. And ſo you will find what elevations aje proper for each velocity; and from thence the ratio of AB to AC, or of the weight to the friction.

There is a way to make the experiment, by drawing the body along a horizontal plane, by weights hung at a ſtring, which goes over a pulley; but the method here deſcribed is more eaſy and ſimple.

Scholium.

From what has been before laid down, it will be eaſy to underſtand the nature of engines, and how to contrive one for any purpoſe aſſigned. And likewiſe having any engine before us, we can by the ſame rules, compute its powers and operations.

Engines are of various kinds; ſome are fixed in a particular place, where they are to act; as windmills and water-mills for corn, fire engines for drawing water, gins for coal pits, many ſorts of mills; pumps, cranes, &c. others are movable from one place to another, and may be carried to any place where they are wanted, as blocks, pullies and tackles for raiſing weights, the lifting jack, and lifting ſtock, clocks, watches, ſmall bellows, ſcales, ſteelyards, and an infinite number of others. Another ſort of engines are ſuch as are made on purpoſe to move from one place to another, ſuch as boats, ſhips, coaches, carriages, waggons, &c. If any of theſe are urged forward by the help of levers, wheels, &c. By having the acting power given, the moving force that drives it forward, is eaſily found by the properties of
theſe

46.

B   F

E   I   F   D

A   E   C   47.

I

F   B

D

F   G   A

H   50.

W

C

53.

P

A   D   B

Q

C

F

B

P

G

Q

B   A   B   A   C   57.

C   O   I   B

D   D

W   56.   W   D

W

c

60.   D

A   B

Pl. IV. ралов of
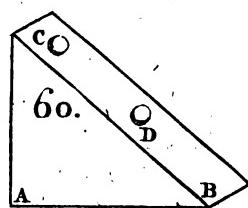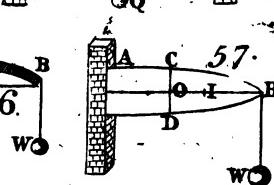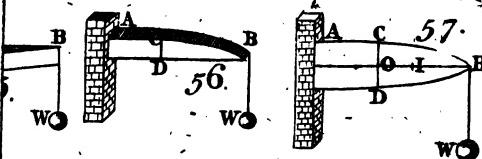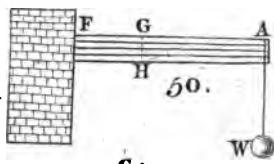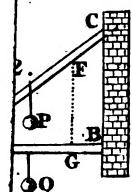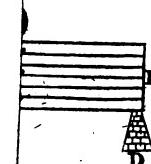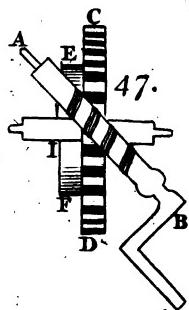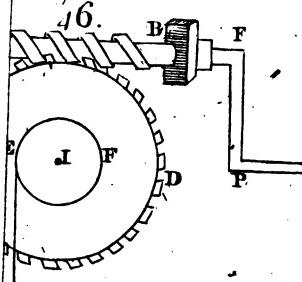
thefe machines.   Only obferve, if the firft acting Fig.
power be external, as wind, water, horfes, &c.
you muft not forget to add or fubtract it, to or
from the moving force before found ; according as
that firft acting power confpires with, or oppofes
the motion of the machine ; and the refult is the
true force it is driven forward with.   I have only
room to defcribe a very few engines, but thofe that
defire it may fee great variety in my large book of
Mechanics.

A WHEEL CARRIAGE.

   AB is a cart or carriage, going upon two wheels 61.
as CD, and fometimes upon four, as all waggons
do.   The advantages of wheel carriages is fo great,
that no body who has any great weight to carry,
will make ufe of any other method.   Was a great
weight to be dragged along upon a fledge or any
fuch machine without wheels, the friction would be
fo great, that a fufficient force in many cafes could
not be got to do it. But by applying wheels to carri-
ages, the friction is almoft all of it taken away.
And this is occafioned by the wheel's turning round
upon the ground, inftead of dragging upon it. And
the reafon of the wheel's turning round is the refif-
tance the earth makes againft it at O where it
touches. For as the carriage goes along, the wheel
meets with a refiftance at the bottom O, where it
touches the ground ; and meeting with none at the
top at C, to balance it ; that force at O muft make
it turn round in the order ODC, fo that all the
parts of the circumference of the wheel are fuccef-
fively applied to the earth.   In going down a fteep
bank it is often neceffary to tie one wheel faft, that
it cannot turn round, this will make it drag ; and
by the great refiftance it meets with, ftops the too
violent motion, the carriage would otherwife have,
in defcending the hill.

                                        But

Fig.   But although all forts of wheels very much dimi-
61. nifh the friction; yet fome have more then others;
and it may be obferved that great wheels, and fmall
axles have the leaft friction.   To make the friction
as little as poffible, fome have applied friction
wheels, which is thus; EG is the friction wheel
running upon an axis I which is fixed in the piece
of timber ES, which timber is fixed to the fide of
the carriage.   KL is the axle of the carriage,
which is fixed in the wheel CD, fo that both
turn round together. Then inftead of the carriage
lying upon the axle KL, the friction wheel FG
lies upon the axle; fo that when the wheel CD
turns round, the axle caufes the friction wheel,
with the weight of the carriage upon it, to turn
round the center I, which diminifhes the friction in
proportion to the radius IG: and there is the fame
contrivance for the wheel on the other fide.   But
the wheel CD need not be fixed to the axle; for
it may turn round on the axle KL, and alfo the
axle turn round under the carriage.

In paffing over any obftacles, the large wheels
have the advantage.   For let MN be an obftacle;
then drawing the wheel over this obftacle, is the
fame thing as drawing it up the inclined plane MP,
which is a tangent to the point M; but the greater
the wheel CD is, the lefs is that plane inclined to
the horizon.

Likewife great wheels do not fink fo deep into
the earth as fmall ones, and confequently require
lefs force to pull them out again.

But there are difadvantages in great wheels ; for
in the firft place they are more eafily overturned ;
and fecondly, they are not fo eafy to turn with,
in a ftrait road as fmall wheels.

The tackle of any carriage ought to be fo fixed,
that the horfe may pull partly upwards, or lift, as
well as pull forwards; for all hills and inequalities
in

in the road, being like so many inclined planes, Fig.
the weight is moft eafily drawn over them, when 61.
the power draws at an equal elevation.

A carriage with four wheels is more advan-
tageous, than one with two only, but they are bad
to turn; and therefore are obliged to make ufe of
fmall fore wheels.   Broad wheels which are lately
come into fafhion, are very advantageous, as they
fink but little into the earth.   But there is a dif-
advantage attends them, for they take up fuch a
quantity of dirt by their great breadth, as fenfibly
retards the carriage by its weight, and the like
may be faid of their own weight.

. The under fide of the axle where the wheels are,
muft be in a right line; otherwife if they flant
upwards, the weight of the carriage will caufe them
to work toward the end, and prefs againft the
runners and lin pin.   And as the ends of the axle
are conical, this caufes the wheels to come nearer
together at bottom, and be further diftant at the
top; by which means the carriage is fooner over-
turned.   To help this, the ends of the axle muft
be made as near a cylindrical form as poffible, to
get the wheels to fit, and to move free.

A HAND MILL,

Fig 62. is a hand mill for grinding corn, A, B 62.
the ftones included in a wooden cafe.   A the up-
per ftone, being the living or moving ftone. B the
lower ftone, or the dead ftone, being fixed immov-
able.   The upper ftone is 5 inches thick, and a
foot and three quarters broad ; the lower ftone is
broader.   C is a cog-wheel, with 16 or 18 cogs ;
DE its axis.   F is a trundle with 9 rounds, fixed
to the axis G, which axis is fixed to the upper
ftone A, by a piece of iron made on purpofe.   H
is the hopper, into which the corn is put; I the fhoe,
to carry the corn by little and little through a hole

at

Fig.
62. at K, to fall between the two ftones. L is the mill
eye, being the place where the flour or meal comes
out after it is ground. The under ftone is fup-
ported by ftrong beams not drawn here. And the
fpindle G ftands on the beam MN, which lies up-
on the bearer O, and O lies upon a fixed beam
at one end, and at the other end has a ftring fixed,
and tied to the pin P. The under ftone is not flat,
but rifes a little in the middle, and the upper one
is a little hollow. The ftones very near touch at
the outfide, but are wider towards the middle to
let the corn go in.

When corn is to be ground, it is put into the
hopper H, a little at a time, and a man turns the
handle D, which carries round the cog-wheel C,
and this carries about the trundle F, and axis G,
and ftone A. The axis G is angular at K; and
as it goes round, it fhakes the fhoe I, and makes
the corn fall gradually through the hole K. And
the upper ftone going round grinds it, and when
ground it comes out at the mill eye L, where there
is a fack or tub placed to receive it. Another
handle may be made at E like that at D, for two
men to work, if any one pleafes. In order to
make the mill grind coarfer or finer, the upper
ftone A may be lowered or raifed, by means of the
ftring going from the bearer O; for turning round
the pin P, the ftring is lengthened or fhortened,
and thereby the timbers O, M are lowered or
raifed, and with them the axle G and ftone A.
For the fpindle G goes through the ftone B, and
runs upon the beam MN. The fpindle is made
fo clofe and tight, by wood or leather, where it goes
through the under ftone, that no meal can fall
through. The under fide of the upper ftone is cut
into gutters in the manner reprefented at Q. It is a
pity fome fuch like mills are not made at a cheap
rate for the fake of the poor, who are much dif-
treffed by the roguery of the millers.   Fig.

C F E

I

S

L G

B

K

D

M

N P O

Q

62.

C

D

N
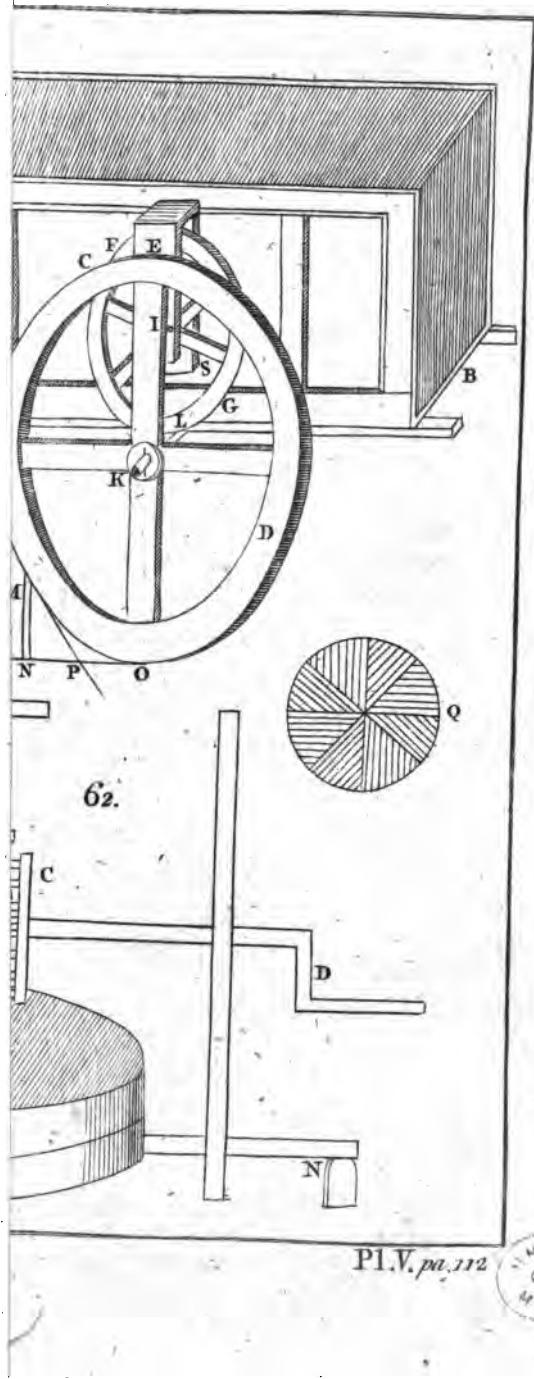
Fig. 63. is a fort of crane, BC an upright poft, Fig.
AB a beam fixed horizontally at top of it ; thefe 63.
turn round together on the pivot C, and within
the circle S, which is fixed to the top of the frame
PQ. EF is a wooden roller, or rather a roller
made of thin boards, for lightnefs, and all nailed
to feveral circular pieces on the infide. GH a
wheel fixed to the roller, about which goes the rope
GR. IK, LN, two other ropes; fixed with one
end to the crofs piece AB, and the other end to
the roller EF. W a weight equal to the weight
of the wheel and roller, which is faftened to a rope
which goes over the pulley O, and then is faftened
to a collar V, which goes round the roller. ET
is another rope with a hook at it to lift up any
weight, the other end of the rope being fixed to
the roller; here are in all five ropes.

To raife any weight as M, hang it upon the
hook T, then pulling at the rope R which goes
about the wheel GH, this caufes the wheel and
roller to turn round, and the ropes IK, LN to
wind about it, by which means the wheel and axle
rifes ; and by rifing, folds the rope TE about the
roller the contrary way, and fo raifes the weight
M. When the weight M is raifed high enough, a
man muft take hold of the rope T with a hook,
by which the whole machine may be drawn about,
turning upon the centers C and S. And then the
weight M may be let down again. The weight of
the wheel and roller do not affect the power draw-
ing at R, becaufe it is balanced by the weight W.
There is no friction in this machine but what is
occafioned by the collar V, and the bending of the
ropes. And the power is to the weight in this
crane, as the diameter of the roller to the radius
of the wheel GH.

H   An

Fig.   An ENGINE for raifing WEIGHTS.

64.   Fig. 64. is an engine compofed of a perpetual
fcrew AB, and a wheel DE with teeth, and a fin-
gle pulley H. FG is an axle, about which a rope
goes, which lifts the pulley and weight W.   BC is
the winch, to turn'it round withal.   As the fpin-
dle AB is turned about, the teeth of it takes the
teeth of the wheel DE, and turns it about, toge-
ther with the axle FG, which winds up the rope,
and raifes the pulley H, with the weight W. The
power at C, is to the weight W, as diameter FG
 × by the breadth of one tooth, is to twice the di-
ameter DE × circumference of the circle defcribed
by C.

A   FULLING   MILL.

65.   Fig. 65. is a fulling mill.   AB a great water
wheel, carried about by a ftream of water, com-
ing from the trough C, and falling into the buck-
ets D, D, D whofe weight carries the wheel about;
this is a breaft mill, becaufe the water comes no
higher than the middle or breaft of the wheel ;
EF is its axis ; I, I ; K, K, two lifters going
through the axle, which raife the ends G, G of the
wooden mallets GH, GH, as the wheel goes about;
and when the end G flips off the cog or lifter K
or I, the mallet falls into the trough L, and each
of the mallets makes two ftrokes for one revolu-
tion of the wheel.   The mallets move about the
centers M, M.   Thefe troughs L, L, contain the
ftuff which is to be milled, by the beating of the
mallets.   N, N, is a channel to carry the water,
being juft wide enough to let the wheel go round.
And the wheel may be ftopt, by turning the
trough C afide, which brings the water.   In this
engine more mallets may be ufed, and then more
pins or lifters muft be put through the axis EF.

Fig.

22

64.

65.
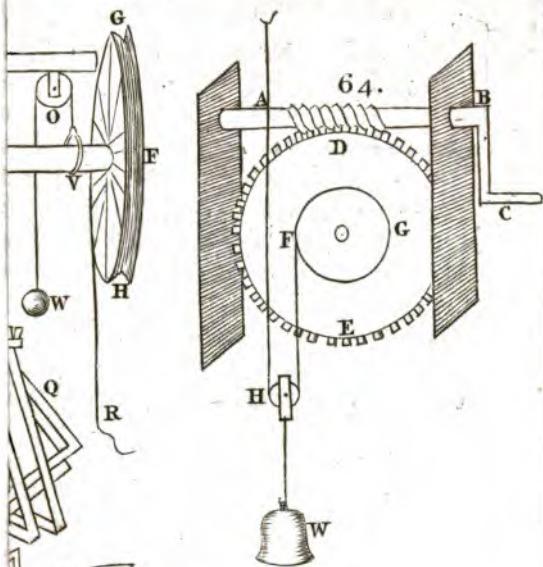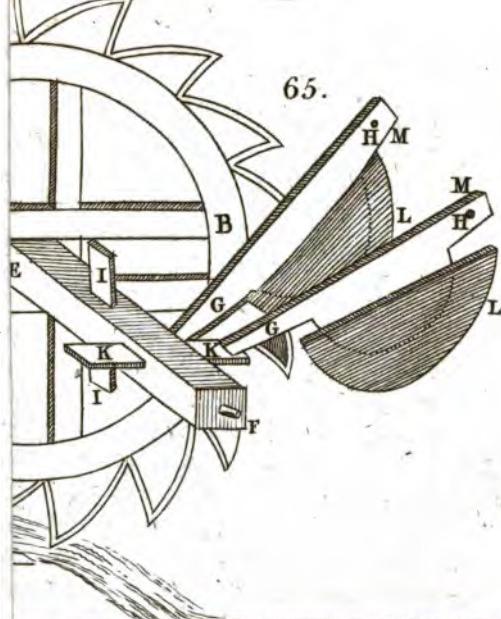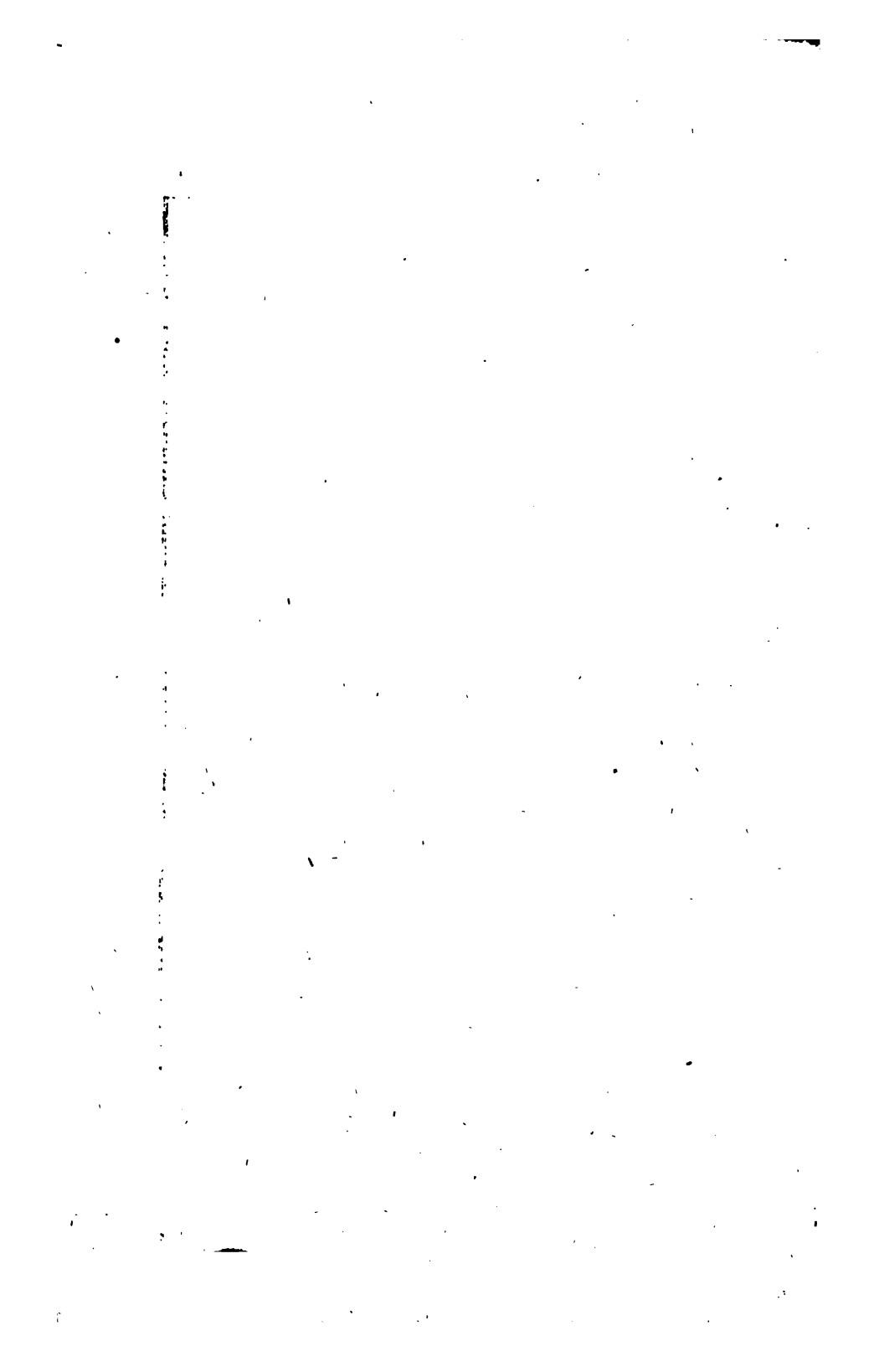
Fig. 66. is a common *Pocket Watch*. AA the *balance*, BB the *verge*; C, C, two *palats*. D the *crown wheel* acting againſt the palats C, C; E its pinion. F the *contrate wheel*, G its pinion. H the *third wheel*, I its pinion. K the ſecond wheel or *center wheel*, L its pinion. M the *great wheel*, N the *fuſee* turning round upon the ſpindle of M. O the *ſpring box*, having a ſpring included in it. PP the *chain* going round the ſpring box O, and the fuſee N. This work is within the watch between the two plates. Here the face is downward, and in the watch the wheel K is placed in the center, and the others round about it. Here I have placed them ſo as beſt to be ſeen, which ſignifies nothing to the motion. The balance AA is without the plate, covered by the cock X. The minute hand Q goes upon the axis of the wheel K.

Then between the upper plate and the face, we have V the *cannon pinion* or *pinion of report*. Z the *dial wheel*. T the *minute wheel*. S the *pinion* or *nut*, fixed to it. The ſocket of the cannon pinion V goes into the ſocket of the wheel Z, and are movable about one another, and both go through the face; on the ſocket of the pinion Z is fixed the hour hand R; and on the ſocket of V is fixed the minute hand Q. Likewiſe the ſocket of V is hollow, and both go upon the arbor of the wheel K, which reaches through the face, and are faſtened there. The wheel and ſocket, T, S are hollow, and go upon a fixed axle on which they turn round.

When the chain PP is wound up, upon the fuſee N; the ſpring included in the box O, draws the chain PP, which forces about the wheel M, the fuſee being kept from ſlipping back, by a catch on purpoſe. Then M drives L and K, and K drives I, and H drives G, and F drives E, and the teeth of the crown wheel D, act againſt the palats C, C alternately, and cauſe the balance AA

Fig. 66.

Fig. to vibrate back and forward, and thus the watch
      is kept going.

66.   The cannon pinion and dial wheel V and Z,
      and the hands Q, R, being put upon the arbor
      of K at W; and faftened there by means of a
      fhoulder which is upon the axis, and a brafs fpring;
      as the wheel K goes round, it carries with it the
      pinion V with the minute hand, and V drives T to-
      gether with S; and S drives Z with the hour hand.

      The numbers of the wheels and pinions, (that
      is the teeth in them) are, M = 48, L = 12, K =
      54, I = 6, H = 48, G = 6, F = 48, E = 6,
      D = 15, and 2 palats. The *train*, or number of
      beats in an hour, is 17280, which is about $4\frac{4}{5}$
      beats in a fecond. Alfo V = 10, Z = 36, S = 12,
   T = 40.

      The wheel M goes round 6 times in 24 hours,

      therefore K goes round $\left(\dfrac{48}{12}\right)$ 4 times as much;

      that is, 24 times, or once in an hour, and the hand
      Q along with it; therefore Q will fhew minutes.
      Then as V goes round once in an hour, T will go

      round $\left(\dfrac{10}{40}\right)$ $\frac{1}{4}$ of that, or $\frac{1}{4}$ the circumference;

      and as S goes $\frac{1}{4}$, Z will go $\left(\dfrac{12}{36}\right)$ $\frac{1}{3}$ of that, or $\frac{1}{12}$

      of the circumferenee in an hour, and therefore as
      R goes along with it, R will fhew the hours. The
      wheels and pinions T, Z, and S, V, are drawn
      with the face upwards. And the whole machine
      included in a cafe is but about two inches diameter.
      There is a fpiral fpring fixed under the balance
      AB, called the *regulator*, which gives it a regu-
      lar motion; and likewife abundance of fmall parts
      helpful to her motion, too long to be defcribed
      here.

                                                    The

The way of writing down the numbers, is thus, Fig.
66.

48
12—54   10Q
  6—48   40—12
    6—48   36R.
      6—15
       2

Explanation. The wheel with 48 drives a pinion
of 12, and a wheel of 54 on the same arbor. The
wheel 54 drives the pinion 6 with the wheel 48 on
the same arbor.   The wheel 48 drives the pinion
6 and wheel 48 on the same arbor.   The wheel
48 drives the pinion 6 and wheel 15 on the same
arbor.   And the wheel 15 drives the two palats.

Again the wheel 54 has the pinion 10 on its
arbor, and the hand Q; and the pinion 10 drives
the wheel 40, with the pinion 12.   And the
pinion 12 drives the wheel 36 with the hand R.

As this machine is moved by a spring, it is sub-
ject to very great inequalities of motion, occasi-
oned by heat and cold. For hot weather so relaxes,
softens, and weakens the main spring, that it loses
a great deal of its strength, which causes the watch
to lose time and go too slow. On the other hand,
cold frosty weather so affects the spring, and it is
so condensed and hardened, that it becomes far
stronger; and by that means accelerates the mo-
tion of the watch, and makes her go faster. The
difference of motion in a watch, thus occasioned
by heat and cold, will often amount to an hour,
and more in 24 hours. To remedy this, there is
a piece of machinery, called the *Slide*, placed near
the regulating spring; which being put forward or
backward, shortens or lengthens the spring, so as
to make her keep time truly.

Some people have been so silly as to think,
that the greater strength of a spring arises wholly

from

Fig.
67.

from its being made ſhorter, as this happens to be one of the effects of cold. But it is eaſily demonſtrated that this is not the cauſe. For let AB be a ſpring as it is dilated by heat, and *ab* the ſame ſpring contracted by cold. Now if the ſpring has been contracted in length, it muſt be proportionally contracted in all dimenſions. Let *l, b, d,* denote the length, breadth, and depth, in its cold, and leaſt dimenſions; and *rl, rb, rd,* the length, breadth, and depth, in its hot and greateſt dimenſions. Then (Prop. LIV.) the ſtrength of the longer, to the ſtrength of the ſhorter, will be as $\dfrac{rb \times rrdd}{rl}$ to $\dfrac{bdd}{l}$ (conſidering it weakened by the length) and that is as *rr* to 1, or as AB^2 to ab^2. So that the longer ſpring, upon account of its being affected with heat, is ſo far from being weaker, than the ſhorter affected with cold, that it is the ſtronger of the two. And therefore this difference is not to be aſcribed merely to the lengthning or ſhortning thereof; but muſt be owing to the nature, texture and conſtitution of the ſteel, as it is ſome way or other affected and changed by the heat and cold.

And that there is ſome change induced by the cold, into the very texture of the metal, it is evident from this, that all ſorts of tools made of iron or ſteel, as ſprings, knives, ſaws, nails, &c. very eaſily ſnap and break in cold froſty weather, which they will not do in hot weather. And that property of ſteel ſprings is the true cauſe, that theſe ſorts of movements can never go true.

66.   To make a calculation of the different forces requiſite to make a watch gain or loſe any number of minutes, as ſuppoſe half an hour in 24; and I have often experienced it to be more. By Cor. 4. Prop. VI. the product of the force and ſquare of

<div align="right">the</div>

Fig. 66.

the time, is as the product of the body and space described, which here is a given quantity. For the matter of the balance remains the same in hot as cold weather; and so does the length of the swing, which here is the space described. Therefore the force is reciprocally as the square of the time of vibrating, or directly as the square of the number of vibrations in 24 hours. Therefore the force with the warm spring, is to the force with the cold one; as the square of 23 ½ hours, to the square of 24; that is, nearly as 23 to 24. So that if a spring was to contract half an inch in a foot in length, without altering its other dimensions, it would but be sufficient to account for that phæ-nomenon; but this is forty times more than the lengthening and shortening by heat and cold, for that does not alter so much as a thousandth part, as is plain from experiments.

The case being thus, a clock or watch going by a spring, can never be made to keep time truly, except it be always kept to the same degree of heat or cold, which cannot be done without constant attendance. And if any sort of mechanism be con-trived to correct this; yet as such a thing can only be made by guess, it cannot be trusted to at sea, but only for short voyages. But no motion however regular, can ever answer at sea, where the irregu-lar motion of the ship will continually disturb it; add to this, that the small compass a watch is con-tained in, makes it easier disturbed, than a larger machine would be; but to suppose that any regu-lar motion can subsist among ten thousand irregular motions, and in ten thousand different directions, is a most glaring absurdity. And if any one with such a machine would but make trial of it to the East Indies, he would find the absurdity and disappoint-ment. And therefore I never expect to see such a time keeper, or any such thing as a watch or clock

going

Fig. going by a fpring, to keep true time at fea. But
66. time will difcover all things.

As to pendulum clocks, their irregularity in the
fame latitude is owing to nothing but the length-
ning or fhortning of the pendulum; which is a
mere trifle to the other. But then they would be
infinitely more difturbed at fea, than a watch; and
in a ftorm could not go at all. In different lati-
tudes too, another irregularity attends a pendulum,
depending on the different forces of gravity.
Though this amounts but to a fmall matter, yet it
makes a confiderable variation, in a great length
of time. For in fouth latitudes, where the gravity
is lefs, a clock lofes time. And in north latitudes,
where the gravity is greater, it gains time. So that
none of thefe machines are fit to meafure time at
fea, although ten times ten thoufand pounds fhould
be given away for making them.

A DESCENDING CLOCK.

68.   Fig. 68. is a clock defcending down an inclined
plane. This confifts of a train of watch work,
contained between two circular plates AB, CD,
4 inches diameter, fixed together by a hoop an
inch and half broad, inclofing all the work. The
inner work confifts of 5 wheels, the fame as in a
watch, only there is a fpur wheel inftead of the
contrate wheel, as 4; *b* is the balance, whofe pa-
lats play in the teeth of the crown wheel 5. Here
is no fpring to give it motion, but inftead thereof,
the weight W is fixed to the wheel 1, and fo ad-
jufted for weight, that it may balance the lower
fide, and hinder it from rolling down the plane.
Now whilft the weight W moves the wheel 1, this
wheel by moving about, caufes the weight W to
defcend, by which it ceafes to be a balance for the
oppofite fide, and therefore that fide begins to de-
fcend,

fcend, till the weight W be raifed high enough Fig.
again to become a balance, which muft be about 68.
the pofition it appears in the figure.   Thus whilft
wheels move gradually about, the weight W def-
cends gradually, which makes the body of the
machine turn gradually round, and defcend down
the inclined plane PQ; making one revolution in
12 hours. And therefore to have her to go 24 or
30 hours; the length of the plane PQ muft be 2
or 2½ circumferences of the plates.   Before the
weight W is· fixed to the wheel 1, fome lead or
brafs muft be foldered on the fide E oppofite to
the wheels 2, 3, 4, &c. for the wheel 1 muft be
in the center.   And then the lead or brafs muft
be filed away till the center of gravity of the ma-
chine be in the center of the plates.   And to hin-
der the machine from fliding, the edges of the
plates muft be lightly indented.   The inclined
plane PQ may be a board, which muft be elevated
10 or 12 degrees, but that is to be found by trials;
for if fhe go too flow the end P muft be raifed;
but if too faft it muft be lowered.   When the
clock has gone the length of the board to Q, it
muft be fet again at P.   The fore fide CD is di-
vided into hours, and a pin is fixed in the center
at G, on which the hand FGH, always hangs
loofely in a perp. pofition, with the heavy end H
downward.   And the end F fhews the hour of
the day.   So that the hours come to the hand,
and not the hand to the hours.

   The board PQ muft be perfectly ftraight from
one end to the other, or elfe fhe will go fafter in
fome places and flower in others.

   The circle with hours ought to be a narrow
rim of brafs, movable round about, by the help
of one or more pins placed in it; fo that it may
be fet to the true time.

                                          The

Fig.   The weight W ferves for two ufes, 1, to be a
68. counterpoife to the fide A; and 2, by its weight
to put the clock in motion.

The weight W muft be fo heavy as to make
the clock keep time, when it has a proper de-
gree of elevation, as 45 degrees; and then the
board muft have an elevation of 10 or 12 de-
grees.   If fhe go too faft, with thefe pofitions,
take fomething off the weight; if too flow, add
fomething to it.

SECT.

68.
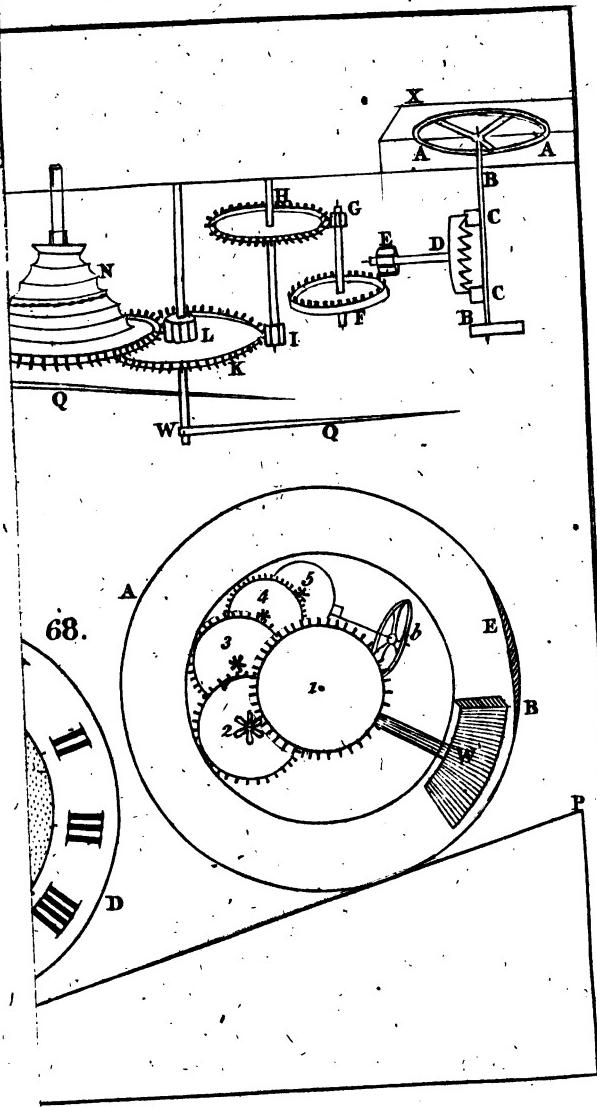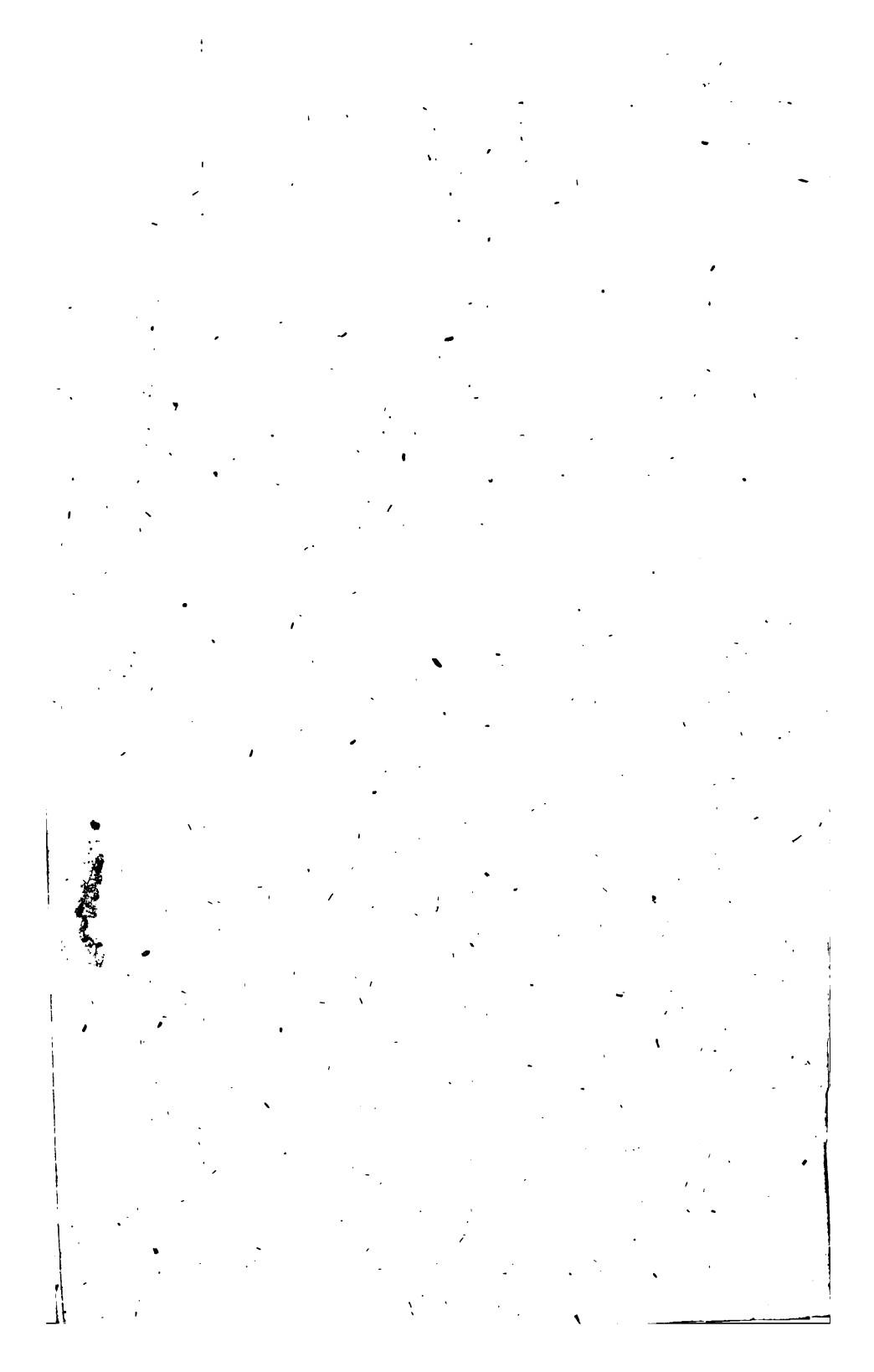
SECT. VI.

HYDROSTATICS *and* PNEUMATICS,

DEFINITION I.

A FLUID is fuch a body whofe parts are eafily moved among themfelves, and yield to any force acting againft them.

DEF. II.

Hydroftatics, is a fcience that demonftrates the properties of fluids.

DEF. III.

Hydraulics, is the art of raifing water by engines.

DEF. IV.

Pneumatics, is that fcience which fhews the properties of the air.

DEF. V.

A fountain or *jet d'eau*, is an artificial fpout of water.

PROP. LXIII.

If one part of a fluid be higher than another, the higher parts will continually defcend to the lower places, and will not be at reft, till the furface of it is quite level.

For the parts of a fluid being movable every way, if any part is above the reft, it will defcend by its own gravity as low as it can get. And afterwards other parts that are now become higher,
will

Fig. will defcend as the other did, till at laft they will all be reduced to a level or horizontal plane.

Cor. 1. *Hence water that communicates by means of a channel or pipe, with other water, will fettle at the fame height in both places.*

Cor. 2. *For the fame reafon, if a fluid gravitates towards a center; it will difpofe itfelf into a fpherical figure, whofe center is the center of force. As the fea in refpect of the earth.*

PROP. LXIV.

If a fluid be at reft in a veffel whofe bafe is parallel to the horizon; equal parts of the bafe are equally preffed by the fluid.

For upon every part of the bafe there is an equal column of the fluid fupported by it. And as all thefe columns are of equal weight, they muft prefs the bafe equally; or equal parts of the bafe will fuftain an equal preffure.

Cor. 1. *All parts of the fluid prefs equally at the fame depth.*

For imagine a plain drawn through the fluid parallel to the horizon. Then the preffure will be the fame in any part of that plane, and therefore the parts of the fluid at the fame depth fuftain the fame preffure.

Cor. 2. *The preffure of a fluid at any depth, is as the depth of the fluid.*

For the preffure is as the weight, and the weight is as the height of a column of the fluid.

PROP.

PROP. LXV.

If a fluid is compreſſed by its weight or otherwiſe; at any point it preſſes equally, in all manner of directions.

This ariſes from the nature of fluidity; which is, to yield to any force in any direction.   If it cannot give way to any force applied, it will preſs againſt other parts of the fluid in direction of that force.   And the preſſure in all directions will be the ſame.   For if any one was leſs, the fluid would move that way, till the preſſure be equal every way.

Cor. *In any veſſel containing a fluid; the preſſure is the ſame againſt the bottom, as againſt the ſides, or even upwards, at the ſame depth.*

PROP. LXVI.

The preſſure of a fluid upon the baſe of the containing veſſel, is as the baſe, and perpendicular altitude; whatever be the figure of the veſſel that contains it.

Let ABIC, EGKH be two veſſels.   Then 69. (Prop. LXIV. Cor. 2.) the preſſure upon an inch on the baſe AB = height CD × 1 inch.   And the preſſure upon an inch on the baſe HK is = height FH × 1 inch.   But (Prop. LXIV.) equal parts of the baſes are equally preſſed, therefore the preſſure on the baſe AB is CD × number of inches in AB; and preſſure on the baſe HK is FH × number of inches in HK.   That is, the preſſure on AB is to the preſſure on HK; as baſe AB × height CD, to the baſe HK × height FH.

Cor. 1. *Hence if the heights be equal, the preſſures are as the baſes.   And if both the heights and baſes be*
$$\textit{equal;}$$

Fig.
69.
*equal; the preſſures are equal in both; though their
contents be never ſo different.*

For the reaſon that the wider veſſel EK, has no
greater preſſure at the bottom, is, becauſe the
oblique ſides EH, GK, take off part of the weight.
And in the narrower veſſel CB, the ſides CA, IB,
re-act againſt the preſſure of the water, which is all
alike at the ſame depth; and by this re-action the
preſſure is encreaſed at the bottom, ſo as to be-
come the ſame every where.

Cor. 2. *The preſſure againſt the baſe of any veſſel,
is the ſame as of a cylinder of an equal baſe and height.*

70.   Cor. 3. *If there be a recurve tube* ABF, *in which
are two different fluids* CD, EF. *Their heights in
the two legs* CD, EF, *will be reciprocally as their
ſpecific gravities, when they are at reſt.*

For if the fluid EF be twice or thrice as light
as CD; it muſt have twice or thrice the height, to
have an equal preſſure, to counterbalance the other.

PROP. LXVII.

71.   *If a body of the ſame ſpecific gravity of a fluid,
be immerſed in it, it will reſt in any place of it. A
body of greater denſity will ſink; and one of a leſs
denſity will ſwim.*

Let A, B, C be three bodies; whereof A is
lighter bulk for bulk than the fluid; B is equal; and
C heavier. The body B, being of the ſame den-
ſity, or equal in weight as ſo much of the fluid,
it will preſs the fluid under it juſt as much as if
the ſpace was filled with the fluid. The preſſure
then will be the ſame all around it, as if the fluid
was there, and conſequently there is no force to
put it out of its place. But if the body be lighter,
<div align="right">the</div>

the preſſure of it downwards will be leſs than be- Fig.
fore, and leſs than in other places at the ſame 71.
depth ; and conſequently the leſſer force will give
way, and it will riſe to the top.   And if the body
be heavier, the preſſure downwards will be greater
than before ; and the greater preſſure will prevail
and carry it to the bottom.

Cor. 1. *Hence if ſeveral bodies of different ſpecific*
gravity be immerſed in a fluid, the heavieſt will get
the loweſt.
For the heavieſt are impelled with a greater
force, and therefore will go faſteſt down.

Cor. 2. *A body immerſed in a fluid, loſes as much*
weight, as an equal quantity of the fluid weighs. And
the fluid gains it.
For if the body is of the ſame ſpecific gravity
as the fluid ; then it will loſe all its weight.   And
if it be lighter or heavier, there remains only the
difference of the weights of the body and fluid,
to move the body.

'Cor. 3. *All bodies of equal magnitudes, loſe equal*
weights in the ſame fluid.   And bodies of different
magnitudes loſe weights proportional to the magnitudes.

Cor. 4. *The weights loſt in different fluids, by im-*
merging the ſame body therein, are as the ſpecific gra-
vities of the fluids.   And bodies of equal weight, loſe
weights in the ſame fluid, reciprocally as the ſpecific
gravities of the bodies.

Cor. 5. *The weight of a body ſwimming in a*
fluid, is equal to the weight of as much of the fluid,
as the immerſed part of the body takes up.
For the preſſure underneath the ſwimming body
is juſt the ſame as ſo much of the immerſed fluid ;
and therefore the weights are the ſame.

<div align="right">Cor.</div>

Fig.
71.
Cor. 6. *Hence a body will fink deeper in a lighter fluid than in a heavier.*

Cor. 7. *Hence appears the reafon why we do not feel the whole weight of an immerfed body, till it be drawn quite out of the water.*

PROP. LXVIII.

72. *If a fluid runs through a pipe, fo as to leave no vacuities ; the velocity of the fluid in different parts of it, will be reciprocally as the tranfverfe fections, in thefe parts.*

Let AC, LB be the fections at A and L. And let the part of the fluid ACBL come to the place *acbl*. Then will the folid ACBL = folid *acbl* ; take away the part *ac*BL common to both ; and we have ACc*a* = LB*bl*. But in equal folids the bafes and heights are reciprocally proportional. But if D*f* be the axis of the pipe, the heights D*d*, F*f*, paffed through in equal times, are as the velocities. Therefore, fection AC : fection LB : : velocity along F*f* : velocity along D*d*.

PROP. LXIX.

73. *If AD is a veffel of water or any other fluid; B a hole in the bottom or fide. Then if the veffel be always kept full; in the time a heavy body falls through half the height of the water above the hole AB, a cylinder of water will flow out of the hole, whofe height is AB, and bafe the area of the hole.*

The preffure of the water againft the hole B, by which the motion is generated, is equal to the weight of a column of water whofe height is AB, and bafe the area B (by Cor. 2. Prop. LXVI.).But equal forces generate equal motions ; and fince a
cylinder

cylinder of water falling through ¼AB by its gra- Fig.
vity, acquires such a motion, as to pass through 73.
the whole height AB in that time; therefore in
that time the water running out muſt acquire the
ſame motion.   And that the effluent water may
have the ſame motion, a cylinder muſt run out
whoſe length is AB; and then the ſpace deſcribed
by the water in that time will alſo be AB, for that
ſpace is the length of the cylinder run out.
Therefore this is the quantity run out in that time.

Cor. 1. *The quantity run out in any time is equal
to a cylinder or priſm, whoſe length is the ſpace deſ-
cribed in that time by the velocity acquired by falling
through half the height, and whoſe baſe is the hole.*
For the length of the cylinder is as the time of
running out.

Cor. 2. *The velocity a little without the hole, is
greater than in the hole; and is nearly equal to the
velocity of a body falling through the whole height AB.*
For without the hole the ſtream is contracted by
the water's converging from all ſides to the cen-
ter of the hole.   And this makes the velocity
greater in about the ratio of 1 to √2.

Cor. 3. *The water ſpouts out with the ſame velo-
city, whether it be downwards, or ſideways, or up-
wards.   And therefore if it be upwards, it aſcends
nearly to the height of the water above the hole.*

Cor. 4. *The velocities and likewiſe the quantities of
the ſpouting water, at different depths; will be as
the ſquare roots of the depths.*

Scholium.

From hence are derived the rules for the con- 74.
ſtruction of fountains or jets. Let ABC be a reſer-
voir of water, CDE a pipe coming from it, to
                               I   bring

Fig. bring water to the fountain which fpouts up at E,
74. to the height EF, near to the level of the refervoir
AB.   In order to have a fountain in perfection, the
pipe CD muft be wide, and covered with a thin
plate at E with a hole in it, not above the fifth or
fixth part of the diameter of the pipe CD.   And
this pipe muft be curve, having no angles.   If the
refervoir be 50 feet high, the diameter of the hole
at E may be an inch, and the diameter of the pipe
6 inches.   In general, the diameter of the hole E,
ought to be as the fquare root of the height of the
refervoir.   When the water runs through a great
length of pipe, the jet will not rife fo high.   A
jet never rifes to the full height of the refervoir; in
a 5 feet jet it wants an inch, and it falls fhort by
lengths which are as the fquares of the heights; and
fmaller jets lofe more. No jet will rife 300 feet high.

75.   A fmall fountain is eafily made by taking a
ftrong bottle A, and filling it half full of water;
cement a tube BI very clofe in it, going near the
bottom of the bottle.   Then blow in at the top B,
to comprefs the air within; and the water will
fpout out at B. If a fountain be placed in the funfhine
and made to play, it will fhew all the colours of
the rainbow, if a black cloth be placed beyond it.

A jet goes higher if it is not exactly perpendi-
cular; for then the upper part of the jet falls to
one fide without refifting the column below.   The
refiftance of the air will alfo deftroy a deal of its
motion, and hinder it from rifing to the height of
the refervoir.   Alfo the friction of the tube or pipe
of conduct has a great fhare in retarding the motion.

78.   If there be an upright veffel as AF full of wa-
ter, and feveral holes be made in the fide as B, C,
D: then the diftances, the water will fpout, upon
the horizontal plane EL, will be as the fquare
roots of the rectangles of the fegments, ABE,
ACE, and ADE.   For the fpaces will be as the
                                    velocities

velocities and times. But (Cor. 4.) the velocity of Fig.
the water flowing out of B, will be as $\sqrt{AB}$, and 78.
the time of its moving (which is the same as the
time of its fall) will be (by Prop. XIII.) as
$\sqrt{BE}$; therefore the distance EH is as $\sqrt{AB \times BE}$;
and the space EL as $\sqrt{ACE}$. And hence if two
holes are made equidistant from top and bottom,
they will project the water to the same distance,
for if AB = DE, then ABE = ADE, which
makes EH the same for both, and hence also it
follows, that the projection from the middle point
C will be furthest; for ACE is the greatest rect-
angle. These are the proportions of the distances;
but for the absolute distances, it will be thus. The
velocity through any hole B, will carry it through
AB in the time of falling through $\frac{1}{2}$AB; then to
find how far it will move in the time of falling
through BE. Since these times are as the square
roots of the heights, it will be, $\sqrt{\frac{1}{2}AB} : AB : :$
$\sqrt{BE} : EH = AB\sqrt{\dfrac{BE}{\frac{1}{2}AB}} = \sqrt{2ABE}$; and so
the space $EL = \sqrt{2ACE}$. It is plain, these curves
are parabolas. For the horizontal motion being
uniform; EH will be as the time; that is, as
$\sqrt{BE}$, or BE will be as EH^2, which is the pro-
perty of a parabola.

If there be a broad vessel ABDC full of water, 76.
and the top AB fits exactly into it; and if the
small pipe FE of a great length be soldered close
into the top, and if water be poured into the top
of the pipe F, till it be full; it will raise a great
weight laid upon the top, with the little quantity
of water contained in the pipe; which weight will
be nearly equal to a column of the fluid, whose
base is the top AB; and height, that of the pipe
EF. For the pressure of the water against the
top AB, is equal to the weight of that column of wa-

Fig.
76. ter, by Prop. LXV. and Cor.　And Prop. LXVI.
Cor. 2.

But here the tube muſt not be too ſmall.　For in capillary tubes the attraction of the glaſs will take off its gravity.　If a very ſmall tube be immerſed with one end in a veſſel of water, the water will riſe in the tube above the ſurface of the water; and the higher, the ſmaller the tube is. But in quickſilver, it deſcends in the tube below the external ſurface, from the repulſion of the glaſs.

To explain the operation of a ſyphon, which is
77. a crooked pipe CDE, to draw liquors off.　Set the ſyphon with the ends C, E, upwards, and fill it with water at the end E till it run out at C; to prevent it, clap the finger at C, and fill the other end to the top, and ſtop that with the finger.　Then keeping both ends ſtopt, invert the ſhorter end C into a veſſel of water AB, and take off the fingers, and the water will run out at E, till it be as low as C in the veſſel; provided the end E be always lower than C.　Since E is always below C, the height of the column of water DE is greater than that of CD, and therefore DE muſt out weigh CD and deſcend, and CD will follow after, being forced up by the preſſure of the air, which acts upon the ſurface of the water in the veſſel AB.

The ſurface of the earth falls below the horizontal level only an inch in 620 yards; and in other diſtances the deſcents are as the ſquares of the diſtances.

And to find the nature of the curve DCG, form-
79. ing the jet IDG.　Let AK be the height or top of the reſervoir HF, and ſuppoſe the ſtream to aſcend without any friction, or reſiſtance.　By the laws of falling bodies the velocity in any place B, will be as $\sqrt{AB}$.　Put the ſemidiameter of the hole at D -- d, and AD $= h$.　Then ſince the ſame water paſſes through the ſections at D and B; therefore (Prop. LXVIII.) the velocity will be reciprocally

as

as the section; whence $\sqrt{b} : \frac{1}{dd} :: \sqrt{AB} : \frac{1}{BC^2}$; Fig. 79.

therefore $\frac{\sqrt{b}}{BC^2} = \frac{\sqrt{AB}}{dd}$, and $dd\sqrt{b} = BC^2\sqrt{AB}$,

whence $AB \times BC^4 = bd^4$; which is a paraboliform figure whose afymptote is AK, for the nature of the cataractic curve DCG. And if the fluid was to defcend through a hole, as IC; it would form itfelf into the fame figure GCD in defcending.

PROP. LXX.

The refiftance any body meets with in moving through a fluid is as the fquare of the velocity.

For if any body moves with twice the velocity of another body equal to it, it will ftrike againft twice as much of the fluid, and with twice the velocity; and therefore has four times the refiftance; for that will be as the matter and velocity. And if it moves with thrice the velocity, it ftrikes againft thrice as much of the fluid in the fame time, with thrice the velocity, and therefore has nine times the refiftance. And fo on for all other velocities.

Cor. *If a ftream of water whofe diameter is given, ftrike againft an obftacle at reft; the force againft it will be as the fquare of the velocity of the ftream.*

For the reafon is the fame; fince with twice or thrice the velocity, twice or thrice as much of the fluid impinges upon it, in the fame time.

PROP. LXXI.

The force of a ftream of water againft any plane obftacle at reft, is equal to the weight of a column of water, whofe bafe is the fection of the ftream; and height, the fpace defcended through by a falling body, to acquire that velocity.

For let there be a refervoir whofe height is that fpace fallen through. Then the water (by Cor. 2.

Fig. Prop. LXIX.) flowing out at the bottom of the refervatory, has the fame motion as the ftream; but this is generated by the weight of that column of water, which is the force producing it. And that fame motion is deftroyed by the obftacle, therefore the force againft it is the very fame : for there is required as much force to deftroy as to generate any motion.

Cor. *The force of a ftream of water flowing out at a hole in the bottom of a refervatory, is equal to the weight of a column of the fluid of the fame height and whofe bafe is the hole.*

PROP. LXXII. *Prob.*

To find the fpecific gravity of folids or fluids.

1. *For a folid heavier than water.*

Weigh the body feparately, firft out of water, and then fufpended in water. And divide the weight out of water by the difference of the weights, gives the fpecific gravity; reckoning the fpecific gravity of water **1**.

For the difference of the weights is equal to the weight of as much water (by Cor. 2. Prop. LXVII.); and the weights of equal magnitudes, are as the fpecific gravities; therefore the difference of thefe weights, is to the weight of the body, as the fpecific gravity of water 1, to the fpecific gravity of the body.

2. *For a body lighter than water.*

Take a piece of any heavy body, fo big as being tied to the light body, it may fink it in water. Weigh the heavy body in and out of water, and find the lofs of weight. Alfo weigh the compound both in and out of water, and find alfo the lofs of weight.

weight.   Then divide the weight of the light Fig.
body (out of water), by the difference of thefe
loffes, gives the fpecific gravity; the fpecific gra-
vity of water being 1.

For the laft lofs is = weight of water equal in
                      magnitude to the com-
                      pound.

And the firft lofs is = weight of water equal in
                      magnitude to the heavy
                      body.

Whence the diff. loffes is = weight of water equal in
                      magnitude to the light
                      body.

and the weights of equal magnitudes, being as the
fpecific gravities; therefore the difference of the
loffes, (or the weight of water equal to the light
body) : weight of the light body : : fpecific gravity
of water 1 : fpecific gravity of the light body.

3. *For a fluid of any fort.*

Take a piece of a body whofe fpecific gravity
you know; weigh it both in and out of the
fluid; take the difference of the weights, and mul-
tiply it by the fpecific gravity of the folid body,
and divide the product by the weight of the body
(out of water), for the fpecific gravity of the fluid.

For the difference of the weights in and out of
water, is the weight of fo much of the fluid as
equals the magnitude of the body. And the weight
of equal magnitudes being as the fpecific gravities;
therefore, weight of the folid : difference of the
weights (or the weight of fo much of the fluid) : :
fpecific gravity of the folid : to the fpecific gravity
of the fluid.

Example to Cafe 1.

I weighed a piece of lead ore, which was 124
grains; and in water it weighed 104 grains, the

Fig. difference is 20; then $\frac{124}{20} = 6.2$, the specific

gravity of the ore.

A table of specific gravities.

Fine gold	19.640
Standard gold	18.888
Quicksilver	14.000
Lead	11.340
Fine silver	11.092
Standard Silver	10.536
Copper	9.000
Copper half-pence	8.915
Gun metal	8.784
Fine brass	8.350
Cast brass	8.100
Steel	7.850
Iron	7.644
Pewter	7.471
Tin	7.320
Cast iron	7.000
Lead ore	6.200
Copper ore	5.167
Lapis calaminaris	5.000
Load stone	4.930
Antimony	4.000
Diamond	3.517
Island cryftal	2.720
Stone, hard	2.700
Rock cryftal	2.650
Glass	2.600
Flint	2.570
Common stone	2.500
Cryftal	2.210
Brick	2.000
Earth	1.984
Horn	1.840

Ivory

Ivory	1.820
Chalk	1.793
Allum	1.714
Clay	1.712
Oil of vitriol	1.700
Honey	1.450
Lignum vitæ	1.327
Treacle	1.290
Pitch	1.150
Rosin	1.100
Mahogany	1.063
Amber	1.040
Urine	1.032
Milk	1.031
Brazil	1.031
Box	1.030
Sea water	1.030
Ale	1.028
Vinegar	1.026
Tar	1.015
Common clear water	1.000
Bees wax	.955
Butter	.940
Linseed oil	.932
Brandy	.927
Sallad oil	.913
Logwood	.913
Ice	.908
Oak	.830
Ash	.830
Elm	.820
Oil of turpentine	.810
Walnut tree	.650
Fir	.580
Cork	.238
New fallen snow	.086
Air	.0012

Cor.

Fig.   Cor. 1. *As the weight loſt in a fluid, is to the abſolute weight of the body; ſo is the ſpecific gravity of the fluid, to the ſpecific gravity of the body.*

Cor. 2. *Having the ſpecific gravity of a body, and the weight of it; the ſolidity may be found thus; multiply the weight in pounds by 62¼. Then ſay as that product to 1; ſo is the weight of the body in pounds, to the content in feet. And having the content given, one may find the weight, by working backwards.*

For a cubic foot of water weighs 62¼lb. averdupois; and therefore a cubic foot of the body weighs 62¼ × by the ſpecific gravity of the body. Whence the weight of the body, divided by that product, gives the number of feet in it. Or as 1, to that product; ſo is the content, to the weight.

SCHOLIUM.

80.   The ſpecific gravities of bodies may be found with a pair of ſcales; ſuſpending the body in water, by a horſe hair. But there is an inſtrument for this purpoſe called the *Hydroſtatical Balance*, the conſtruction of which is thus. AB is the ſtand and pedeſtal, having at the top two cheeks of ſteel, on which the beam CD is ſuſpended, which is like the beam of a pair of ſcales, and muſt play freely, and be itſelf exactly in equilibrio. To this belongs the glaſs bubble G, and the glaſs bucket H, and four other parts E, F, I, L. To theſe are loops faſtened to hang them by. And the weights of all theſe are ſo adjuſted, that E = F + the bubble in water, or = I + the bucket out of water, or = I + L + the bucket in water. Whence L = difference of the weights of the bucket in and out of water. And if you pleaſe you may have a weight K, ſo that K + bubble in water = bubble out of water; or elſe find it in grains.

grains. The piece L has a flit in it to flip it upon Fig.
the fhank of I.   80.

It is plain the weight K = weight of water as big
as the bubble, or a water bubble.

Then to find the specific gravity of a solid.

Hang E at one end of the balance, and I and
the bucket with the folid in it, at the other end;
and find what weight is a balance to it.

Then flip L upon I, and immerge the bucket
and folid in the water, and find again what weight
balances it.   Then the firft weight divided by the
difference of the weights, is the fpecific gravity
of the body; that of water being 1.

For fluids.

Hang E at one end, and F with the bubble at
the other; plunge the bubble into the fluid in the
veffel MN. Then find the weight P which makes
a balance. Then the fpecific gravity of the fluid
is $= \dfrac{K + P}{K}$, when P is laid on F; or $= \dfrac{K - P}{K}$,
when P is laid on E.

For E being equal to I + the bucket; the firft
weight found for a balance, is the weight of the
folid.   Again, E being equal to I + L + the
bucket in water; the weight to balance that, is
the weight of the folid in water; and the difference,
is = to the weight of as much water.   Therefore
(Cor. 1.) the firft weight divided by that difference,
is the fpecific gravity of the body.

Again, fince E is = to F + the bubble in wa-
ter; therefore P is the difference of the weights,
of the fluid and fo much water; that is, P = dif-
ference of K and a fluid bubble; or P = fluid − K,
when the fluid is heavier than water, or when P is
laid on F.   And therefore P = K − the fluid
                                        bubble,

Fig.   Cor. 1. *As the weight loſt in a flu* A bubble
abſolute weight of the body; ſo is th A. And the
of the fluid, to the ſpecific gravity of theſe equal
                                          ſpecific gravity

   Cor. 2. *Having the ſpecifi* $\dfrac{K+P}{K}$ the ſpecific
and the weight of it; the ſoli
multiply the weight in pour
that product to 1; *ſo i* here if P be o, it is the
pounds, to the content
tent given, one m
backwards.   PROP. LXXIII.

   For a cubic
dupois; an   *heavy body, and gravitates on all*
weighs 6   *ſurface of the earth.*

Whenc
prod   *that the* air is a fluid is very plain, as it yields
to   *the* leaſt force that is impreſſed upon it,
         without making any ſenſible reſiſtance.   But if it
         moved briſkly, by ſome very thin and light
         body, as a fan, or by a pair of bellows, we become
     very ſenſible of its motion againſt our hands or
     face, and likewiſe by its impelling or blowing away
     any light bodies, that lie in the way of its motion.
Therefore the air being capable of moving other
bodies by its impulſe, muſt itſelf be a body; and
muſt therefore be heavy like all other bodies, in
proportion to the matter it contains; and will
conſequently preſs upon all bodies placed under it.
And being a fluid, it will dilate and ſpread itſelf
all over upon the earth : and like other fluids will
gravitate upon, and preſs every where upon its
ſurface. The gravity and preſſure of the air is alſo
evident from experiments. For (fig. 70.) if water,
&c. be put into the tube ABF, and the air be
drawn out of the end F by an air-pump, the water
will aſcend in the end F, and deſcend in the end
A, by reaſon of the preſſure at A, which was
taken off or diminiſhed at F. There are number-
leſs experiments of this ſort. And though theſe pro-
                                             perties

₵s and effects are certain, yet the air is a fluid Fig.
fine and subtle, as to be perfectly transpa- 70.
quite invisible to the eye.

> The air, like other fluids, will, by its
> fluidity, insinuate itself into all the cavi-
> ,orners within the earth; and there press
> so much greater force, as the places are deeper.

Cor. 2. *Hence the atmosphere, or the whole body of*
air surrounding the earth, gravitates upon the sur-
faces of all other bodies, whether solid or fluid, and
that with a force proportional to its weight or quan-
tity of matter.

For this property it must have in common with
all other fluids.

Cor. 3. *Hence the pressure, at any depth of water,*
or other fluid, will be equal to the pressure of the fluid
together with the pressure of the atmosphere.

Cor. 4. *Likewise all bodies, near the surface of the*
earth, lose so much of their weight, as the same bulk of
so much air weighs. And consequently, they are something
lighter than they would be in a vacuum. But being
so very small it is commonly neglected; though in strict-
ness, the true or absolute weight is the weight in vacuo.

PROP. LXXIV.

The air is an elastic fluid, or such a one, as is ca-
pable of being condensed or expanded. And it observes
this law, that its density is proportional to the force
that compresses it.

These properties of the air, are proved by ex-
periments, of which there are innumerable. If you
take a syringe, and thrust the handle inwards, you
will feel the included air act strongly against your
                                               hand;

Fig. hand; and the more you thruft, the further the piston goes in, but the more it refifts; and taking away your hand, the handle returns back to where it was at firft. This proves its elafticity, and alfo that air may be driven into a lefs fpace, and condenfed.

75. Again, take a ftrong bottle, and fill it half full of water, and cement a pipe BI, clofe in it, going near the bottom; then inject air into the bottle through the pipe BI. Then the water will fpout out at B, and form a jet; which proves, that the air is firft condenfed, and then by its fpring drives out the water, till it become of the fame denfity as at firft, and then the fpouting ceafes.

81. Likewife if a veffel of glafs AB be filled with water in the veffel CD, and then drawn up with the bottom upwards; if any air is left in the top at A, the higher you pull it up, the more it expands; and the further the glafs is thruft down into the veffel CD, the more the air is condenfed.

82. Again, take a crooked glafs tube ABD open at the end A, and clofe at D; pour in mercury to the height BC, but no higher, and then the air in DC is in the fame ftate as the external air. Then pour in more mercury at A, and obferve where it rifes to in both legs, as to G and H. Then you may always fee that the higher the mercury is in the leg BH, the lefs the fpace GD is, into which the air is driven. And if the height of the mercury FH be fuch as to equal the preffure of the atmofphere, then DG will be half DC; if it be twice the preffure of the atmofphere, DG will be $\frac{1}{3}$DC, &c. So that the denfity is always as the weight or compreffion. And here the part CD is fuppofed to be cylindrical.

Cor. 1. *The fpace that any quantity of air takes up, is reciprocally as the force that compreffes it.*

Cor.

Pl. VIII. page 181
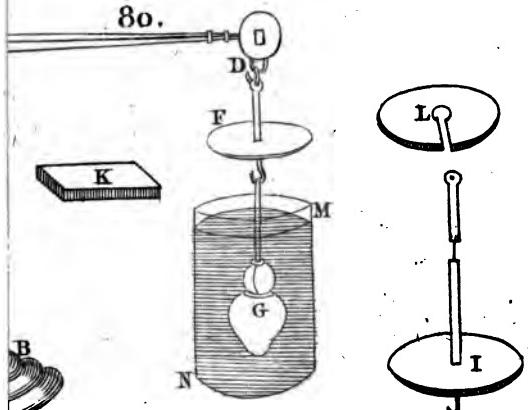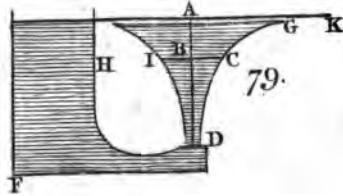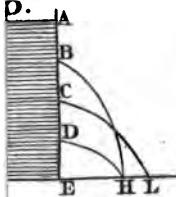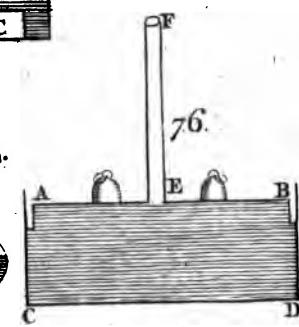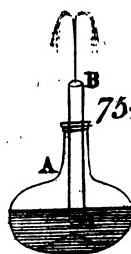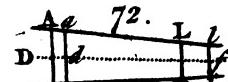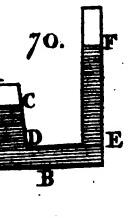
Fig. by its elaftic force, drive all the water out of the
81. glafs, and a good part of the air will follow, by
continuing the veffel there. Many more experi-
ments may be produced proving the fame thing.

PROP. LXXVI.

*The air will prefs upon the furfaces of all fluids,
with any force; without paffing through them, or
entering into them.*

If this was not fo, no machine, whofe ufe or
actions depends upon the preffure of the atmofphere,
could do its bufinefs. Thus the weight of the at-
mofphere preffes upon the furface of water, and
forces it up into the barrel of a pump, without any
air getting in, which would fpoil its working.
Likewife the preffure of the atmofphere keeps
mercury fufpended at fuch a height, that its weight
is equal to that preffure; and yet it never forces
itfelf through the mercury into the vacuum above,
though it ftand never fo long. And whatever be
the texture or conftitution of that fubtle invifible
fluid we call air, yet it is never found to pafs
through any fluid, though it be made to prefs
never fo ftrongly upon it. For though there be
fome air inclofed in the pores of almoft all bodies,
whether folid or fluid; yet the particles of air
cannot by any force be made to pafs through the
body of any fluid; or forced through the pores
of it, although that force or preffure be continued
never fo long. And this feems to argue that
the particles of air are greater than the particles
or pores of other fluids; or at leaft are of a ftruc-
ture quite different from any of them.

PROP.

PROP. LXXVII.

The weight or preſſure of the atmoſphere, upon any baſe at the earth's ſurface; is equal to the weight of a column of mercury of the ſame baſe, and whoſe height is from 28 to 31 inches; ſeldom more or leſs.

This is evident from the barometer, an inſtrument which ſhews the preſſure of the air; which at ſome ſeaſons ſtands at a height of 28 inches, ſometimes at 29, and 30, or 31. The reaſon of this is not becauſe there is at ſome times more air in the atmoſphere, than at others; but becauſe the air being an extremely ſubtle and elaſtic fluid, capable of being moved by any impreſſions, and many miles high; it is much diſturbed by w·nds, and by heat and cold; and being often in a tumultuous agitation; it happens to be accumulated in ſome places, and conſequently depreſſed in others; by which means it becomes denſer and heavier where it is higher, ſo as to raiſe the column of mercury to 30 or 31 inches. And where it is lower, it is rarer and lighter, ſo as only to raiſe it to 28 or 29 inches. And experience ſhews, that it ſeldom goes without the limits of 28 and 31.

Cor. 1. *The air in the ſame place does not always continue of the ſame weight; but is ſometimes heavier, and ſometimes lighter; but the mean weight of the atmoſphere, is that when the quickſilver ſtands at about $29\frac{1}{2}$ inches.*

Cor. 2. *Hence the preſſure of the atmoſphere upon a ſquare inch at the earth's ſurface, at a medium, is very near 15 pounds, averdupoiſe.*
For an inch of quickſilver weighs 8.102 ounces.

Cor. 3. *Hence alſo the weight or preſſure of the atmoſphere, in its lighteſt and heavieſt ſtate, is equal to*

·Fig. *the weight of a column of water, 32 or 36 feet high; or at a medium 34 feet.*

For water and quickſilver are in weight nearly as 1 to 14.

Cor. 4. If the air was of tho ſame denſity to the top of the atmoſphere, as it is at the earth; its height would be about 5⅓ miles at a medium.

For the weight of air and water are nearly as 12 to 10000.

Cor. 5. The denſity of the air in two places diſtant from each other but a few miles, on the earth's ſurface and in the ſame level; may be looked on to be the ſame, at the ſame time.

Cor. 6. The denſity of the air at two different altitudes in the ſame place, differing only by a few feet; may be looked on as the ſame.

Cor. 7. If the perpendicular height of the top of a ſyphon from the water, be more than 34 feet, at a mean denſity of the air; the ſyphon cannot be made to run.

For the weight of the water in the legs will be greater than the preſſure of the atmoſphere, and both columns will run down, till they be 34 feet high.

Cor 8. Hence alſo the quickſilver riſes higher in the barometer, at the bottom of a mountain, than at the top; and at the bottom of a coal pit, than at the top of it.

SCHOLIUM.

Hence the denfity of the air may be found at any height from the earth as in the following table.

Miles	denfity	Miles	denfity
$\frac{1}{4}$	·9564	10	.1700
$\frac{1}{2}$	.9146	20	.02917
$\frac{3}{4}$	.8748	30	.005048
1	.8372	40	.000881
2	.7012	50	.000155
3	.5871	100	.0000000298
4	·4917		
5	.4119		

The firft and third columns are the height in miles from the furface of the earth. And the fecond and fourth columns, fhew the denfity at that height; fuppofing the denfity at the furface of the earth, to be 1.

The denfity at any height is eafily calculated by this feries. Put r = radius of the earth, h = height from the furface, both in feet. Then the denfity at the height h, is the number belonging to the logarithm, denoted by this feries $-\dfrac{h}{68444}-$

$\dfrac{h}{r}A - \dfrac{h}{r}B - \dfrac{h}{r}C$ &c. where A, B, C, &c. are the preceding terms. The terms here will be alternately negative and affirmative. But the firft term alone is fufficient when the height is but a few miles.

By the weight and preffure of the atmofphere, the operations of pneumatic engines may be accounted for and explained. I fhall juft mention one or two.

A PUMP.

Fig. 83. is a *common pump.* AB the barrel or body of the pump, being a hollow cylinder, made

K 2   of

Fig.
83.
of wood or lead. CD the handle movable about the pin E. DF an iron rod moving about a pin D; this rod is hooked to the bucket or fucker FG, which moves up and down within the pump. The bucket FG is hollow, and has a valve or clack L at the top opening upwards. H a plug fixed at the bottom of the barrel, being likewife hollow, and a valve at I opening alfo upwards. BK the bottom going into the well at K; the pipe below B need not be large, being only to convey the water out of the well into the body of the pump. The plug H muft be fixed clofe that no water can get between it and the barrel; and the fucker FG, is to be armed with leather, to fit clofe that no air or water can get through between it and the barrel.

When the pump is firft wrought, or any time in dry weather when the water above the fucker is wafted, it muft be primed, by pouring in fome water at the top A to cover the fucker, that no air get through. Then raifing the end C of the handle, the bucket F defcends, and the water will rife through the hollow GL, prefing open the valve L. Then putting down the end C raifes the bucket F, and the valve L fhuts by the weight of the water above it. And at the fame time the preffure of the atmofphere forces the water up through the pipe KB, and opening the valve I, it paffes through the plug into the body of the pump. And when the fucker G defcends again, the valve I fhuts, and the water cannot return, but opening the valve L, paffes through the fucker GL. And when the fucker is raifed again, the valve L fhuts again, and the water is raifed in the pump. So that by the motion of the piiton up and down, and the alternate opening and fhutting of the two valves; water is continually raifed into the body of the pump, and difcharged at the fpout M.

The

The diftance KG, from the well to the bucket, Fig.
muft not be above 32 feet; for the preffure of the 83.
atmofphere will raife the water no higher, and if
it is more, the pump will not work.   It is evident
a pump will work better when the atmofphere is
heavy than when it is light, there being a twelfth
or fifteenth part difference, at different times. And
when it is lighteft it is only equal to 32 feet. Where-
fore the plug H muft always be placed fo low, as
that the fucker GL may be within that compafs.

A BAROMETER.

Fig. 84. is a *Barometer*, or an inftrument to mea- 84.
fure the weight of the air.   It confifts of a glafs
cone ABC hollow within, filled full of mercury,
and hermetically fealed at the end C, fo that no
air be left in it. When it is fet upright, the mercury
defcends down the tube BC, into the bubble A,
which has a little opening at the top A, that the
air may have free ingrefs and egrefs.   At the top
of the tube C, there muft be a perfect vacuum.
This is fixed in a frame, and hung perpendicular
againft a wall.   Near the top C, on the frame, is
placed a fcale of inches, fhewing how high the
mercury is in the tube BC, above the level of it
in the bubble A, which is generally from 28 to 31
inches, but moftly about 29 or 30. Along with
the fcale of inches, there is alfo placed· a fcale of
fuch weather as has been obferved to anfwer the fe-
veral heights of the quickfi!ver. Such a fcale you
have annexed to the 84th figure. In dividing the
fcale of inches, care muft be taken to make proper
allowance for the rifing or falling of the quickfilver
in the bubble A, which ought to be about half
full, when it ftands 29½, which is the mean
height.   For whilft the quickfilver rifes an inch at
C, it defcends a little in the bubble A, and that

K 3   defcent

Fig.
84.
defcent muſt be deducted, which makes the divi-
ſions be ſomething leſs than an inch. Theſe inches
muſt be divided into tenth parts, for the more exact
meaſuring the weight of the atmoſphere. For the
pillar of mercury in the tube is always equal to the
weight of a pillar of the atmoſphere of the ſame
thickneſs. And as the height of the quickſilver
increaſes or decreaſes the weight of the air in-
creaſes or decreaſes accordingly. The tube muſt
be near 3 feet long, and the bore not leſs than $\frac{1}{5}$
or $\frac{1}{6}$ of an inch, in diameter, or elſe the quickſil-
ver will not move freely in it.

By help of the barometer, the height of moun-
tains may be meaſured by the following table. In
which the firſt column is the height of the moun-
tain, &c. in feet or miles; the ſecond the height
of the quickſilver; and the third the deſcent of the
quickſilver in the barometer; and this at a mean
denſity of the air.

Feet

Feet	High Barom.	Defcent	Feet	High Barom	Defcent
0	29.500				
100	29.400	.100	2600	27.028	2.472
200	29.301	.199	2700	26.938	2.562
300	29.203	.297	2800	26.848	2.652
400	29.105	.395	2900	26.758	2.742
500	29.007	.493	3000	26.668	2.832
600	28.910	.590	3100	26.578	2.922
700	28.812	.688	3200	26.489	3.011
800	28.716	.784	3300	26.400	3.100
900	28.619	.881	3400	26.311	3.189
1000	28.523	.977	3500	26.222	3.278
1100	28.428	1.072	3600	26.136	3.364
1200	28.332	1.168	3700	26.049	3.451
1300	28.237	1.263	3800	25.961	3.539
1400	28.143	1.357	3900	25.874	3.626
1500	28.048	1.452	4000	25.786	3.714
1600	27.954	1.546	4100	25.699	3.801
1700	27.860	1.640	4200	25.613	3.887
1800	27.766	1.734	4300	25.527	3.973
1900	27.672	1.828	4400	25.441	4.059
2000	27.579	1.921	4500	25.355	4.145
2100	27.487	2.013	4600	25.270	4.230
2200	27.394	2.106	4700	25.185	4.315
2300	27.302	2.198	4800	25.101	4.399
2400	27.210	2.290	4900	25.017	4.483
2500	27.119	2.381	5000	24.933	4.567

PNEUMATICS.

The Table continued in MILES.

Miles	H. Barom	D. fcent	Miles	H. Barom.	Defcent
0.	29.50				
0.25	28.21	1.29	3.25	16.57	12.93
0.50	26.98	2.52	3.50	15.85	13.65
0.75	25.80	3.70	3.75	15.16	14.34
1.	24.70	4.80	4.	14.50	15.00
1.25	23.62	5.88	4.25	13.87	15.63
1.50	22.60	6.90	4.50	13.27	16.23
1.75	21.62	7.88	4.75	12.70	16.80
2.	20.68	8.82	5.	12.15	17.35
2.25	19.78	9.72	5.25	11.62	17.88
2.50	18.93	10.57	5.50	11.12	18.38
2.75	18.11	11.39	5.75	10.64	18.86
3.	17.32	12.18	6.	10.18	19.32

This table is made from a table of the air's denfity, made as in Schol. Prop. LXXVII. And then multiplying all the numbers thereof by 29.5 the mean denfity of the air. For the denfity of the air at any height above the earth is as the weight of the atmofphere above it, (by Prop. LXXIV.); and that is as the height of the mercury in the barometer.

I lately contrived another fort of barometer, which fhews the afcent and defcent of the mercury at the bottom.

85. ABC is a recurve tube, clofe at the top, where the bucket C is, and open at the end A. The length of CB is 32 or 33 inches, and of AB 6 or 7. The bucket C fhould contain about as much as the end AB. And the bucket and end CB muft be quite filled with mercury, as far as B, a little beyond the turn. The wider the bucket C is, the better. The fcale fet to the end AB muft be graduated downwards, for the mercury falls in
this

this, when it rises in the other sort.   This being Fig.
placed against a wall, will shew the height of the 85.
mercury, as in the common ones.   And this way
is more commodious, as it saves the labour of
clambering up upon chairs to see it, as one must
do, in the common sort, to see exactly.

A WATER BAROMETER.

A barometer may also be made of water as in 86.
fig. 86, which is a water barometer.   AB is a
glass tube open at both ends, and cemented close
in the mouth of the bottle EF, and reaching very
near the bottom.   Then warming the bottle at the
fire, part of the air will fly out; then the end A
is put into a vessel of water mixed with cochineal,
which will go through the pipe into the bottle as
it grows cold.   Then it is set upright; and the
water may be made to stand at any point C, by
sucking or blowing at A.   And if this barometer
be kept to the same degree of heat, by putting
it in a vessel of sand, it will be very correct for
taking small altitudes; for a little alteration in the
weight of the atmosphere, will make the water at
C rise or fall in the tube very sensibly.   But if it
be suffered to grow warmer, the water will rise
too high in the tube, and spoil the use of it; so
that it must be kept to the same temper.

If a barometer was to be made of water put
into an exhausted tube, after the manner of quick-
silver; it would require a tube 36 feet long or
more; which could hardly find room within doors.
But then it would go 14 times more exact than
quicksilver; because for every inch the quicksilver
rises, the water would rise 14; from whence every
minute change in the atmosphere would be dis-
cernable.

And the water barometer above described will
shew the variation of the air's gravity as minutely

Fig. as the other, if the bottle be large to hold a great
86. quantity of air. And in any cafe, by reducing
the botttle (fo far as the air is contained) to a cylin-
der; and put $D =$ diameter of the bottle, $d =$
diameter of the pipe, $p =$ height of air, $x =$ rifing
in the pipe, all in inches. Then the height of a
hill in feet will be nearly $1 + \dfrac{408dd}{pDD} \times 71x$. And
if $y =$ height of the hill or any afcent, $Q = \dfrac{408dd}{pDD}$. Then $x = \dfrac{y}{1 + Q \times 71}$ very near, at a
mean denfity of the air.

A THERMOMETER.

87. Fig. 87. is a *thermometer*, or an inftrument to
meafure the degrees of heat and cold. AB is a
hollow tube near two foot long, with a ball at the
bottom; it is filled with fpirits of wine mixed with
cochineal, half way up the neck; which done, it is
heated very much, till the liquor fill the tube, and
then it is fealed hermetically at the end A. Then
the fpirit contracts within the tube as it cools. It
is inclofed in a frame, which is graduated into de-
grees, for heat and cold. For hot weather dilates
the fpirit, and makes it run further up the tube;
and cold weather on the contrary, contracts it, and
makes it fink lower in the tube. And the parti-
cular divifions, fhew the feveral degrees of heat
and cold; againft the principal of which, the words
heat, cold, temperate, &c. are written.

They that would fee more machines defcribed,
may confult my large book of Mechanics, where
he will meet with great variety.

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Tem- perate
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B
87

Pl.IX. pa 154

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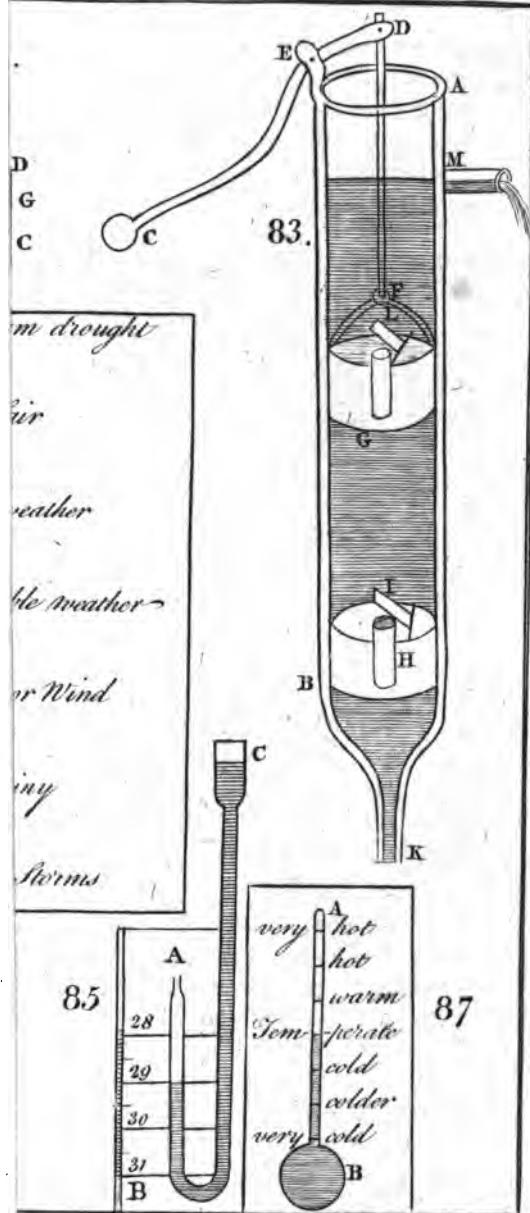
PROJECTION

OF THE

S P H E R E,

ORTHOGRAPHIC, STEREOGRAPHIC,
and GNOMONICAL.

Both demonftrating the

P R I N C I P L E S,

And explaining the

P R A C T I C E

Of thefe three feveral Sorts of
PROJECTION.

In Minimis Ufus————
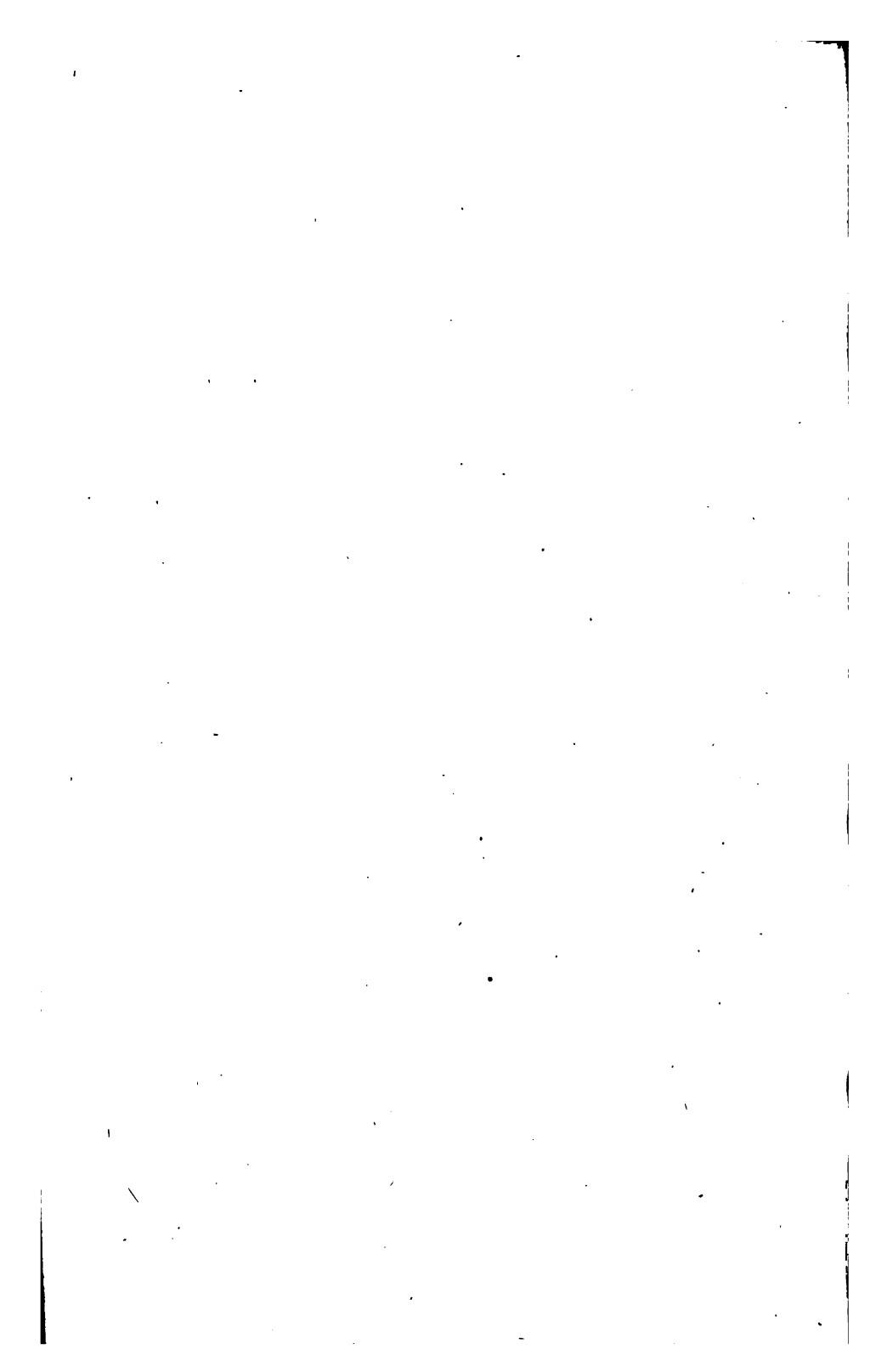
PREFACE.

THE *Projection of the Sphere, or of its*
Circles, has the same relation to Spherical
Trigonometry, that practical Geometry has to
plane Trigonometry. For as the one saves a
deal of Calculation, by drawing a few right
Lines, so does the other by drawing a few
Circles. The Projection of the Sphere gives a
Learner a good Idea of the Sphere and all its
Circles, and of their several Positions to one
another, and consequently of Spherical Triangles,
and the Nature of Spherical Trigonometry.

I have here delivered the Principles of three
sorts of Projection, in a small compass; and yet
the Reader will find here, all that is essential to
the subject; and yet nothing superfluous; for I
think no more need be said, or indeed can be said
about it, to make it intelligible and practicable.
For here is laid down, not only the whole The-
ory, but the Practice likewise. Yet the practical
Part is entirely disengaged from the Theory; so
that any body (though he has no desire or leisure
to attain to the Theory) may nevertheless, by help
of the Problems, make himself Master of the
Practice. For which end I have endeavoured to
make all the rules relating to practice, plain, short
and easy, and at the same time full and clear.

It is true the solution of Problems this way,
must be allowed to be imperfect; for there will
always be some errors in working, as well as
in the instruments we work with. But nobody

in

PREFACE.

in seeking an accurate solution to a Problem, will trust to a Projection by scale and compass; because this cannot be depended on in cases of great nicety. Yet where no great exactness is required, it will be found very ready and useful; and besides it will serve to prove and confirm the solution obtained by Calculation.

But then this defect is abundantly recompensed by the easiness of this method. For by scale and compass only, all sorts of Problems belonging to the Sphere, as in Astronomy, Geography, Dialing, &c. may be solved with very little trouble, which require a great deal of time and pains, to work out trigonometrically by the tables. It likewise affords a great pleasure to the mind, that one can, in a little time, describe the whole furniture of Heaven and Earth, and represent them to the eye, in a small scheme of paper.

But its principal use is for such persons (and that is by far the greater number) as having no opportunity for learning Spherical Trigonometry, have yet a desire to resolve some Problems of the Sphere. For such as these, this small Treatise will be of particular service, because the practical rules, especially of any one sort of Projection, may be learned in a very little time, and are easily remembered. So that I have some hopes I shall please all my readers, whether theoretical or practical.

<div align="right">

W. E.

</div>

THE

PROJECTION

OF THE

SPHERE IN PLANO.

DEFINITIONS.

1. *PROJECTION* of the fphere is the repre-
fenting its furface upon a pláne, called the
Plane of Projection.

2. *Orthographic* Projection, is the drawing the
circles of the fphere upon the plane of fome great
circle, by lines perpendicular to that plane, let fall
from all the points of the circles to be projected.

3. The *Stereographic* Projection, is the drawing
the circles of the fphere upon the plane of one of
its great circles, by lines drawn from the pole of
that great circle to all the points of the circles to be
projected.

4. The *Gnomonical* Projection, is the drawing the
circles of an hemifphere, upon a plane touching it
in the vertex, by lines or rays iffuing from the cen-
ter of the hemifphere, to all the points of the cir-
cles to be projected.

5. The *Primitive* circle is that on whofe plane
the fphere is projected. And the pole of this cir-
cle is called the *Pole* of Projection. The point from
whence the *projecting* right lines iffue, is the *project-
ing Point.*

6. The

6. The *Line of Measures* of any circle is the common interfection of the plane of projection, and another plane that passes through the eye, and is perpendicular both to the plane of projection, and to the plane of that circle.

SCHOLIUM.

There are other Projections of the Sphere, as the *Cylindrical*, the *Scenographhic* which belongs to Perspective, the *Globical* which belongs to Geography, *Mercator's*, for which see Navigation, &c.

AXIOM.

The *Place* of any visible point of the Sphere upon the plane of projection, is where the projecting line cuts that plane.

Cor. *If the eye be applied to the projecting point, it will view all the circles of the Sphere, and every part of them, in the projection, just as they appear from thence in the Sphere itself.*

SCHOLIUM.

The Projection of the Sphere is only the shadow of the circles of the Sphere upon the plane of Projection, the light being in the place of the eye or projecting point.

The Signification of some Characters.

+ added to.
− subtracting the following quantity.
∠ an angle.
= equal to.
⊥ perpendicular to.
‖ parallel to.
: : a proportion.

SECT.

SECT. I.

The Orthographic Projection of the SPHERE.

PROP. I.

*I*F *a right line* AB *is projected upon a plane, it is projected into a right line; and its length will be* Fig. *to the length of the projection, as radius to the cosine* 3. *of its inclination above that plane.*

For let fall the perpendiculars A*a*, B*b* upon the plane of projection; then *ab* will be the line it is projected into; but by trigonometry AB : is to A*o* or *ab* :: as radius : to the sine of B or cosine of *o*AB.

Cor 1. *If a right line is projected upon a plane, parallel thereto, it is projected into a right line parallel and equal to itself.*

Cor. 2. *If an angle be projected upon a plane which is parallel to the two lines forming the angle; it is projected into an angle equal to itself.*

Cor. 3. *Any plane figure projected upon a plane parallel to itself, is projected into a figure similar and equal to itself.*

Cor. 4. *Hence also the area of any plane figure, is to the area of its projection :: as radius, to the cosine of its elevation or inclination.*

PROP. II.

A circle perpendicular to the plane of projection, is projected into a right line equal to its diameter.

For projecting lines drawn through all the points of the circle fall in the common section of the planes

L   of

Fig. of the circle and of projection, which is a right
line (Geom. V. 3.), and equal to the diameter of
the circle; becaufe the planes interfect in that dia-
meter. $\mathcal{Q}$. E. D.

Cor. *Hence any plane figure, perpendicular to the
plane of projection is projected into a right line. For
the perpendiculars from every point, will all fall in
the common interfection of the figure with the plane of
projection.*

PROP. III.

1. *A circle parallel to the plane of projection is pro-
jected into a circle equal to itfelf, and concentric with
the primitive.*

Let BOD be the circle, I its center, C the cen-
ter of the fphere; the points I, B, O, D, are pro-
jected into the points C, L, F, G, and therefore
OICF, and BICL are rectangled parallelograms.
Confequently LC = BI = OI = FC, (Geom.
III. 1.). $\mathcal{Q}$. E. D.

Cor. *The radius* CL *or* CF *is the cofine of the cir-
cle's diftance from the primitive, for it is the fine of* AB.

PROP. IV.

2. *An inclined circle is projected into an ellipfis whofe
tranfverfe axis is the diameter of the circle.*

Let ADBH be the inclined circle, P its center;
and let it be projected into *adbh*; draw the plane
ABFC*a* through the center C of the fphere, per-
pendicular to the plane of the given circle and
plane of projection, to interfect them in the lines
AB, *ab*; draw GPH, DE, perpendicular, and DQ
parallel to AB; then becaufe the line GP, and
the plane of projection are both perpendicular to
the

the plane ABF; therefore GH is parallel to the Fig.
plane of projection, and therefore to gb.   2.

In the circle ADB, $DQ^2 = GQH = gqb$, and
$BP^2 = GP^2 = gp^2$. And (Geom. V. 12.) BP :
EP or DQ :: bp : ep or dq, and BP^2 : DQ^2 ::
bp^2 : dq^2; that is, gp^2 : gqb :: bp^2 : dq^2; and there-
fore $agbb$ is an ellipsis, whose transverse gb is the
diameter of the circle. *Q. E. D.*

Cor. 1. *Since ab is perpendicular to gb, therefore
ab is the conjugate axis, and is twice the sine of the
∠ ABb to the radius gp; that is, the conjugate axis
is equal to twice the cosine of the inclination, to the
radius of the circle.*

Cor. 2. *The transverse axis is equal to twice the
cosine of its distance from its parallel great circle. For
gb = GH = 2AP = twice the sine of AK.*

Cor. 3. *The extremities of the conjugate axis are
distant from the center of the primitive, by the sines
of the circle's nearest and greatest distance from the
pole of the primitive. Thus aC is the sine of AN,
and bC the sine of BN.*

Cor. 4. *Hence also it is plain that the conjugate axis
always passes through the center C of the primitive;
and is always in the line of measures of that circle.*

SCHOLIUM.

Every circle in the projection represents two equal 3.
circles, parallel and equidistant from the primitive.
Every right line represents two semicircles, one
towards the eye, the other in the opposite side.
Every ellipsis represents two equal circles, but
contrarily inclined, as AB, CD; one above the
primitive, the other below it.

And now the Theory being laid down, it re-
mains only to deduce thence, some short rules for
practice, as follows.

PROP. V. *Prob.*

5.   *To projeᐸ a circle parallel to the primitive.*

Rule.

Take the complement of its diftance from the primitive, and fet it from A to E ; and with the center C and radius CD = perpendicular EF, defcribe the circle D*g*G.

By the plane fcale.

Take the fine of its diftance from the pole of the primitive; with that radius and the center C defcribe the circle.

PROP. VI. *Prob.*

4.   *To projeᐸ a right circle, or one that is perpendicular to the plane of projeᐸion.*

Rule.

Through the center C of the primitive, draw the diameter AB, and take the diftance from its parallel great circle, and fet from A to E, and from B to D, and draw ED, the right circle required.

By the fcale.

Take the fine of the circle's diftance from its parallel great circle AB, and at that diftance draw a parallel ED for the circle required.

PROP. VII. *Prob.*

To projeᐸ a given oblique circle.

Rule.

6.   Draw the line of meafures AB, and take the circle's neareft diftance from the primitive and fet
from

from B to D; upwards if it be above the primitive; Fig.
or downward, if below; likewise take its greatest 6.
distance, and set from A to E, and draw ED, and
let fall the perpendiculars EF, DG; and bisect FG
in H, and erect the perpendicular KHI, making
KH = HI = half ED; then describe an ellipsis
(by the Conic Sections) whose transverse is IK and
conjugate FG; and that shall represent the circle
given.

By the scale.

Draw the line of measures AB; and take the 6.
sines of the circle's nearest and greatest distance
from the pole of the primitive, and set them from
the center C to F and G, (both ways if the circle
encompass the pole, but the same way if it lie on
one side the pole;) bisect FG in H, and erect HK,
HI perpendicular to FG, and = to the radius of
the circle given, or the sine of its distance from its
own pole; about the axes FG, KI describe an el-
lipsis, and it is done.

Scholium.

An ellipsis great or small may be described by 10.
points, thus; through the center D of the circle and
ellipsis, draw BD ⊥ the transverse AR; and on
AR erect a sufficient number of perpendiculars IK,
ik &c. and make as DB or DA : DE : : IK:IF : :
ik : *if* &c. then through all the points E, F, *f*, &c.
draw a curve.   See Prop. 76. ellipsis.

PROP. VIII. *Prob.*

To find the pole of a given ellipsis.

Rule.

Through the center of the primitive C, draw the 7.
conjugate of the ellipsis; on the extreme points
F, G, erect the perpendiculars FE, GD, or set the

transverse

Fig. tranfverfe IK from E to D, and bifect ED in R,
7. and let fall RP perpendicular to AB, then is P
the pole.

By the fcale.

7. Take CF, and CG, and apply to the fines, and
find the degrees anfwering, or the fupplements;
then take the fine of half the fum of thefe degrees,
if F, G be both on one fide of C, or the fine of
half the difference, if they lie on contrary fides;
and fet it from C to the pole P.

Or thus; apply the femi-tranfverfe IH to the
fines, and fet the degrees from E to R; and draw
RP ⊥ to AB; and P is the pole.

PROP. IX. *Prob.*

*To meafure an arch of a parallel circle; or to fet
any number of degrees on it.*

Rule.

5. With the radius of the parallel, and one foot
in C defcribe a circle Gg, draw CGB, and Cgb;
then Bb will meafure the given arch Gg; or Gg
will contain the given number of degrees fet from
B to b. So that either being given finds the other.

PROP. X. *Prob.*

To meafure any part of a right circle.

Rule.

8. In the right circle ED, let EA = AD; and
let AB be to be meafured. Make CF = AE;
with extent BA = FG defcribe the arch GI; draw
CGK to touch it in G; then is HK the meafure of
AB. For FG = S. ∠ HCK to the radius CF or
AE, and BA is the fame, by Cor. Prop. III.

Otherwife

Fig.
8.

Otherwise thus.

On the diameter ED, defcribe the femicircle END, draw AN, BO, LP perpendicular to ED, then ON is the meafure of BA, and NP of AL; and ON or NP may be meafured as in Prop. IX.

By the fcale.

Let AL be to be meafured. Draw CD: and LM parallel to AC, then CM applied to the fines gives the degrees. For radius CD : AD : : CM : AL.

Cor. *If the right circle paffes through the center, there is no more to do, but to raife perpendiculars on it, which will cut the primitive as required. Or apply the part of the right circle to the line of fines.*

PROP. XI. *Prob.*

To fet off any number of degrees upon a right circle, DE.

Rule.

Draw CA ⊥ DE, and make the ∠ HCK = 8. the degrees given, make CF = radius AE, take FG the neareft diftance, and fet from A to B; then AB = ∠HCK, the degrees propofed.

Otherwife thus.

On ED defcribe the femicircle END; then by Prop. IX. fet off NP = degrees given, draw PL perpendicular to ED; then AL contains the degrees required.

Or thus by the fcale.

Draw CD, take the given degrees off the fines, and fet from C to M, and draw ML parallel to CA, then AL = arch required.

PROP.

PROP. XII. *Prob.*

9.   *To meafure an arch of an ellipfis; or to fet any*
10. *number of degrees upon it.*

Rule.

About AR the tranfverfe axis of the ellipfis,
defcribe a circle ABR; erect the perpendiculars
BED, KFI, on AR; then BK is the meafure of
EF, or EF is the reprefentation of the arch BK.
And BK is to be meafured, or any degrees fet up-
on it, as in Prop. IX.

Scholium.

Thefe Problems are all evident from the three
firft propofitions, and need no other demonftration.
If the fphere be projected on any plane parallel to
the primitive, the projection will be the very fame;
for being effected by parallel lines, which are al-
ways at the fame diftance, there will be produced
the fame figure, or reprefentation.   Of all ortho-
graphic projections, thofe on the meridian, or on
the folftitial colure, commonly called the Analem-
ma, is moft ufeful; becaufe a great many of the
circles of the fphere fall into right lines or circles,
whereas in the projections upon other planes, they
are projected into ellipfes, which are hard to de-
fcribe ; which makes thefe forts of projection to be
neglected

And by the fame rules that the circles of the
fphere are projected upon a plane, any other figure
may likewife be orthographically projected; by let-
ting fall perpendiculars upon the plane from all the
angles, or all the points of the figure, and joining
thefe points with right or curve lines, as they are
in the figure itfelf.

By this kind of projection, either the convex or
concave fide of the fphere, may be projected;
which

2.

5.

6.

10.
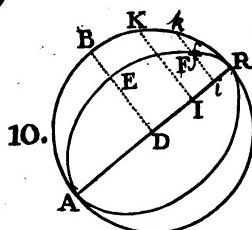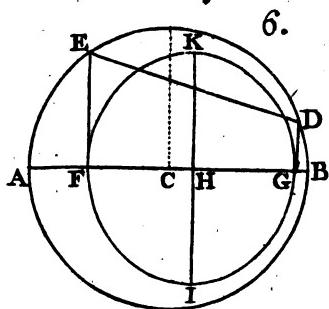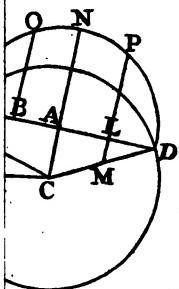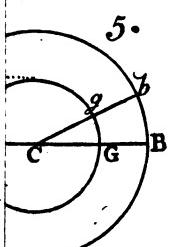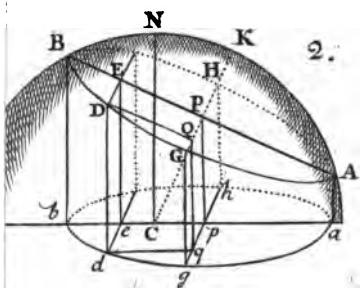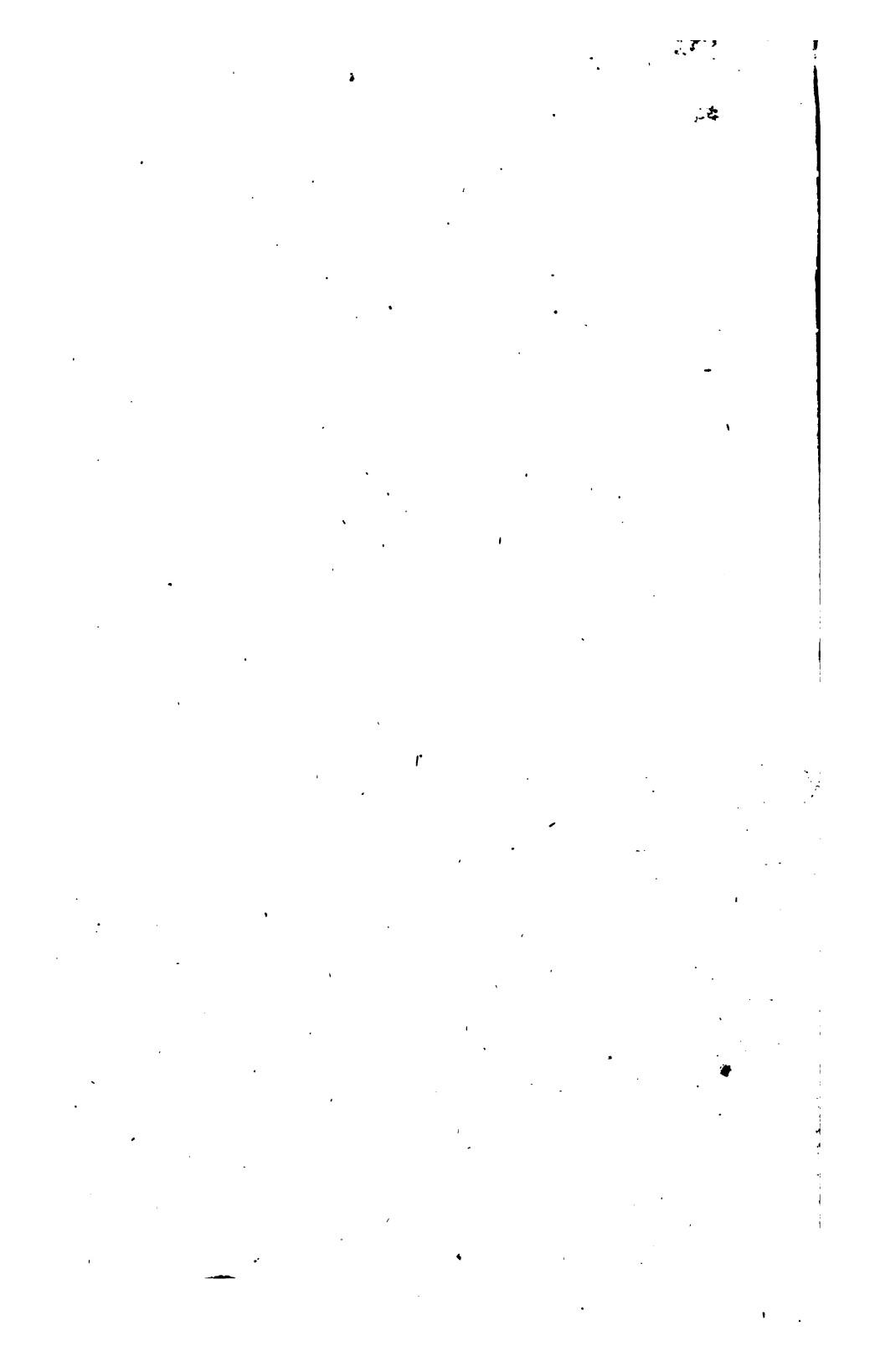
which is peculiar to this fort of projection; that Fig.
is, either the hemisphere towards you, or that from 9.
you, may be projected upon the plane of its great
circle.   And since in fome cafes they both have the
fame appearance, it ought to be mentioned whe-
ther it is.

But if both the convex and concave fides of
the *fame hemifphere* be projected; that is, if you
make two projections, one for the convex, the
other for the concave fide ; the circles in one will
be inverted in refpect of the other, the right to
the left, &c.   Becaufe in looking at the fame he-
mifphere, it will not have the fame appearance,
when you look at the contrary fides of it ; becaufe
you look contrary ways at it, to fee the external
and internal furfaces.

SECT. II.

The Stereographic Projection of the SPHERE.

PROP. I.

*A*NY *circle paſſing through the projeɛting point, is projeɛted into a right line.*

For all lines drawn from the projecting point, to this circle, paſs through the interſection of this circle and plane of projection, which is a right line.

Cor. 1. *A great circle paſſing through the poles of the primitive is projeɛted into a right line paſſing through the center.*

Cor. 2. *Any circle paſſing through the projeɛting point is projeɛted into a right line perpendicular to the line of meaſures, and diſtant from the center, the ſemitangent of its neareſt diſtance from the pole oppoſite to the projeɛting point. Thus the circle* AE *is pro-jeɛted into a right line paſſing through* G, *and perpen-dicular to* BC, *the line of meaſures, and* GC *is the ſemitangent of* EM.

12.

PROP. II.

Every circle (that paſses not through the projeɛting point) is projeɛted into a circle.

11. *Caſe* I. Let the circle EF be parallel to the primitive BD; lines drawn to all points of it from the projecting point A, will form a conic ſurface, which being cut parallel to the baſe by the plane BD, the ſection GH (into which EF is projected) will be a circle by the conic ſections.

Caſe

Cafe II. Let BH be the line of meafures to the Fig.
circle EF, draw FK parallel to BD, then arch AK 12.
= AF, and therefore ∠AFK or AHG = AEF;
therefore in the triangles AEF, AGH, the angles
at E and H are equal, and the angle A common;
therefore the angles at F and G are equal. There-
fore the cone of rays AEF (whofe bafe EF is a
circle) is cut by fubcontrary fection, by the plane
of projection BD, and therefore, by the conic
fections, the fection GH (which is the projection
of the circle EF) will alfo be a circle.   *Q. E. D.*

Cor. *When* AF *is equal to* AG, *the circle* EF *is
projected into a circle equal to itfelf.*
    For then the fimilar triangles AHG and AEF,
will alfo be equal, and GH = EF.

PROP.   III.

*Any point on the fphere's furface is projected into
a point, diftant from the center, the femi-tangent of
its diftance from the pole oppofite to the projecting point.*

Thus the point E is projected into G, and F 12.
into H; and CG is the femi-tangent of EM, and
CH of MF.

Cor. 1. *A great circle perpendicular to the primi-
tive, is projected into a line of femi-tangents paffing
through the center, and produced infinitly.*
    For MF is projected into CH its femi-tangent,
and EM into the femi-tangent CG.

Cor. 2. *Any arch* EM *of a great circle perp. to the
primitive, is projected into the femi-tangent of it.*
    Thus EM is projected into GC.

Cor. 3. *Any arch* EMF *of a great circle, is pro-
jected into the fum of its femi-tangents of its greateft*
                                                        and

Fig. *and leaft diftances from the oppofite pole* M, *if it lie*
12. *on both fides of* M, *or the dif. of the femi-tangents,*
when all on one fide.

PROP. IV.

13.   *The angle made by two circles on the furface of the*
fphere, is equal to that made by their reprefentatives
upon the plane of projection.

Let the angle BPK be projected. Through the
angular point P and the center C, draw the plane
of a great circle PED perpendicular to the plane
of projection EFG. Let a plane PHG touch the
fphere in P; then fince the circle EPD is per-
pendicular both to this plane and to the plane of
projection, therefore it is perpendicular to their
interfection GH. The angles made by circles are
the fame as thofe made by their tangents, therefore
in the plane PGH, draw the tangents PH, PF, PG
to the arches, PB, PD, PK; and thefe will be
projected into the lines pH, pF, pK: Now I fay the
∠ HPG = ∠ HpG.   For the angle CPF = a
right angle = CpA + CAp; therefore taking away
the equal angles CPA and CAP, and ∠ pPF
= CpA or PpF; confequently pF = PF. There-
fore in the right angled triangles PFG and pFG,
there are two fides equal and the included ∠right;
therefore hypothenufe PG = pG.   And for the
fame reafon in the right angled triangles PFH and
pFH, PH = pH.   Laftly in the triangles PHG
and pHG, all the fides are refpectively equal, and
therefore ∠P = ∠p.   Q. E. D.

. Cor. 1. *The rumb lines projected make the fame*
angles with the meridians as upon the globe; and there-
fore are logarithmic fpirals on the plane of the equi-
noctial.

For every particle of the rumb coincides with
fome great circle.

Cor.

Cor. 2. *The angle made by two circles on the* Fig. *sphere, is equal to the angle made by the radii of their projections at the point of interfection.*

For the angle made by two circles on a plane, is the fame with that made by their radii drawn to the point of interfection.

PROP. V.

The center of a projected (leffer) circle perpendicular to the primitive, is in the line of meafures diftant from the center of the primitive, the fecant of the leffer circles diftance from its own pole; and its radius is the tangent of that diftance.

Let A be the projecting point, EF the circle to 14. be projected, GH the projected diameter. From the centers C, D draw CF, DF, and the triangles CFI, DFI are right angled at I; then $\angle$ IFC = $\angle$ FCA = 2 $\angle$ FEA or 2FEG = 2 $\angle$ FHG = $\angle$ FDG, therefore IFC + IFD = FDG + IFD = a right angle; that is CFD is a right angle, and the line CD is the fecant of BF, and the radius FD is the tangent of it. *Q. E. D.*

Cor. *If thefe circles be actually defcribed, it is plain the radius* FD *is a tangent to the primitive at* F, *where the leffer circle cuts it.*

PROP. VI.

The center of Projection of a great circle is in the line of meafures, diftant from the center of the pri-mitive, the tangent of its inclination to the primitive; and its radius is the fecant of its inclination.

Let A be the projecting point, EF the great 15. circle, GH the projected diameter, D the center;

draw

Fig. draw DA. The angle EAF being in a femicir-
15. cle is right. In the right angled triangle GAH,
AC is perpendicular to GH, therefore ∠ GAC =
AHC and their double, ECB = ADC, and their
complements ECI = CAD. Therefore CD is the
tangent of ECI, and radius AD its fecant. *Q.E.D.*

16.   Cor. 1. *If the great oblique circle AGBH be actu-*
ally defcribed upon the primitive AIB. I fay, all great
circles pafsing through G will have the centers of their
projections in the line RS drawn through the center D,
perpendicular to the line of meafures IH.

For fince all great circles cut one another at a fe-
micircle's diftance, all circles paffing through G muft
cut at the oppofite point H; and therefore their
centers muft be in the line RDS.

Cor. 2. *Hence alfo if any oblique circle GLH be*
required to make any given angle with another circle
BGAH, it will be projected the fame way with re-
gard to GAH confidered as a primitive, and RS its
line of meafures; as the circle BGA is on the primi-
tive BIA, and line of meafures ID. And therefore
the tangent of the angle AGL to the radius GD, fet
from D to N gives the center of GL.

For the ∠ NGD will then be equal to AGL,
by Cor. 2. Prop. IV. and therefore GLH is rightly
projected.

Cor. 3. *And for the fame reafon, if N be the cen-*
ter of the circle GgHR; the centers of all circles paff-
ing through g and R, will be in the line rNs perpendi-
cular to RS; fo n is the center of grR. But then as
g, R do not reprefent oppofite points of the circle GgH,
therefore all circles paffing through g, R, (as grR)will
be leffer circles, except GgHR.

SCHOLIUM.

Fig.
16.

SCHOLIUM.

Of all great circles in the projection, the primitive is the leaft. For the radius of any oblique great circle (being the fecant of the inclination) is greater than the radius of the primitive; as the fecant is always greater than the radius. Therefore every oblique great circle in the projection is greater than the primitive.

PROP. VII.

The projected extremities of the diameter of any circle, are in the line of meafures, diftant from the center of the primitive circle, the femi-tangents of its neareft and greateft diftances from the pole of projection oppofite to the projecting point.

For the diameter of the circle EF is projected 15. into GH, from the projecting point A. But GC 17. is the femi-tangent of EB, and CH the femi-tangent of BF. *2. E. D.*

Cor. 1. *The points where an inclined great circle* 15. *cuts the line of meafures, within and without the primitive, is diftant from the center of the primitive, the tangent and co-tangent of half the complement of the circle's inclination to the primitive.*
For CG = tangent of half EB, or of half the complement of IE the inclination. And (becaufe the ∠EAF is right) CH is the co-tangent of GAC or half EB.

Cor 2. *Hence the center D of a projected circle is* 17. *in the line of meafures, diftant from the center of the* 18. *primitive, half the difference of the femi-tangents of its neareft and greateft diftance from the oppofite pole, if it encompafses that pole; but half the fum of the femi-tangents if it lye on one fide the pole of projection.*

Cor.

Fig. Cor. 3. *And the radius is half the sum of the se-*
mi-tangents, if the circle encompasses the pole ; or half
the difference if it lyes on one side.

17.   Cor. 4. *Hence also if p, q, be the projected poles, it*
will be qG : pG :: qH : pH.

For draw G*n* parallel to *q*A, and since P, Q are
the poles, therefore *q*A*p* is a right angle, and since
the angles GA*p* and *p*AH are equal, and G*n* per-
pendicular to A*p*, therefore GA = A*n*; whence
by similar triangles *q*G : *q*H :: A*n* or AG : AH ::
G*p*, *p*H (Geom. II. 25.)   And consequently the
line *q*H is cut harmonically in the points G, *p*.

PROP. VIII.

The projected poles of any circle are in the line of
measures, within and without the primitive, and dis-
tant from its center the tangent and co-tangent of half
its inclination to the primitive.

19.   The poles P, *p* of the circle EF are projected
into D and *d*; and CD is the tangent of CAD or
half BCP, that is, of half GCI, the inclination of
the circle ICK, parallel to EF.   Likewise C*d* is
the tangent of CA*d*, or the co-tangent of CAD.
                                                    Q. E. D.

Cor. 1. *The pole of the primitive is its center; and*
the pole of a right circle is in the primitive.

19.   Cor. 2. *The projected center of any circle is always*
between the projected pole (nearest to it on the sphere)
and the center of the primitive; and the projected
centers of all circles lye between the projected poles.

For the middle point of EF or its center is
projected into S; and all the points in P*p* (in
which are all the centers) are projected into D*d*.

                                                    Cor.

14.

16.

18.
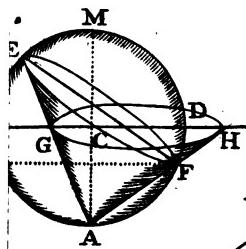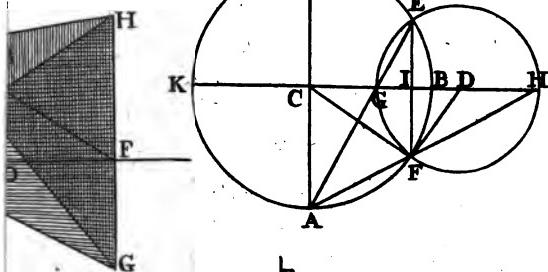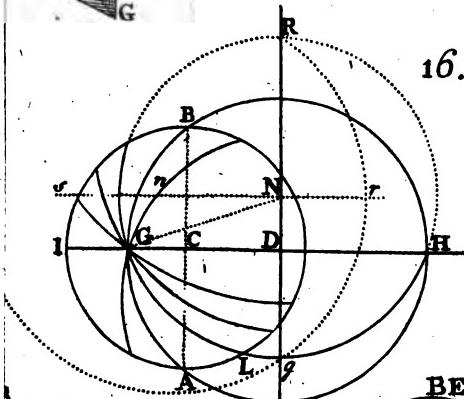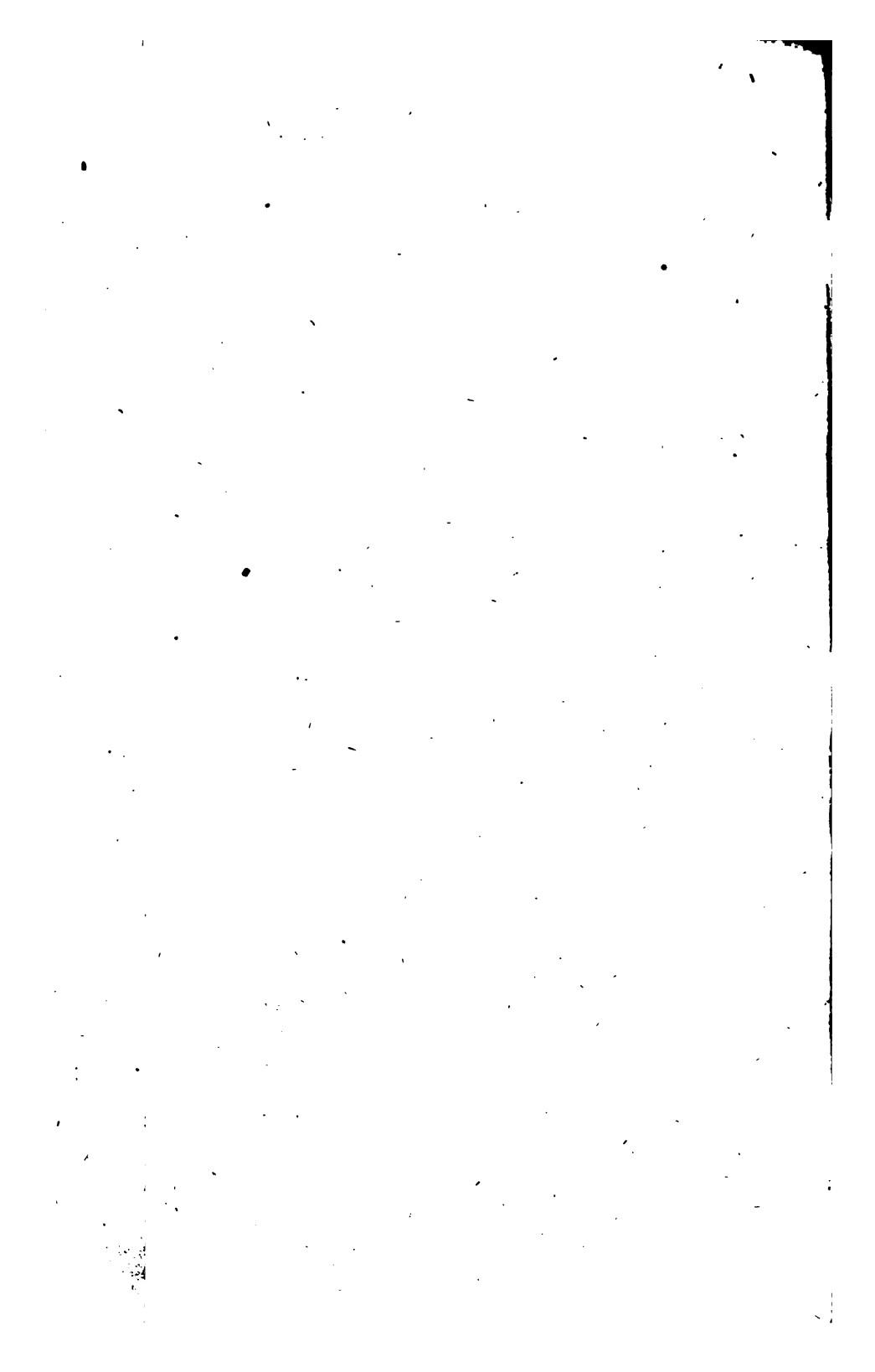
Cor. 3. *If* P *be the projected center of any circle* Fig.
EFG, *any right lines* EG, FH *passing through* P *will* 20.
intercept equal arches EF, GH.

For in any circle of the sphere, any two lines,
passing through the center, intercept equal arches ;
and these are projected into right lines, passing
through the projected center P, and therefore EF,
GH, represent equal arches.

PROP. IX.

If EFGH, *efgh represent two equal circles, where-* 20.
of EFG *is as far distant from its pole* P, *as efg is* 21.
from the projecting point. I say, any two right lines
(eEP, and fFP,) being drawn through P, *will intercept*
equal arches (in representation) of these circles; on
the same side, if P *falls within the circles; but on the*
contrary side, if without; that is, EF = *ef, and*
GH = *gh.*

For by the nature of the section of a sphere ;
any two circles passing through two given points or
poles on the surface of the sphere, will intercept
equal arches of two other circles 'equidistant from
these poles.   Therefore the circles EFG and *efg*
on the sphere, are equally cut by the planes of any
two circles passing through the projecting point and
the pole P, on the sphere.   But these circles (by
Prop. I.) are projected into the right lines P*e* and
P*f*, passing through *p*.   And the intercepted arches
representing equal arches on the sphere, are there-
fore equal, that is, EF = *ef*, and GH = *gh*.

Cor. 1. *If a circle is projected into a right line* EF, 22.
perpendicular to the line of measures EG ; *and if from*
the center C *a circle ef* P *be described passing through*
its pole P, *and* P*f be drawn; then arch ef* = EF. *And*
if any other circle be described whose vertex is P, *the*
arch ef will always be equal to EF.

Fig.   *Cor.* 2. *Hence alſo, if from the pole of a great circle there be drawn two right lines, the intercepted arch of the projected great circle will be equal to the intercepted arch of the primitive.*

23.   *Cor.* 3. *After the ſame manner if there be two*
24. *equal circles* EF, *ef, whereof one is as far from the pole* P, *as the other is from the pole of projection* C, *oppoſite to the projecting point. Then any circle drawn through the points* P, C, *will intercept equal arches* EF = *ef; and* GH = *gh, between it and the line of meaſures* PCG.

For this is true on the ſphere, and their pro-
jections are the ſame.

Cor. 4. *If from an angular point be drawn two right lines through the poles of its ſides; the intercepted arch of the primitive, will be equal to that angle.*

For the diſtance of the poles is equal to that angle.

PROP. X.

25.   *If* QH, NK *be two equal circles, whereof* NK
26. *is as far from the projecting point as* QH *from its pole* P; *and if they be projected into the circles whoſe radii are* MC *or* CL, *and* DF *or* FG, F *being the center of* DG, *and* E *the projected pole. I ſay, the pole* E *will be diſtant from their centers in proportion to the radii of the circles; that is,* CE : EF : : CL : DF *or* FG.

For ſince NK and ML are parallel, and arch
NI = PH, therefore ∠ELI = NKI (or *n*KI) =
GIP; therefore the triangles IEL and IEG are
ſimilar, whence EL : EI : : EI : EG. Again the
angle EMI = KNI = PIQ, and therefore the
triangles IEM and IED are ſimilar, whence EM :
EI : : EI : ED. Therefore EI^2 = EL × EG =
<div align="right">EM</div>

20.

f F
e E C P g G
h H

22.

H
e G

F
E e C P f G

24.

E P f F C
e E g G
h
H
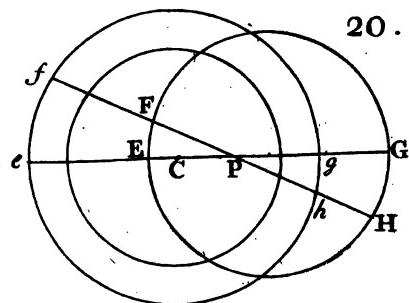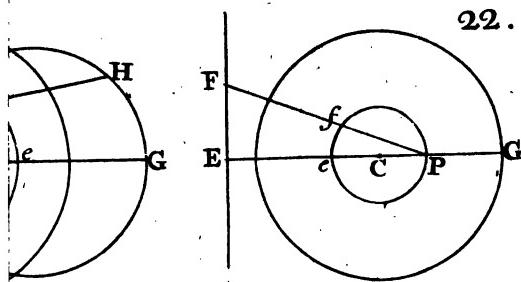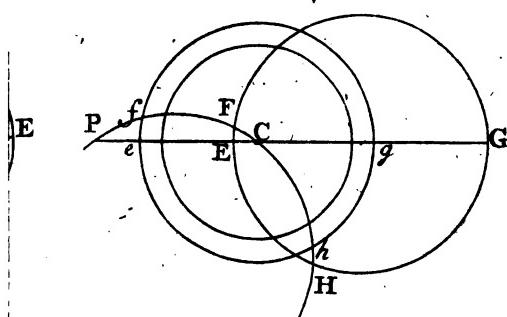
EM × ED. Confequently EM : EL : : EG : ED; Fig.

and by compofition $\dfrac{EM + EL}{2} : \dfrac{EM - EL}{2} ::$

$\dfrac{EG + ED}{2} : \dfrac{EG - ED}{2}$; that is, CM : EC : : FG

: EF. $\mathcal{Q}. E. D.$

Cor. 1. *Hence if the circle* KN *be as far from* 25.
the projecting point, as QH *is from either of its poles,* 26.
and if E, O, *be its projected poles; then will* EL :
EM : : ED : : EG : : OD : OG.

This follows from the foregoing demonftration,
and Cor. 4. Prop. VII.

Cor. 2. *Hence alfo if* E *be the center, and* FD *the* 25.
radius of any circle QH, *and* E, O *the projected* 26.
poles; then EF : DF : : DF : FO.

For it follows from Cor. 1, that $\dfrac{EG + ED}{2}$:

$\dfrac{EG - ED}{2} :: \dfrac{OG + OD}{2} : \dfrac{OG - OD}{2}$.

Cor. 3. *Hence if the circle* DBG *be as far from* 27.
its projected pole P, *as* LMN *is from the projecting* 28.
point; and if any right lines be drawn through P, *as*
MPG, NPK, *they will cut off fimilar arches* GK,
MN *in the two circles.*

For from the centers C, F, draw the lines CN,
FK, then fince the angles CPN, and FPK are
equal, and by this Prop. CP : CN : : FP : FK;
therefore (Geom. II. 16.); the triangles PCN and
PFK are fimilar; and the angle PCN = ∠PFK;
therefore the arches MN and GK are fimilar.

Cor. 4. *Hence alfo if through the projected pole* P 27.
of any circle DBG, *a right line* BPK *be drawn.* 28.
Then I fay the degrees in the arch GK *fhall be the*
meafure of DB *in the projection. And the degrees in*
DB, *fhall be the meafure of* GK *in the projection. For*

M 2   For

Fig.   For (by Prop. IX.) the arch MN is the meafure of DB, and therefore GK which is fimilar to MN will alfo be the meafure of it.

Cor. 5. *The centers of all projected circles are all beyond the projected poles (in refpect to the center of the primitives) ; and none of their centers can fall between them.*

20.   Cor. 6. *Hence it follows (by Cor. 5. and Pr. VIII. Cor. 3.) that all circles that are not parallel to the primitive have equal arches on the fphere reprefented by unequal arches on the plane of projection.*

For if P be the projected center, then GH is greater than EF.

Scholium.

It will be eafy by the foregoing propofitions to defcribe the reprefentation of any circle, and the reverfe will eafily fhow what circle of the fphere any projected circle reprefents. What follows hereafter is deduced from the foregoing propofitions and will eafily be underftood without any other demonftration.

If the fphere was to be projected on any plane parallel to the primitive, it is all the fame thing. For the cones of rays iffuing from the projecting point, are all cut by parallel planes into fimilar fections, it only makes the projections bigger or lefs, according to the diftance of the plane of projection, whilft they are ftill fimilar; and amounts to no more than projecting from different fcales upon the fame plane. And therefore the projecting the fphere on the plane of a leffer circle is only projecting it upon the great circle parallel thereto, and continuing all the lines of the fcheme to that lefler circle.

PROP.

PROP. XI. *Prob.*

To draw a circle parallel to the primitive at a given distance from its pole.

Rule.

Through the center C draw two diameters AB, 29. DE, perpendicular to one another.   Take in your compasses the distance of the circle from the pole of the primitive oppofite to the projecting point, and fet it from D to F; from E draw EF to interfect AB in I; with the radius CI, and center C, defcribe the circle GI required.

By the plane Scale.

With the radius CI, equal to the femi-tangent of the circle's diftance from the pole of projection oppofite the projecting point, defcribe the circle IG. Here the radius of projection CA, is the tangent of 45°, or the femi-tangent of 90°.

PROP. · XII. *Prob.*

To draw a leffer circle perpendicular to the primitive at a given diftance from the pole of that circle.

Rule.

Through the pole B draw the line of meafures 30. AB, make BG the circle's diftance from its pole, and draw CG, and GF perpendicular to it; with the radius FG defcribe rhe circle GI required.

By the Scale.

Set the fecant of the circle's diftance from its pole from C to F, gives the center.   With the tangent of that diftance for a radius, defcribe the circle GI.

Or thus, make BG the circle's diftance from its pole; and GF its tangent, fet from G, gives F the

center;

Fig. center; through G defcribe the circle GI from the
30. center F.

Cor. *Hence a great circle perpendicular to the pri-
mitive, is a right line* CDE *drawn through the center
perpendicular to the line of meafures.*

Scholium.

When the center F lyes at too a great a diftance;
draw EG, to cut AB in H; or lay the femi-tan-
gent of DG from C to H. And through the three
points G, H, I, draw a circle with a bow.

PROP. XIII. *Prob.*

*To defcribe an oblique circle at a given diftance from
a pole given.*

Rule.

31.   Draw the line of meafures AB through the given
point *p*, if that point is given; and draw DE ⊥
to it, alfo draw E*p*P. Or if the point *p* is not gi-
ven, fet the height of the pole above the primitive'
from B to P. Then from P fet of PH = PI = cir-
cle's diftance from its pole; and draw EH, EI, to
interfect AB in F and G. About the diameter FG
defcribe the circle required.

By the Scale.

If the point P is given, apply C*p* to the femi-tan-
gents, and it gives the diftance of the pole from D,
the pole of projection oppofite to the projecting
point. This diftance being had, you will eafily find
the greateft and neareft diftances of the circle from
the pole of the primitive oppofite to the projecting
point; take the femi-tangents of thefe diftances and
fet from C to G and F, both the fame way if the
circle lye all on one fide, but each its own way, if
on different fides of D. And then FG is the dia-
meter of the circle required to be drawn.

Cor.

26.
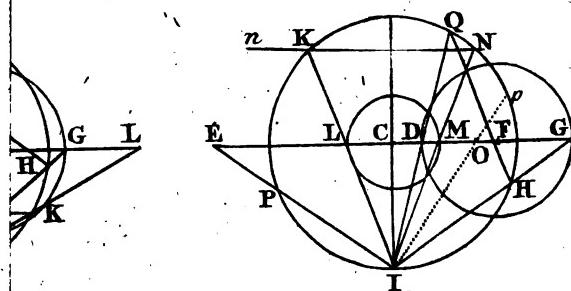
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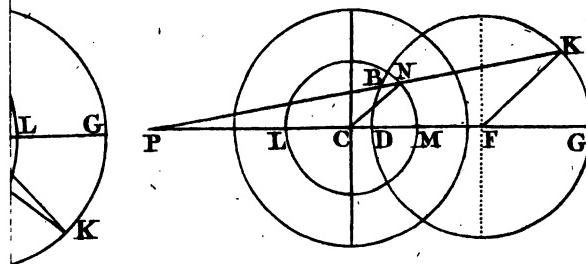
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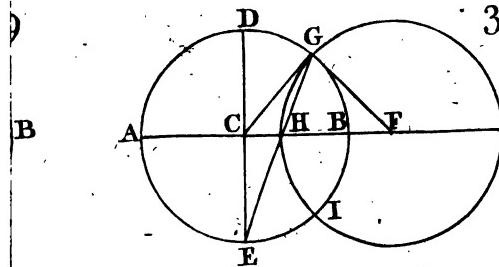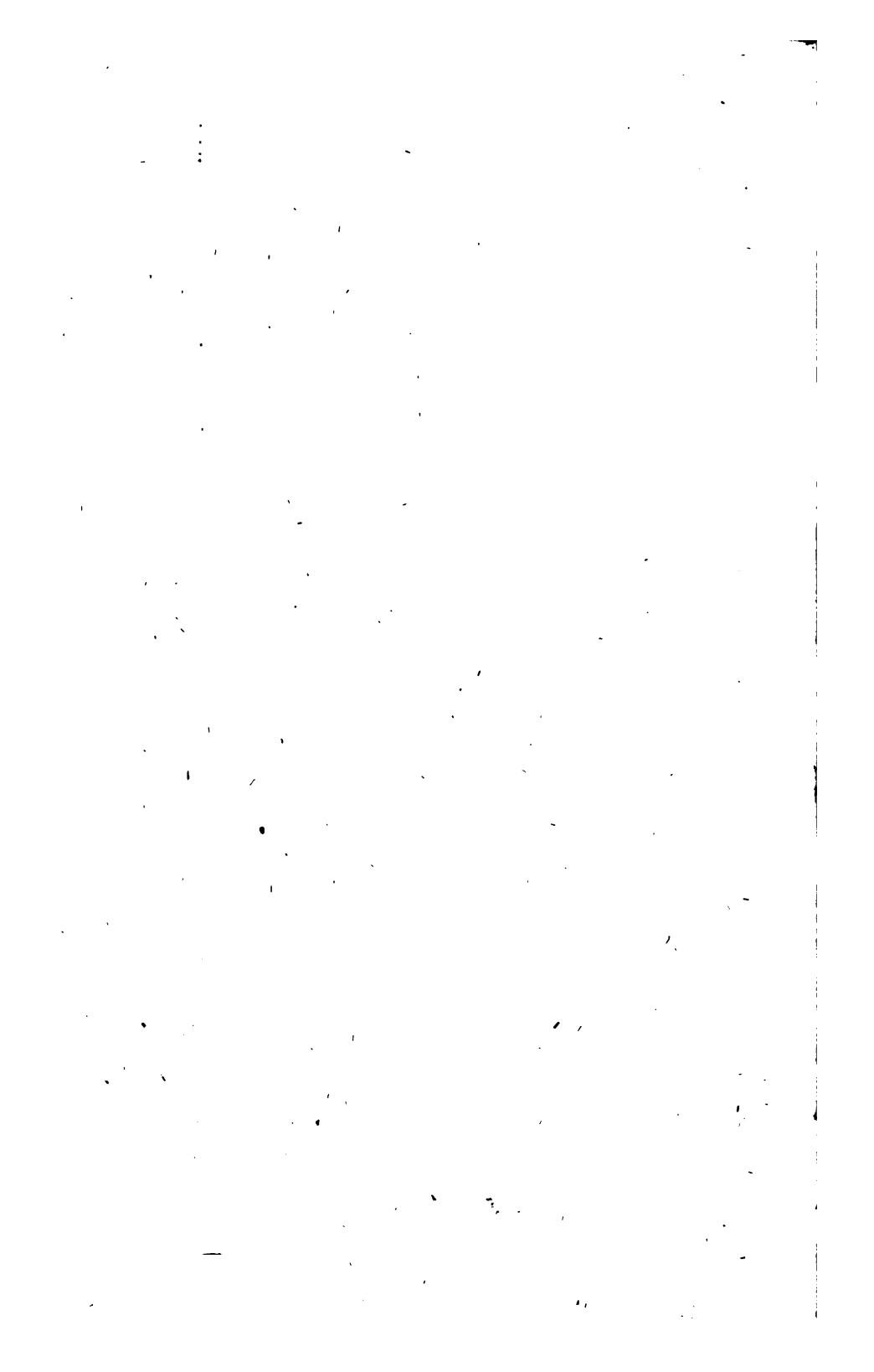
Cor. 1. *If* F *be the pole of a great circle, as of* Fig.
DLE. *Draw* EFH, *and make* HP = DH, *and* 31.
draw EpP, *and then* P *is its center.*

Or thus, draw EFH *through the pole* F, *make* HK
90 *degrees; draw* EK *cutting the line of meafures in*
L. *Through the three points* D, L, E, *draw the great
circle required.*

Cor. 2.   *Hence it will be eafy to draw one circle ·
parallel to another.*

PROP. XIV. *Prob.*

Through two given points A, B *to draw a great circle.*

Rule.

Through one of the points A, draw a line through 32.
the center, ACG; and EF perpendicular to it.
Then draw AE, and EG perpendicular to it.
Through the three points A, B, G draw the circle
·required.

Or thus; From E (found as before) draw EH,
and then HCI, and laftly EIG, gives G a third
point, through which the circle muft pafs.

By the fcale.

Draw ACG; and apply AC to the femi-tangents,
find the degrees, fet the femi-tangent of its fupple- .
ment from C to G, for a third point. ·

Or thus; Apply AC to the tangents, and fet the
tangent of its complement from C to G. And
through the three points ABG, defcribe the circle
required.

For fince HEI or AEG is a right angle, there-
fore A, G are oppofite points of the fphere; and
therefore all circles paffing through A and G are
great circles.

Scholium.

If the points A, B, G lie nearly in a right line,
then you may draw a circle through them with a
bow.   *s*

M 4   PROP.

PROP. XV. *Prob.*

About a pole given, to defcribe a circle through a given point.

Rule.

33.   Let **P** be the pole, and **B** the given point; through P, B defcribe the great circle AD (by Prop. XIV.) whofe center is E; through the center C draw CPH; and from the center E, draw EB to the point B, and BF perpendicular to it, cutting CH in F. To the center F, and radius FB defcribe the circle BGH required.

PROP. XVI. *Prob.*

To find the poles of any circle FNG.

Rule.

31.   Through its center drawn the line of meafures AG, and DE perpendicular to it. Draw EFH, and fet its diftance (from its own pole) from H to P, and draw E*p*P, then *p* is the pole.

Or thus, Draw EFH, EIG, and bifect HI in P, and draw E*p*P, and *p* is the internal pole. Laftly draw PCQ, and EQ *q*, and *q* is the external pole.

In a great circle DLE, draw ELK, and make DH = AK, or KH = AD, and draw EFH, and F is the pole.

By the Scale.

Apply CF to the femi-tangents, and note the degrees. Take the fum of thefe degrees and of the circle's diftance from its pole, if the circle lie all on one fide, but their difference if it encompaffes the pole of projection; fet the femi-tangent of this fum or difference from C to the internal pole *p*. And the femi-tangent of its fupplement C*q*, gives the external pole *q*.

Or

Or thus, Apply CF and CG to the femi-tangents, Fig. fet the femi-tangent of half the fum of the degrees 31. (if the circle lies all one way) or of half the difference (if it encompaffes the pole of projection), from C to the pole *p;* and the femi-tangent of the fupplement, C*q* gives the external pole *q.*

In a great circle as DLE, draw the line of meafures AB perp. to DE; and fet the tangent and co-tangent of half its inclination, from the center C, different ways to F and *f;* which gives the internal and external poles F and *f.*

PROP. XVII. *Prob.*

To draw a great circle at any.given inclination above the primitive; or making any given angle with it, at a given point.

Rule.

Draw the line of meafures AB; and DCE per- 34. pendicular to it.   Make EK = 2HD = twice the complement of the circle's inclination; (or DK = 2AH = twice the inclination); and draw EKF, then F is the center of EGD, the circle required.

Or thus; Draw DE and AB perp. to it, and let D be the point given.   Make AH the inclination and draw EGH and HCN; and ENO, to cut AB in O.   Then bifect GO in F, for the center of the circle required.

By the Scale.

Set the tangent of the inclination in the line of meafures from C to F, then F is the center.   Set the femi-tangent of the complement from C to G; then GF or DF is the radius.

Or the fecant of the inclination fet from G or D to F gives the center.

Cor.

Fig.   Cor. *To draw an oblique circle to make a given an-*
gle with a given oblique circle DGE *at* D.

Draw EGH, and fet the given angle from H to
I, and draw ELI.   Through D, L, E defcribe a
great circle.

But if the circle DLE was to lie on the other
fide of DGE, the arch HI muft be taken on the
other fide of H.

PROP. XVIII. *Prob.*

Through a given point P, *to draw a great circle,*
to make a given angle with the primitive.

Rule.

35.   Through the point given P, and the center C
draw the line AB; and DE perpendicular to it. Set
the given angle from A to H and from H to K,
and draw BGK; with radius CG, and center C de-
fcribe the circle GIF; and with radius BG and cen-
ter P crofs that circle in F.   Then with radius FP
and center F, defcribe the circle LPM required.

By the Scale.

With the tangent of the given angle and one
foot in C, defcribe the arch FG.   With the fecant
of the given angle and one foot in the given point
P, crofs that arch at F.   From the center F def-
cribe a circle through the point P.

PROP. XIX. *Prob.*

To draw a great circle to make a given angle with
a given oblique circle FPR, *at a given point* P, *in*
that circle.

Rule.

36.   Through the center C and the given point P,
draw the right line DE; and AB perpendicular to
it;

it; draw APG and make BM = 2DG; and draw Fig.
AM to cut DE in I. Draw IQ perpendicular to 36.
DE, then IQ is the line wherein the centers of all
circles are found which pass through the point P.'
Find N the center of the given circle FPR, and
make the angle NPL equal to the given angle,
then L is the center of the circle HPK required.

By the Scale.

Through P and C draw DE; apply CP to the
semi-tangents, and set the tangent of its comple-
ment from C to I (or the secant from P to I).
On DI erect the perpendicular IQ. Find the
center N of FPR, and make the angle NPL =
angle given, and L is the center.

Cor. *If one circle is to be drawn perpendicular to
another, it must be drawn through its poles.*

PROP. XX. Prob.

*To draw a great circle through a given point P,
to make a given angle with a given circle DE.*

Rule.

. About the given point P as a pole (by Prop. 13. 37.
Cor. 1.) describe the great circle FG; find I the pole
of the given circle DE, and (by Prop. 16.) about
the pole I (by Prop. 13.) describe the small circle
HKL at a distance equal to the given angle, to
intersect FG in H; about the pole H describe
(by Prop. 13.) the great circle APB required.

PROP. XXI. Prob.

*To draw a great circle to cut two given great circles
abd, ebf at given angles.*

Rule.

Find the poles *s*, *r*, of the two given circles, 50.
by Prop. 16. about which draw two parallels *phk*,
pnk,

Fig. *pnk,* at the diſtances reſpectively equal to the an-
50. gles given by Prop. 13. the point of interſection *p,*
is the pole of the circle *moq* required.

Cor. *Hence, to draw a right circle to make with
an oblique circle, abd, any given angle. Draw a
parallel phk at a diſtance from the pole of the oblique
circle, equal to the given angle. Its interſection f with
the primitive, gives the pole of the right circle gCt
required.*

PROP. XXII. *Prob.*

*To lay any number of degrees on a great circle, or
to meaſure any arch of it.*

Rule.

38.   Let AFI be the primitive; find the internal pole
P of the given circle DEH, or (by Prop. 16.)
lay the degrees on the primitive from A to F, and
draw PA, PF, intercepting the part required DE.
Or to meaſure DE, draw PEF and PDA, and
AF is its meaſure, and applied to the line of
chords ſhows how many degrees it is.

Or *thus*; Find the external pole *p* of the given
circle, ſet the given degrees from I to K, and draw
*p*I, *p*K, intercepting the part DE required. Or
to meaſure DE, through D and E draw *p*I, *p*K,
then KI is the meaſure of DE.

Or *thus*; Through the internal pole P, draw the
lines DPG, and EPL; ſetting the given degrees
from G to L in the circle GL; then DE is the arch
required. Or if DE be to be meaſured, then the
degrees in the arch GL is the meaſure of DE.

Or *thus*; Set the given degrees from G to H in
the circle GL and from the external pole *p,* draw
*p*G, *p*H, intercepting DE the arch required. Or
to meaſure DE, draw *p*DG, *p*EH, then the
degrees in GH, is equal to DE.

For

32.

34.

36.
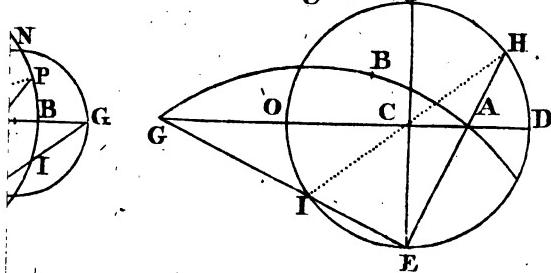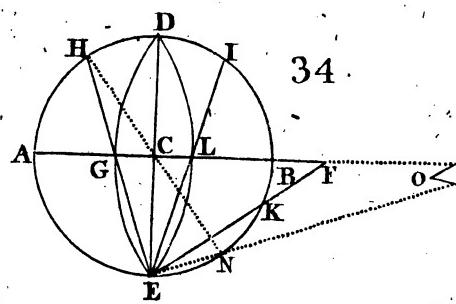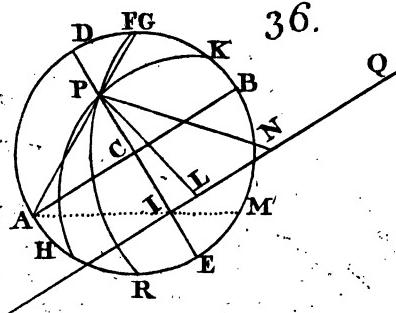
For a right Circle.

Through C (fig. 34.) draw DE perpendicular to AB, then E is the pole of AB. Set the number of degrees from D to H, and draw EGH, then GC contains the degrees required. Or if GC be given, draw EGH, then HD in degrees, is the meafure of GC.

And in like manner AG contains the degrees in AH, or AH is the meafure of AG.

By the Scale for right Circles.

Let CA be the right circle, take the number of 38. degrees off the femi-tangents and fet from C to D for the arch CD. Or if the given degrees are to be fet from A, then take the degrees off the femi-tangents from 90° towards the beginning, and fet from A to D. And if CD was to be meafured, apply it to the beginning of the femi-tangents; and to meafure AD, apply it from 90° backwards, and the degrees intercepted gives its meafure.

SCHOLIUM.

The primitive is meafured by the line of chords, or elfe it is actually divided into degrees.

PROP. XXIII. Prob.

To fet any number of degrees on a leffer circle, or to meafure any arch of it.

Rule.

Let the leffer circle be DEH; find its internal 38. pole P, by Prop. 16. defcribe the circle AFK parallel to the primitive, by Prop. 11. and as far from the projecting point, as the given circle DE is from its internal pole P, fet the given degrees from A to F, and draw PA, PF interfecting the given circle

in

Fig. in D, E; then DE is the arch required. Or to
38. meafure DE, draw PDA, PEF, and AF fhows
the degrees in DE.

Or thus ; Find the external pole p, of the given
circle by Prop. 16. defcribe the leffer circle AFK
as far from the projecting point, as DE the given
circle is from its pole p by Prop. 11. fet the de-
grees from I to K and draw pDI, pEK, then DE
reprefents the given number of degrees. Or to
meafure DE; draw pDI, pEK ; and KI is the mea-
fure of DE.

Or thus; Let O be the center of the given circle
DEH ; through the internal pole P, draw lines
DPG, EPL, divide the quadrant GQ into 90
equal degrees, and if the given degrees be fet from
G to L, and LPE drawn, then DE will reprefent
thefe degrees. Or the degrees in GL will meafure
DE.

Or thus ; Divide the quadrant GR into 90 equal
parts or degrees, and fet the given degrees from G
to H, and draw pDG, pEH, from the external pole
p; then DE will reprefent the given degrees. Or
through D, E drawing pDG, pEH, then the num-
ber of equal degrees in GH is the meafure of DE.

SCHOLIUM.

Any circle parallel to the primitive is divided or
meafured, by drawing lines from the center, to the
like divifions of the primitive. Or by help of the
chords on the fector, fet to the radius of that circle.

PROP. XXIV. Prob.

To meafure any angle.

Rule.

By Cor. 1. Prop. 13. About the angular point as
a pole, defcribe a great circle, and note where it
interfects the legs of the angle; through thefe points
of

of interfection, and the angular point, draw two Fig.
right lines, to cut the primitive; the arch of the
primitive intercepted between them is the meafure
of the angle.   This needs no example.

Or thus; by Prop. 16.   Find the two poles of
the containing fides, (the neareft, if it be an acute
angle, otherwife the furtheft) and through the an-
gular point and thefe poles, draw right lines to the
primitive, then the intercepted arch of the primitive
is the angle required. As if the angle AEL was re- 31.
quired.   Let C and F be the poles of EA and EL.
From the angular point E, draw ECD and EFH.
Then the arch of the primitive DH, is the meafure
of the angle AEL.

<div align="center">SCHOLIUM.</div>

Becaufe in the Stereographic Projection of the
Sphere, all circles are projected either into circles
or right lines, which are eafily defcribed; therefore
this fort of projection is preferred before all others.
Alfo thofe planes are preferred before others to pro-
ject upon, where moft circles are projected into
right lines, they being eafier to defcribe and meafure
than circles are; fuch are the projections on the
planes of the meridian and folftitial colure.

Fig.

SECT. III.

The Gnomonical Projection of the SPHERE.

PROP. I.

39. *E*VERY *great circle as* EAD *is projected into a right line, perpendicular to the line of measures, and distant from the center, the co-tangent of its inclination, or the tangent of its nearest distance from the pole of projection.*

Let CBED be perpendicular both to the given circle BAD and plane of projection, and then the interfection CF will be the line of meafures. Now fince the plane of the circle BD, and the plane of projection are both perpendicular to BCDE, therefore their common fection will alfo be perpendicular to BCDE, and confequently to the line of meafures CF. Now fince the projecting point A is in the plane of this circle, all the points of it will be projected into that fection; that is, into a right line paffing through *d*, and perpendicular to C*d*. And C*d* is the tangent of CD, or co-tangent of C*d*A. *Q. E. D.*

39. Cor. 1. *A great circle perpendicular to the plane of projection is projected into a right line paffing through the center of projection; and any arch is projected into its correfpondent tangent.*

Thus the arch CD is projected into the tangent C*d*.

Cor. 2. *Any point as* D, *or the pole of any circle, is projected into a point* d *diftant from the pole of projection* C, *the tangent of that diftance.*

Cor 3. *If two great circles be perpendicular to each other, and one of them paffes through the pole of projection;*

Fig. 39.

*tion; they will be projected into two right lines per-
pendicular to each other.*

For the reprefentation of that circle which paffes
through the pole of projection is the line of mea-
fures of the other circle.

Cor. 4. *And hence if a great circle be perpendicu-
lar to feveral other great circles, and its reprefentation
pafs through the center of projection; then all thefe cir-
cles will be reprefented by lines parallel to one another,
and perpendicular to the line of meafures or reprefen-
tation of that firft circle.*

PROP. II.

*If two great circles interfect in the pole of projec-
tion; their reprefentations fhall make an angle at the
center of the plane of projection equal to the angle
made by thefe circles on the fphere.*

For fince both thefe circles are perpendicular to
the plane of projection; the angle made by their
interfections with this plane, is the fame as the
angle made by thefe circles. $Q. E. D.$

PROP. III.

*Any leffer circle parallel to the plane of projection
is projected into a circle, whofe center is the pole of
projection, and radius the tangent of the circle's
diftance from the pole of projection.*

Let the circle PI be parallel to the plane GF.
Then the equal arches PC, CI are projected into
the equal tangents GC, CH; and therefore C the
point of contact and pole of the circle PI and of
the projection, is the center of the reprefentation
GH. $Q. E. D.$

Cor. *If a circle be parallel to the plane of projec-
tion, and 45 degrees from the pole, it is projected into*
a circle

Fig. *a circle equal to a great circle of the sphere; and may therefore be looked upon as the primitive circle in this projection, and its radius the radius of projection.*

PROP. IV.

40. *Every lesser circle (not parallel to the plane of projection) is projected into a conic section, whose transverse axis is in the line of measures, and whose nearest vertex is distant from the center of the plane the tangent of its nearest distance from the pole of projection; and the other vertex is distant the tangent of its furthest distance.*

Let BE be parallel to the line of measures *dp*, then any circle is the base of a cone whose vertex is at A, and therefore that cone being produced will be cut by the plane of projection in some conic section; thus the circle whose diameter is DF will be cut by the plane in an ellipsis whose transverse is *df*; and C*d* is the tangent of CAD, and C*f* of CF. In like manner the cone AFE being cut by the plane, *f* will be the nearest vertex; and the other point into which E is projected, is at an infinite distance. Also the cone AFG (whose base is the circle FG) being cut by the plane *f* is the nearest vertex; and GA being produced gives *d* the other vertex. *Q. E. D.*

Cor. 1. *If the distance of the furthest point of the circle be less than 90° from the pole of projection, then it will be projected into an ellipsis.*

Thus DF is projected into *df*, and DC being less than 90°, the section *df* is an ellipsis, whose vertices are at *d* and *f*; for the plane *df* cuts both sides of the cone, *d*A, *f*A.

Cor. 2. *If the furthest point be more than 90 degrees from the pole of projection, it will be projected*
into

into an hyperbola. Thus the circle FG is projected Fig. *into an hyperbola whose vertices are f and d, and* 40. *transverse fd.*

For the plane *dp* cuts only the side A*f* of the cone.   •

Cor. 3. *And in the circle* EF, *where the furthest point* E *is* 90° *from* C; *it will be projected into a parabola, whose vertex is f.*

For the plane *dp* (cutting the cone FAE) is parallel to the side AE.

Cor. 4. *If* H *be the center, and* K, *k*, *l*, *the focus of the ellipsis, hyperbola, or parabola; then* HK = $\dfrac{Ad - Af}{2}$ *for the ellipsis, and* H*k* = $\dfrac{Ad + Af}{2}$ *for the hyperbola; and (drawing fn perpendicular on* AE) *fl* = $\dfrac{nE + Ff}{2}$, *for the parabola; which are the representations of the circles* DF, FG, FE *respectively.*

This all appears from the Conic Sections.

PROP. V.

Let the plane TW *be perpendicular to the plane of* 41. *projection* TV, *and* BCD *a great circle of the sphere in the plane* TW. *And let the great circle* BED *be projected in the right line bek.* *Draw* CQ*s* ⊥ *bk*, *and* C*m* ∥ *to it and equal to* CA; *and make* Q*s* = Q*m*; *then I say any angle* Q*st* = Q*t*.

Suppose the hypothenuse AQ to be drawn, then since the plane ACQ is perpendicular to the plane TV, and *b*Q is ⊥ to the intersection CQ, therefore *b*Q is perpendiclar to the plane ACQ, and consequently *b*Q is perpendicular to the hypothenuse AQ. But AQ=Q*m*=Q*s*, and Q*s* is also perpendicular to *b*Q. Therefore all angles made at S, cut the line *b*Q in the same points as the angles made

at

Fig.
41.

at A; but by the angles at A the circle BED is projected into the line *b*Q. Therefore the angles at *s* are the meafures of the parts of the projected circle *b*Q; and *s* is the dividing center thereof. *Q. E. D.*

Cor. 1. *Any great circle t*Q*b is projected into a line of tangents to the radius* SQ.

For Q*t* is the tangent of the angle QS*t* to the radius QS or Q*m*. . _

Cor. 2. *If the circle b*C *pafs through the center of projection; then* A *the projecting point is the dividing center thereof. And C*b *is the tangent of its correfpondent arch* CB*, to* CA *the radius of projection.*

53-2. Cor. 3. *If* P *be the point where the great circles* PC, PF, *&c. meet (as at the pole); and if* CB, *the radius of projection, be perpendicular to* CP, *at* C *the point of contact. Then the dividing center* A *of* PF, *is diftant from* P, *the length of* PB.

For ICA is perpendicular to PF, and IA = FC; whence PA² = PI² + IA² = PI² + FC² = PI² + FI² + IC² = PC² + FI² = PC² + CB² = PB², and PA = PB.

PROP. VI.

41.   *Let the parallel circle* GEH *be as far from the pole of projection* C *as the circle* FKI *is from its pole* P; *and let the diftance of the poles* C, P *be bifected by the radius* AO, *and draw b*AD *perpendicular to* AO; *then any right line b*e*k drawn through b, will cut off the arches b*l = F*n, and ge = kf (fuppofing f the other vertex), in the reprefentations of thefe equal circles in the plane of projection.*

For let G, E, R, L, H, N, R, K, I be refpectively projected into the points *g, e, r, l, h, n, r, k, f*. Then fince in the fphere, the arch BF = DH, and arch BG = DI. And the great circle BEKD makes the angles at B and D equal, and

is

38.

40.

41.
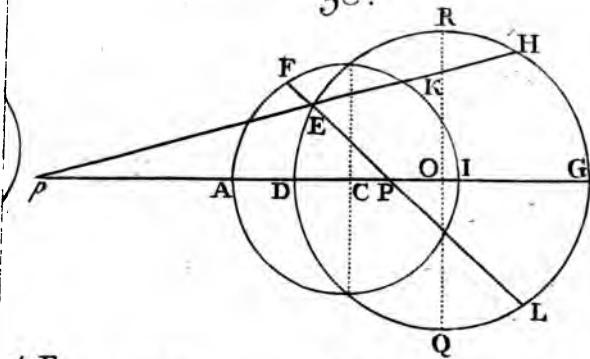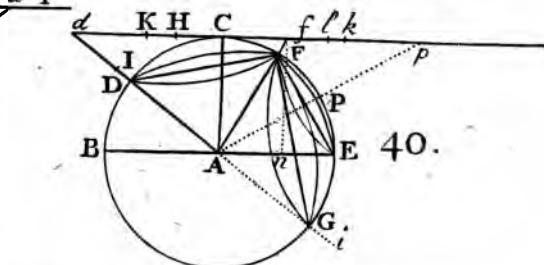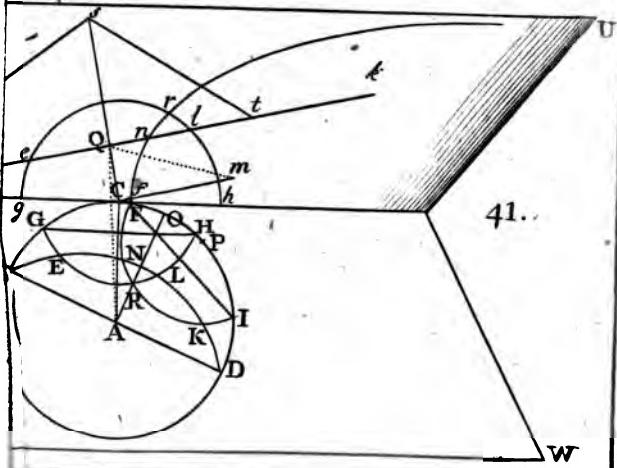
is projected into a right line as *bl*; therefore the Fig.
triangular figures BFN and DHL are similar, and 41.
equal; and likewise BGE and DIK are similar and
equal, and LH=NF, and KI=EG; whence
it is evident their projections *lh = n*F, and *kf = ge*.
Q. E. D.

PROP. VII.

If blg and Fnk be the projections of two equal cir- 42.
cles, whereof one is as far from its pole P *as the other*
from its pole C; *which is the center of projection;*
and if the distance of the projected poles C, *p be*
divided in o, *so that the degrees in Co, op, be equal,*
and the perpendicular oS be erected to the line of
measures gb. I say the lines pn, Cl, drawn from
the poles C, *p through any point* Q *in the line oS,*
will cut off the arch Fn = ∠QCp.

For drawing the great circle GPI, in a plane
perpendicular to the plane of projection. The
great circle AO perpendicular to CP is projected
into *o*S by Prop. I. Cor. 3.   Now let Q be the
projection of *q*, and since *p*Q, CQ are right lines,
therefore they represent the great circles P*q*, C*q*.
But the spherical triangle P*q*C is an isoceles-trian-
gle, and therefore the angles at P and C are equal.
But because P is the pole of FI, therefore the
great circle P*q* continued, will cut an arch off FI
= ∠CP*q* = ∠PC*q* = ∠QC*p* by Prop. II.   That
is (since F*n* represents the part cut off from FI)
arch F*n* = arch *lh* or ∠QC*h*.   *Q. E. D.*

Cor. *Hence if from the projected pole p of any*
circle, a perpendicular be erected to the line of mea-
sures; it will cut off a quadrant from the represen-
tation of that circle.

For that perpendicular will be parallel to *o*S;
Q being at an infinite distance.

PROP. VIII.

42.   *Let Fnk be the projectiou of any circle* FI, *and p
the projected pole* P.   *And if* Cg *be the co-tangent of
. CAP, and gB perpendicular to the line of meafures
gC, and CAP be bifected by* AO, *and the line oB,
be drawn to any point* B, *and alfo pB cutting Fnk in
d.   I fay the angle* goB = *arch* Fd.

For the arch PG is a quadrant, and the $\angle goA$
$= \angle gpA + \angle oAp =$ (becaufe GCA and gAp are
right angles) $gAC + oAp = gAC + CAo = \angle gAo$.
Therefore $gA = go$, confequently o is the dividing
center of gB the reprefentation of GA; and con-
fequently by Prop. V. $\angle goB$ is the meafure of
gB.   But fince pq reprefents a quadrant, therefore
p is the pole of gB, and therefore the great circle
pdB paffing through the pole of the circles gB
and Fn will cut off equal arches in both, that is
F$d = gB = \angle goB$.   *Q. E. D.*

Cor. *The $\angle goB$ is the meafure of the angle gpB.*
For the triangle gpB reprefents a triangle on the
fphere wherein the arch which gB reprefents is
equal to the angle which $\angle p$ reprefents, becaufe
gp is 90 degrees.   Therefore $g$$o$B is the meafure
of both.

Scholium.

Thus far I have treated of the theory; what
follows is the practical part, and depends altoge-
ther on what is above delivered, in which I think.
no difficulty can occur.   In the Gnomonical Pro-
jection, the plane projected on, is fuppofed to touch
the hemifphere to be projected, in its vertex; and
the point of contact will be *the center of projection.*
But if it be required to project upon any plane pa-
rallel

rallel to this touching plane, the procefs will be no Fig.
way different, and is only taking a greater or lefler 42.
radius of projeftion, according to the greater or
lefler diftance; which is in effeft projefting a great-
er or lefler fphere upon its touching plane.

When you have the fphere to projeft this way,
upon a given plane; it will affift the imagination,
if you fuppofe yourfelf placed in the center of the
fphere with your face towards the plane, whofe pofi-
tion is given; and from thence projefting with
your eye, the circles of the fphere upon this plane.

PROP. IX.   *Prob.*

To draw a great circle, through a given point, and 43·
at a given diftance from the pole of projeftion.

Rule.

Defcribe the circle ADB with the radius of pro-
jeftion, and through the given point P draw the
right line PCA, and CE perpendicular to it; make
the angle CAE=given diftance of the circle from
C, and through E defcribe the circle EFG and
through P draw the line PK touching the circle in I,
then is PIK the circle required.

By the plane Scale.

With the tangent of the circle's diftance from
the pole of projeftion C, defcribe the circle EIF,
and draw PK to touch this circle; and PIK is the
circle required.

PROP. X.   *Prob.*

To draw a great circle perpendicular to a given 43·
great circle, which paffes through the pole of projeftion;
and at a given diftance from that pole.

Rule.

Draw the primitive ADB.   Let CI be the given
circle, draw CL perpendicular to CI, and make the

Fig. angle CLI = the given diftance; through I draw
43. KP parallel to CL for the circle required.

By the Scale.

In the given circle CI, fet the tangent of the
given diftance, from C to I; through I draw KP
perpendicular to CI, then KP is the circle required.

PROP. XI. Prob.

44. *To meafure any part of a great circle; or to fet
any number of degrees thereon.*

Rule.

Let EP be the great circle; through C draw
ID perpendicular to EP, and CB parallel to it.
Let EBD be a circle defcribed with the radius of
projeƈion CB, make IA=IB; then A is the divi-
ding center of EP, confequently drawing AP, the
∠IAP=meafure of the given arch IP.

Or thus, Set CD from I to F, and FC from I
to A, and draw AP, &c.

Or if the degrees be given, make the ∠IAP
= the given degrees, which cuts off IP, the arch
correfpondent thereto.

By the Scale.

Draw ICD perpendicular to EP; apply CI to
the tangents, and fet the femi-tangent of its com-
plement from C to A, gives the dividing center of
EP, &c.

PROP. XII. Prob.

51. *To draw a great circle to make a given angle with
a given great circle, at a given point; or to meafure
an angle made by two great circles.*

Rule.

Let P be the given point, and PB the given great
circle. Draw through P, and C the center of pro-
jeƈion,

jection, the line PCG, to which from C draw CA Fig.
perpendicular, and equal to the radius of projec- 51.
tion. Draw PA and AG perpendicular to it, at G
erect BD perpendicular to GC, cutting PB in B;
draw AO bisecting the angle CAP; then at the
point O, make BOD = angle given, and from D
draw the line DP, then BPD is the angle required.

Or if the degrees in the angle BPD be required,
from the points B, D, draw the lines BO, DO;
and the angle BOD is the measure of BPD.

Cor. *If an angle be required to be made at the pole
or center of projection, equal to a given angle; this
is no more than drawing two lines from the center
making the angle required.   And if one great circle
be to be drawn ⊥ to another great circle, it must be
drawn through its pole.*

PROP. XIII. *Prob.*

To project a lesser circle parallel to the primitive. 43.
Rule.

With the radius of projection AC, and center
C, describe the primitive circle ADB, by Cor. Prop.
III. and draw ACB, and GCE perpendicular to it.
Set the circle's distance from its pole from B to
H, and from H to D, and draw AFD.   With
radius CE describe the circle EFG required.

By the Scale.

With the radius CE equal to the tangent of the
circle's distance from its pole, describe the circle
EFG, for the circle required.

PROP. XIV. *Prob.*

To draw a lesser circle perpendicular to the plane 48.
of projection.
Rule.

Through the center of projection C, draw its
parallel great circle TI. At C make the angle ICN
and

Fig. and TCO = the given circle's diſtance from its
48. parallel great circle TI; make CL equal radius of
projection, and draw LM perpendicular to CL.
Set LM from C to V, or CM from C to F. Then
through the vertex V between the aſymptotes CN,
CO deſcribe the hyperbola WVK. Or to the fo-
cus F, and ſemi-tranſverſe CV, deſcribe the
hyperbola; for the circle required.

<div align="center">Otherwiſe by Points.</div>

Through the center of projection C draw the
line of meaſures FC, and TCI perpendicular to it,
draw any number of right lines CV, DE, GH,
IK &c. and PQ, RS, TW, &c. perpendicular to
TI. And by Prop. XI. make CV, DE, GH, &c.
each equal to the diſtance of the given circle from
its parallel great circle; then all the points W, S,
Q, V, E, H, K, &c. joined by a regular curve
will be the repreſentation of the circle required.

<div align="center">Or thus.</div>

Make the angle _iak_= diſtance of the given cir-
cle from its parallel great circle. Then through
the center of projection C, draw the great circle
TCI parallel to the circle given, upon which erect
the perpendicular CA = radius of projection. Alſo
draw any number of right lines CV, DE, GH, IK,
&c. perpendicular to Tl. Then take each of the
diſtances from A to C, D, G, I, &c. and ſet them
from _a_ to _c_, _g_, _d_, _i_, &c. and to _ai_ draw the per-
pendiculars _cv_, _dc_, _gb_, _ik_, &c. and make CV,
DE, GH, IK, &c. reſpectively equal to _cv_, _de_,
_gb_, _ik_, &c. which gives the points V, E, H, K,
&c. after the ſame manner on the other ſide, find
the points Q, S, W, &c. then through all theſe
points W, S, Q, V, E, H, K, &c. draw a
regular curve, which will be an hyperbola repre-
ſenting the circle given.

<div align="right">By</div>

Fig. 48.

By the Scale.

Take the tangent of the circle's diftance from its parallel great circle, and fet it from C (the center of projection) to V, and the fecant thereof from C to F. Then with the femi-tranfverfe CV, and focus F, defcribe the hyperbola WVHK.

PROP. XV. Prob.

To project any leffer oblique circle given.

Rule.

Draw the line of meafures *dp*, and at C the center of projection draw CA ⊥ to *dp* and = radius of projection; with the center A, defcribe the circle DCFG; and draw RAE parallel to *dp*. Then take the greateft and leaft diftances of the circle from the pole of projection and fet from C, to D and F, for the circle DF; and from A, the projecting point, draw AF*f*, and AD*d*, then *df* will be the tranfverfe axis of the ellipfis. But if D fall beyond the line RE, as at G, then draw a line from G backward through A to *d*, and then *df* is the tranfverfe of an hyperbola. But if the point D fall in the line RE as at E, then the line AE no where meets the line of meafures, and the projection of E is at an infinite diftance, and then the circle will be projected into a parabola whofe vertex is *f*. Laftly, bifect *df* in H the center, and for the ellipfis take half the difference of the lines A*d*, A*f*, and fet from H to K for the focus. But for the hyperbola take half the fum of A*d*, A*f*, and fet from H to the focus *k* of the hyperbola. Then with the tranfverfe *df* and focus K or *k* defcribe the el-li.pfis *d*M*f*, or the hyperbola *fm*. For the projection of the circle given.

But for the parabola make EQ = F*f*, and draw *fn* ⊥ AQ, and fet ⅓*n*Q from *f* to K the focus. Then

with

Fig. with the vertex *f* and focus *k* defcribe the parabola *fm*, for the projeċtion of the given circle FE.

Otherwife by Points.

49.   Through the center of projeċtion C, draw the line of meafures CF, paffing through the pole P (if P is given ; but if not, find it, by .fetting off CP = the diftance of that pole, from the center of projeċtion, by Prop. XI.) then fet off PD, PF, equal to the given diftance from its pole, by Prop. XI. Through P draw a fufficient number of right lines, Lλ, Mµ, N*n*, O*o*, R*r*, S*s*, &c. which will all reprefent great circles.   Find the dividing centers of each of thefe lines ; and by Prop. XI. fet off upon each of them from P, the given diftance of the circle from its pole, as PL, Pλ, PM, Pµ, &c. and through all the points L, M, D, O, R, &c. draw a curve line, for the circle required.

Note, the dividing center of all the lines, PM, PL, PN, &c. is always diftant from P the length of PW.   Therefore all the dividing centers are in the circumference of a circle, whofe radius is PW, and center P; fee below, Cor. 2.

Or thus.

49.   Draw the line of meafures .PCG, and by Prop. XI. make CG = the diftance of the parallel great circle from the pole of projeċtion, and draw AGK perpendicular to it, which will reprefent a great circle whofe pole is P. Draw any number of right lines through P to AK, as AP, BP, HP, &c. and by Prop. XI. fet off from AK the parts AL, BM, HO, &c. each equal to the circle's diftance from its pa- rallel great circle.   Then all the points L, M, D, O, &c. being joined by a regular curve, will reprefent the parallel circle required.

Or

Or thus.

Through the center of projection C draw the
line of meafures DCF, and the radius of projection
CW perpendicular to it, and AGK ⊥ GC, for a
great circle whofe pole is **P**.   Draw *vp* = WP,
and *wa* ⊥ to it, draw any number of right lines,
AP, BP, GP, &c. and make *pg*, *pb*, *pa*, &c. =
PG, PB, PA, &c. alfo make the ∠*pwl* and *pwx*
= the circle's diftance from its pole P (or *awl* =
the diftance from its parallel great circle); and
upon PG, PB, PA, &c. make PD, PM, PL, &c.
= *pd*, *pm*, *pl*, &c. refpectively.

Or make GD, BM, AL, &c. = *gd*, *bm*, *al*, &c.
After the fame manner, find the points O, R, &c.
and through all the points R, O, D, M, L, &c.
draw a regular curve, making no angles, which
will reprefent the parallel required. Likewife where
any line *ap* cuts *wx*, that diftance from *p* will give
the point λ, or *px* = Pλ; and fo of any other of
the lines *bp*, *gp*, &c.

*The reafon of this procefs will be plain, if you fup-
pofe the point p, w applied to P, W; and g, b, a, &c.
fucceffively to G, B, A, &c. for then d, m, l, will fall
upon D, M, L, &c.*

'   *By the Scale.*

Take the tangents of the circle's neareft and 45.
furtheft diftance from the pole of projection, and
fet from C to *f* and *d*, gives the vertices, and bifect
df in H; then take half the difference, or half the
fum, of the fecants of the greateft and leaft dif-
tance from the pole of projection, and fet from
H, to K or *k* for the focus of the ellipfis or hy-
perbola, which may then be defcribed.

Cor. 1. *If the curve be required to pafs through a* 49.
given point S; *meafure* PS *by Prop.* XI, *and then
the curve may be drawn by this Problem.*

Cor.

Fig.   Cor. 2. *If* P *be the point where the great circles*
53-3.AP, BP, *&c. meet; then to find the dividing centers*
of all the circles AP, BP, *&c.*

    Through C the center of projection draw PCG.
Through C draw CW perpendicular to PG, and
equal to the radius of projection. Draw PW,
and WG perpendicular to it, to cut GP in G.
At G and the middle of GP, erect the perpendi-
culars GA and DS. With the radius PW defcribe
the circle EWO. Then to find the dividing
center of any circle AP ; with the diftance LP,
and one foot in L, cut the circle OWE in R,
the dividing center required.

    For fince PWG is a right angle, PG is 90
degrees, and fo is PA. Therefore the dividing
center of AP is in a femi-circle defcribed round
AP, and it is alfo in the circle OWR (by Cor.
3. Prop. V.) and therefore at the interfection R.
But DS bifects all thefe circles; and therefore the
center of that femi-circle is in L.

PROP. XVI. *Prob.*

47. *To find the pole of any circle in the projection,* DMF.

Rule.

    From the center of projection C, draw the radius
of projection CA perpendicular to the line of mea-
fures DF. And to A the projecting point, draw
DA, FA, and bifect the angle DAF by the line AP,
then P is the pole. But if the curve be an hyper-
pola, as *fm, fig.* 45, you muft produce *d*A, and bi-
fect the angle *f*AG. And in a parabola, where the
point *d* is at an infinite diftance, bifect the angle *f*AE.

    Or thus; Drawing CA perpendicular to DC, draw
DA, and make the angle DAP = the circle's dif-
tance from its poles, give the pole P.

By

Draw the radius of projection CA ⊥ to the line of meafures DF.   Apply CD, CF to the tangents, and fet the tangent of half the difference of their degrees from C; to P, if D, F lye on contrary fides of C; but half the fum if on the fame fide, gives P the pole.

Or thus; By Prop. XI. fet off from D to P, the circle's diftance from its pole, gives the pole P.

Cor. *If it be a great circle as* BG; *draw the line* 46. *of meafures* GC, *and* CA ⊥ *to it, and equal to the radius of projection; make* GAP *a right angle, and* P *is the pole.*

PROP. XVII. *Prob.*

To meafure any arch of a leffer circle; or to fet any number of degrees thereon.

Rule. .

Let F*n* be the given circle.   From the center of 46. projection C, draw CA perpendicular to the line of meafures GH, and equal to the radius of projection.   To P the pole of the given circle draw AP, and AO bifecting the angle CAP.   And draw AD perpendicular to AO.   Defcribe the circle G*l*H (by Prop. XIII.) as far from the pole of projection C, as the given circle is from its pole P.   And through any given point *n* in the circle F*n*, draw D*nl*, gives H*l* the number of degrees = F*n*.   Or the degrees being given and fet from H to *l*, the line D*l* cuts off F*n* equal thereto.

Or thus; AO being drawn as before, erect OS perpendicular to CO; through the given point *n* draw P*n* cutting OS in Q, then through Q draw C*l*, and the angle QCP is = F*n*.   Or making QCP = the degrees given, draw PQ*n*, and arch F*n* = thefe degrees.

Or

Fig. 46.

Or thus; AO, AP being drawn as before, draw AG perpendicular to AP, and GB perpendicular to GC. Through the given point *n* draw PB cutting GB in B, and draw OB, then the ∠ GOB = arch F*n* Or making ∠ GOB = the given degrees; draw PB, and it cuts off F*n* = the degrees given.

By the Scale.

Let C be the center of projection, P the pole of the given circle, Apply CP to the tangents, and set the tangent of its half from C to O, and the cotangent of its half from C to D; with radius CG = tangent of the degrees in FP the given circle's distance from its pole, describe the circle GSH. Then D*l* drawn through *n* or *l*, cuts off H*l* = F*n*.

Or thus; O being found as before, erect OS perpendicular to CO; through the given point *n* draw PQ*n*, and ∠ QCH = F*n*.

Or thus; Apply CP to the tangents, and set the co-tangent thereof from C to G. Erect GB perpendicular to GC. Through *n* draw P*n*B, and draw BO; then ∠ GOB = F*n*.

48. Cor. *If the leffer circle be perpendicular to the plane of projection as* VHK. *You have no more to do but to draw the perpendiculars* VC, HG, *to its parallel great circle* CI. *Then* CG *(meafured by Prop.* XI.*) will be equal to* VH; *or the degrees fet from* C *to* G, *cuts off* VH *equal thereto.*

SCHOLIUM.

48. ` This fort of projection is little ufed, by reafon feveral of the circles of the fphere fall in ellipfes and hyperbolas, which are very difficult to defcribe. Notwithftanding it is very convenient for folving fome Problems of the fphere, becaufe all the great circles are projected into right lines. And this fort, or the Gnomonic Projection, is the very foundation of all dialling. For if the fphere be projected on

any

43.

P

B

H

F

D I

E C G

K L A

m

M

45.

d K H C f k p

F

D

R A n E Q

G

S

l

Q

O P H

M

47.

D C P F

A

A

VII. pa. 208.
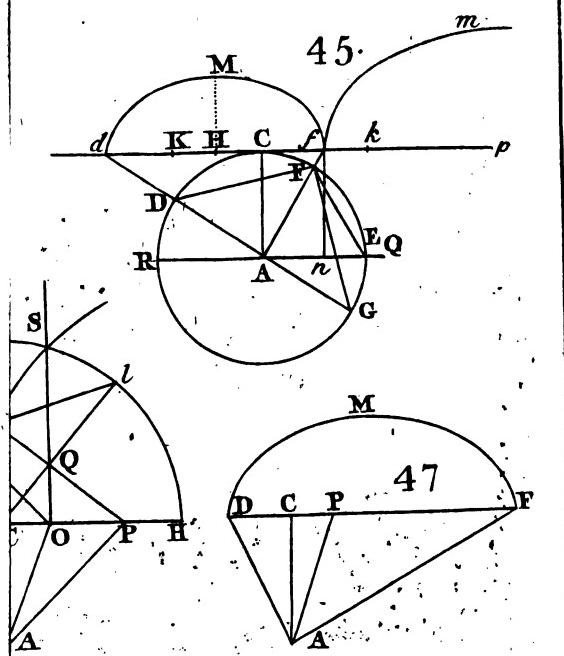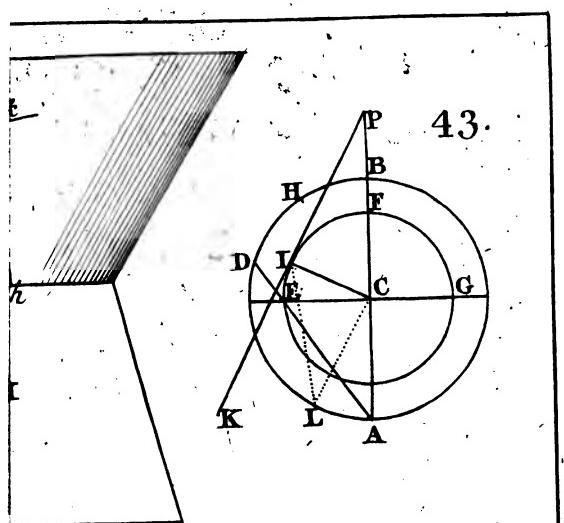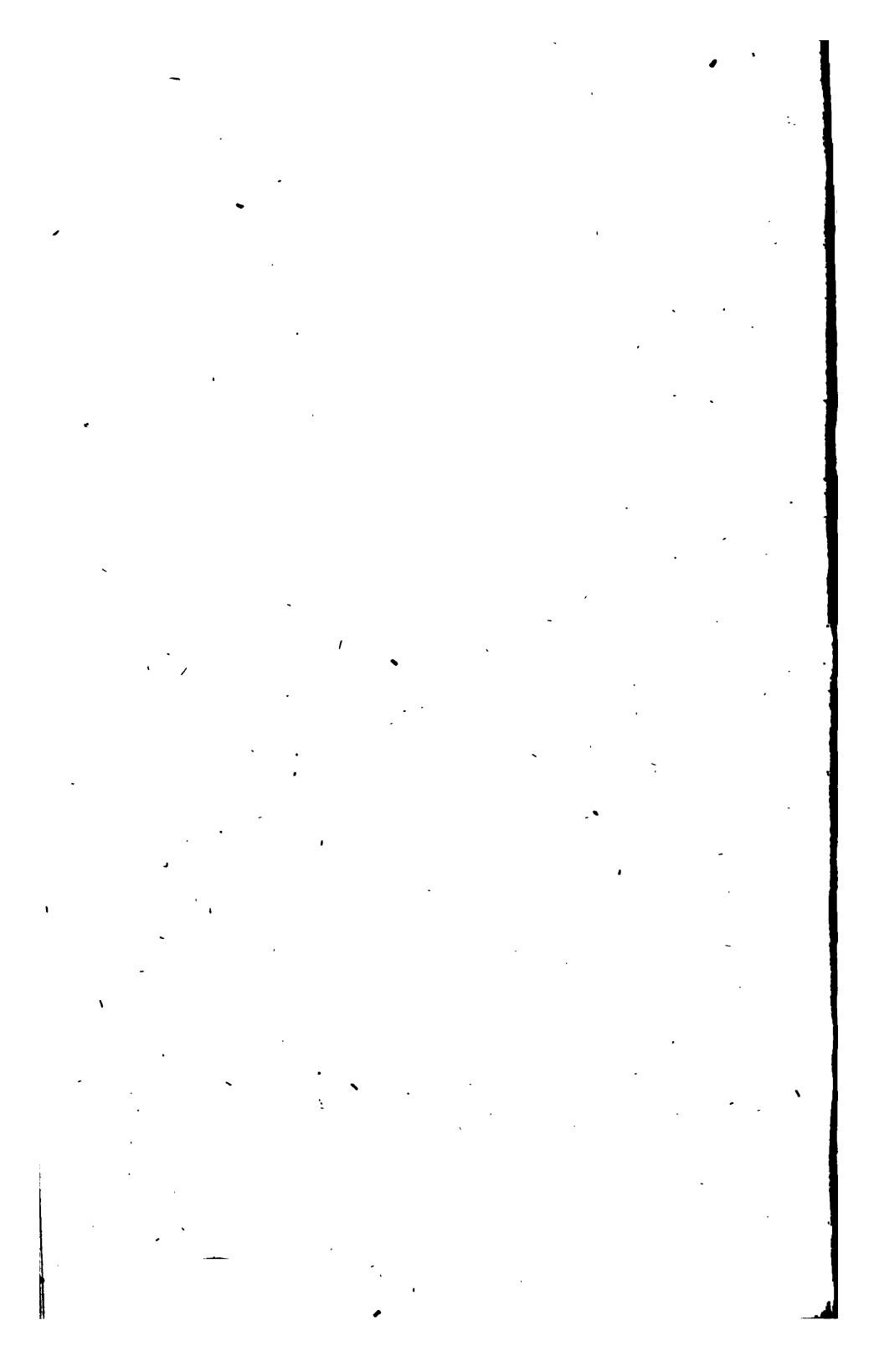
any plane, and upon that fide of it on which the
fun is to fhine; and the projected pole be made
the center of the dial, and the axis of the globe
the Stile or Gnomon, and the radius of projection its
height; you will have a dial drawn with all its fur-
niture. Upon this account it deferves to be more
taken notice of, than at prefent it is. I have in the
foregoing propofitions given, I think, all the fun-
damental principles of this kind of projection, ha-
ving met with little or nothing done upon this
fubject before.

GENERAL PROBLEM.

To project the fphere upon any given plane.

Beforeyou can project the fphere upon any plane,
you muft have a perfect knowledge of all its cir-
cles, and their pofitions in refpect of one another ;
the diftances of the leffer circles from their poles
and from their parallel great circles; the angles
made by great circles, or their inclinations, to one
another, particularly to the primitive circle, on
whofe plane (or a parallel thereto) you are about to
project the fphere. Then, after the primitive cir-
cle is defcribed, you muft defcribe all other circles
concerned in the Problem, according to the rules
of that fort of Projection, you are going to ufe ;
and the interfection of thefe circles will determine
the Problem.

And note, that the projection of the concave fide
of the fphere is more fit for aftronomical purpofes;
for in looking at the heavens, we view the concavity.
But it is better to project the convex hemifphere.
in geography, becaufe we fee the convex fide only.

*The principal Points, Angles and Circles of the Sphere
are as follows.*

I. *Points.*

1. *Zenith* is the point over our heads, Z.

2. *Nadir*

Fig.   2. *Nadir* is the point under our feet, N.

53.   3. *Poles* of the world are two points, round

55. which the diurnal revolution is performed, P the north pole, *p* the fouth pole. A line drawn through the poles, is called the *Axis* of the world, as P*p*.

4. *The Center* of the earth or of the heavens, C.

5. *Equinoctial Points*, are the points of interfection of the Equator and Ecliptic, ♈, ♎.

6. *Solſtitial Points*, are the beginning of Cancer and Capricorn, ♋, ♑.

II. *Great Circles.*

1. *Equinoctial*, is a circle 90 degrees diftant from the poles of the world, as EQ.

2. *Meridians*, or *hour Circles*, are circles paffing through the poles of the world, as P☉*p*, PE*p*, &c.

3. *Solſtitial Colure*, is a meridian paffing through the folftitial points, as P♋*p*.

4. *Equinoctial Colure*, is a meridian paffing through the equinoctial points, PC*p*.

5. *Ecliptic* is the circle through which the fun feems to move in a year, ♋ ♑; it cuts the equinoctial at an angle of 23° 28′, in paffing through the equinoctial points. In this are reckoned the 12 Signs, ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓.

6. *Horizon*, is a circle dividing the upper from the lower hemifphere, as HO, being 90° diftant from the Zenith and Nadir.

7. *Vertical Circles*, are circles paffing through the Zenith and Nadir, Z☉N.

8. *Circles of Longitude* in the heavens, pafs through the poles of the ecliptic and cut it at right angles.

9. *Meridian of a Place*, is that Meridian which paffes through the Zenith, as PZH.

10. *Prime Vertical*, is that which paffes through the eaft and weft points of the horizon.

III.   *Leſſer circles.*

1. *Parallels of Latitude* are parallel to the equinoctial on the earth, *parallels of altitude* are parallel

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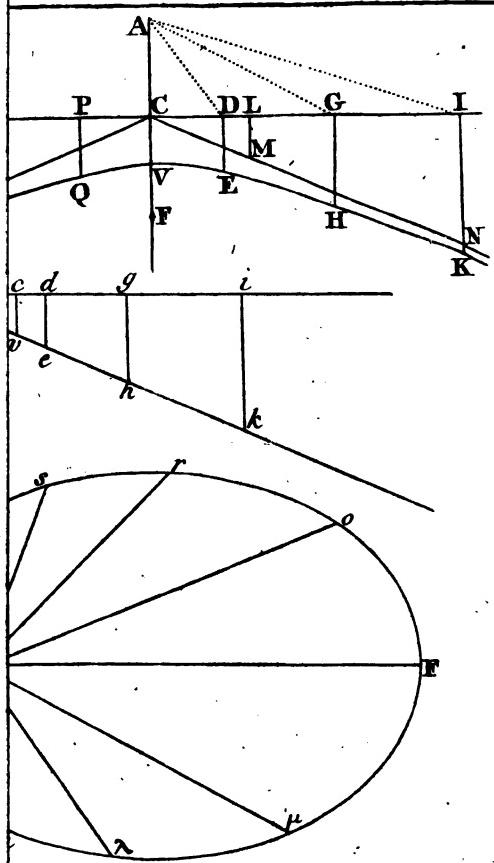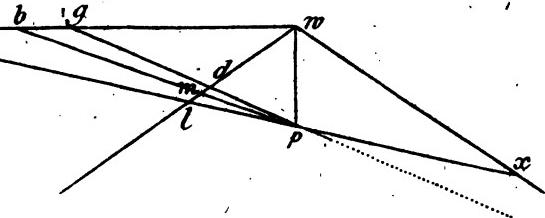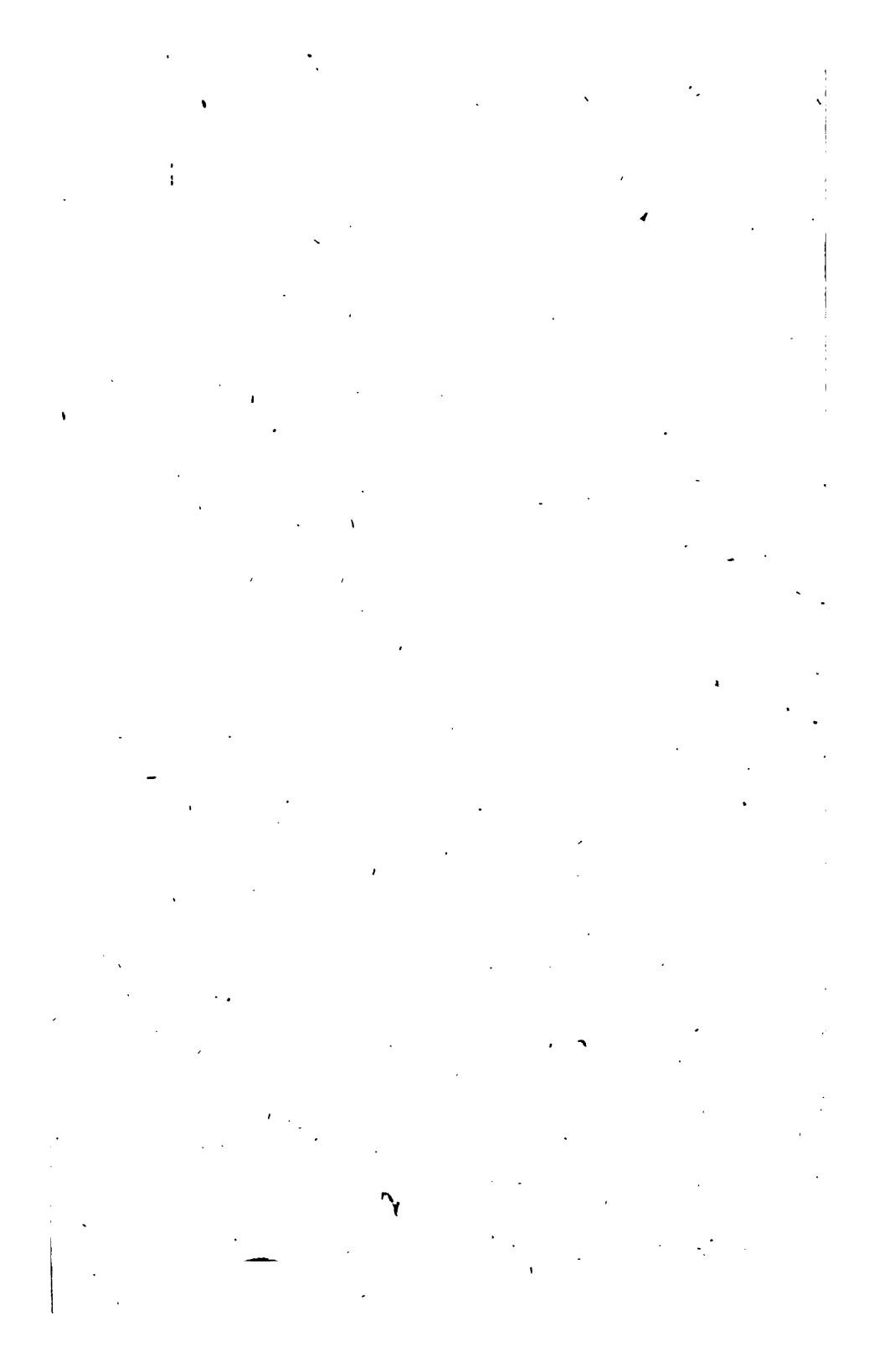
to the horizon, *parallels of declination* are parallel Fig.
to the equinoctial in the heavens.   52.

2. *Tropics*, are two circles distant 23° 28' from 53.
the equinoctial, the tropic of *Cancer* towards the 55.
north, the tropic of *Capricorn* towards the south.

3. *Polar Circles*, are distant 23° 28' from the
poles of the world, the *Arctic* circle towards the
north, the *antarctic* towards the south.

IV. *Angles and Arches of Circles.*

1. *Sun's (or Star's) Altitude*, is an arch of the
Azimuth between the sun and horizon, as ⊙ B.

2. *Amplitude* is an arch of the horizon, between
sun-rising and the east, or sun-setting and the west.

3. *Azimuth*, is an arch of the horizon between
the sun's Azimuth circle, and the north or south,
as HB, or OB, or it is the angle at the zenith,
HZB or OZB.

4. *Right Ascension* is an arch of the equator be-
tween the sun's meridian, and the first point of
Aries, as ♈ K.

5. *Ascensional Difference* is an arch of the equi-
noctial, between the sun's meridian, and that point
of the equinoctial that rises with him, or it is the
angle at the pole between the sun's and the six
o'clock meridian.

6. *Obilque Ascension* or *Descension*, is the sum or
difference of the right ascension and the ascensional
difference.

7. *Sun's Longitude*, is an arch of the ecliptic, be-
tween the sun and first part of Aries, as ♈ ⊙.

8. *Declination* is an arch of the meridian, between
the equinoctial and the sun, as ⊙K.

9. *Latitude of a Star*, is an arch of a circle of
longitude between the star and ecliptic.

10. *Latitude of a place*, is an arch of the meri-
dian between the equinoctial and the place.

11. *Longitude* of a place on the earth is an arch
of the equinoctial, between the first meridian (Isle
of *Ferro*), and the meridian of the place. And

diff.

Fig. *diff. longitude*, is an arch of the equator, between the
52. meridians of the two places, or the angle at the pole.
53.   12. *Hour of the Day*, is an arch of the equinoc-
55. tial, between the meridian of the place and the sun's
meridian, as EK ; or it is the angle they make at
the pole, as EPO.

Example I.

*To project the sphere upon the plane of the meridian,
for May* 12, 1767. *Latitude* 54°½ *north, at a quar-
ter past* 9 *o'clock before noon.*

I. *By the Orthographic Projection.*

52.   Here we will project the convex side of the east-
ern hemisphere.   With the chord of 60 degrees
describe the primitive circle or meridian of the place
HZON.   Through the center C draw the horizon
HO; set the latitude 54½ from O to P and from H to *p*,
and draw P*p* the 6 o'clock meridian. Through C
draw EQ perpendicular to P*p* for the equinoctial.
Make ED, Q*d* 18° 5′ the declination May 12,
and draw D*d* the sun's parallel for that day. By
Prop. XI. make ⊙ G (3¼ hours or) 48° 45′ the
sun's distance from the hour of 6, then ⊙ is the
sun's place. Through ⊙ by Prop. V. draw AL
parallel to H ⊙ for the sun's parallel of altitude.
By Prop. VII. draw the meridian P ⊙ *p* and the
azimuth Z⊙N. Also the ecliptic will be an ellipsis
passing through ⊙, which cannot conveniently be
drawn in this projection. Also draw the parallel S*s*
18° below the horizon, and where it intersects D*d* is
the point of day break, if there is any. Now the sun
is at *d* at 12 o'clock at night, and rises at R, at 6
o'clock is at G, due east at F, at ⊙ a quarter past
9, and is at D in the meridian at 12 o'clock.
    Draw GI parallel to HO.   Then GR measured
by Prop. X. is 27° 14′, and turned into time (allow-
ing 15 degrees for an hour) shows how long the sun
rises before 6, to be 1ʰ 49ᵐ ; GI measured by Prop.
X. gives the azimuth at 6, 79° 16′. CR by Cor.
                                                    Prop.

51.

52.
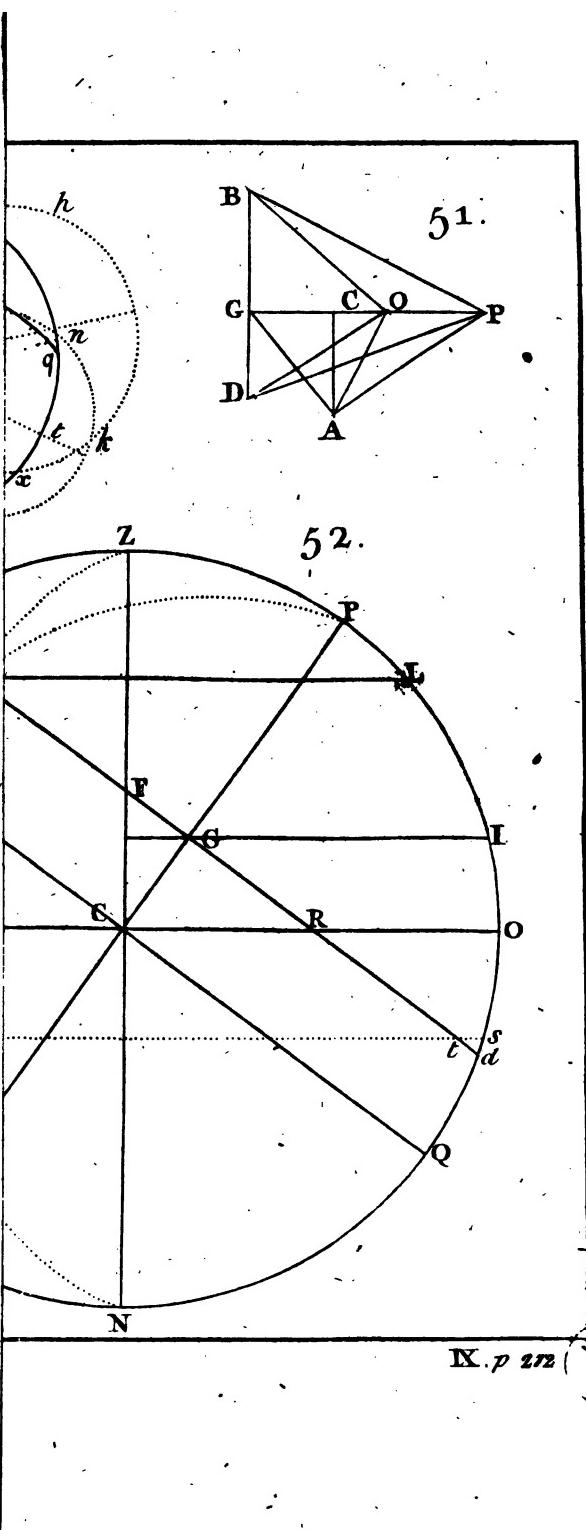
Prop. X. gives the amplitude 32° 19', and CF gives Fig.
his altitude when east 22° 25'. FG 13° 28' (turned 52.
into time) is 54ᵐ, and shews how long after 6 he
is due east. IO is his altitude at 6, 14° 38'. AH
41° 53' is his altitude at ☉, or a quarter past 9 ;
and ☉L measured by Prop. X. is his azimuth
from the north at the same time, 122° 40'. And
thus the place of the moon or star being given, it
may be put into the projection, as at ✳. And its
altitude, azimuth, amplitude, time of rising, &c.
may all be found, as before for the sun.

II. Stereographically.

To project the sphere on the plane of the meri- 53.
dian, the projecting point in the western point of the
horizon; with chord of 60, draw the primitive
circle HZON, and through C draw HO for the ho-
rizon, and ZN perpendicular thereto for the prime
vertical. Set the latitude from O to P, and from
H to p, and draw Pp the 6 o'clock meridian, and
EQ perpendicular thereto for the equinoctial.
Make ED, Qd the declination, and by Prop. XII.
draw DGd, the sun's parallel for the day. Draw
the meridian Pʘp by Prop. XVII. making an an-
gle of 41° 15' with the primitive, to intersect the
sun's parallel in ☉, the sun's place at 9ʰ¼. Through
☉, by Prop. XII. draw the parallel of altitude
A☉L; through ☉ draw, by Prop. XVII. the azi-
muth Z☉N. And by Prop XII. draw the paral-
lel Ssd 18° below the horizon, if it cut Rd, gives
the point of day break. And through G draw the
parallel of altitude GI. Lastly, by Prop. XX.
through ☉ draw the great circle ♈☉♎ cutting the
equinoctial EQ at an angle of 23° 28', and this
is the ecliptic, ♈ the first point of Aries, and ♎
that of Libra.

This done, dR measured by Prop. XXIII. is 62°
46', shows the time of sun' rising; CR by Prop.
XXII. is the amplitude 32° 19'. GI 79° 16' by
Prop. XXIII. the sun's azimuth at 6. IO 14° 38'

Fig. his altitude at 6.   CF 22° 25′ by Prop. XXII. his
5⅓. altitude when eaſt.   GF 13° 28′ the time when he
is due eaſt.   ⊙B 41° 53′ by Prop. XXII. his alti-
tude at a quarter paſt 9; the ∠⊙ZP 122° 40′ by
Prop. XXIV. his azimuth at that time.   Alſo ♈⊙,
by Prop. XXII. is his longitude 51° 7′. ♈K his
right aſcenſion, 48° 40′.

And the place of the moon or a ſtar being given,
it may be put into the ſcheme as at ✶; and its
time of riſing, amplitude, azimuth, &c. found as
before.

III. *Gnomonically.*

54.   To proje� the eaſtern hemiſphere upon a plane
parallel to the meridian.   About the center of pro-
jeᵆion C deſcribe the circle HON with the tangent
of 45 the radius of projeᆴion, for the primitive.
Through C draw the horizon HO, and the prime
vertical ZN perpendicular thereto.   Set the latitude
54½ from H to *a*, and draw the 6 o'clock meri-
dian P*p*, and the equinoᆴial EQ perpendicular to
it.   Set the tangent of 48° 45′ (equal to 3¼ hours)
from C to E, and by Prop. X. draw the meridian
EL parallel to P*p*.   Make E*e*=E*a*, and ∠E*e*⊙
= 18° 5′ the ſun's declination, then by Prop XI.
⊙ is the ſun's place.   Through ⊙ draw the hy-
perbola D⊙*d* (by Prop. XIV.) for the ſun's pa-
rallel of declination; and draw ⊙B perpendicular
to HO, for his azimuth circle.   And draw GI
perpendicular to HO, and RM, FT, ‖ P*p*.   Alſo
the ecliptic is a right line paſſing through ⊙, and
cutting EQ at an angle of 23° 28′, which is
difficult to draw in this projeᆴion.

Alſo by Prop. XIV.   Draw the parallel S*s* 18°
below the horizon, and if it interſeᆴs D*d*, it gives
the point of ſun riſe.

Then if by Prop. XVII. or XI. you meaſure GR
or rather CM, 27° 14′, you have the time of ſun
riſing; GF or CT 13° 28′, the time when he is due
eaſt.   Alſo by Prop. XI. if you meaſure CR, you
   have

53.

53.3
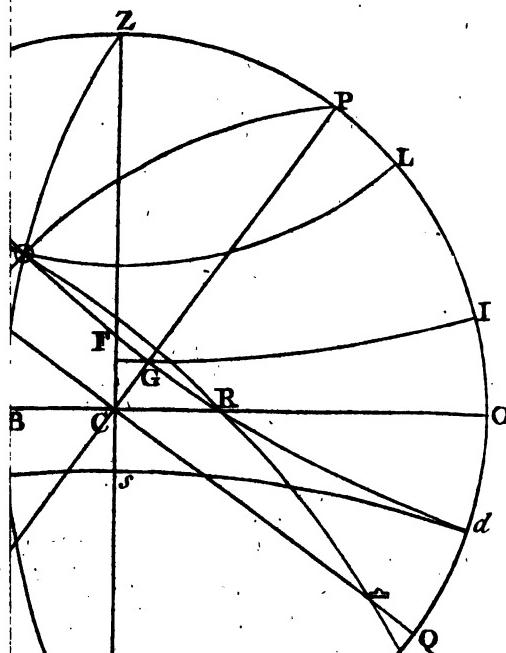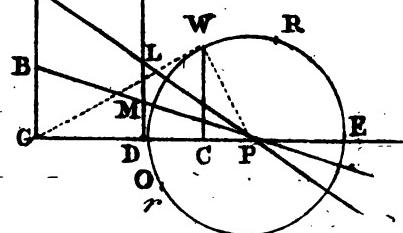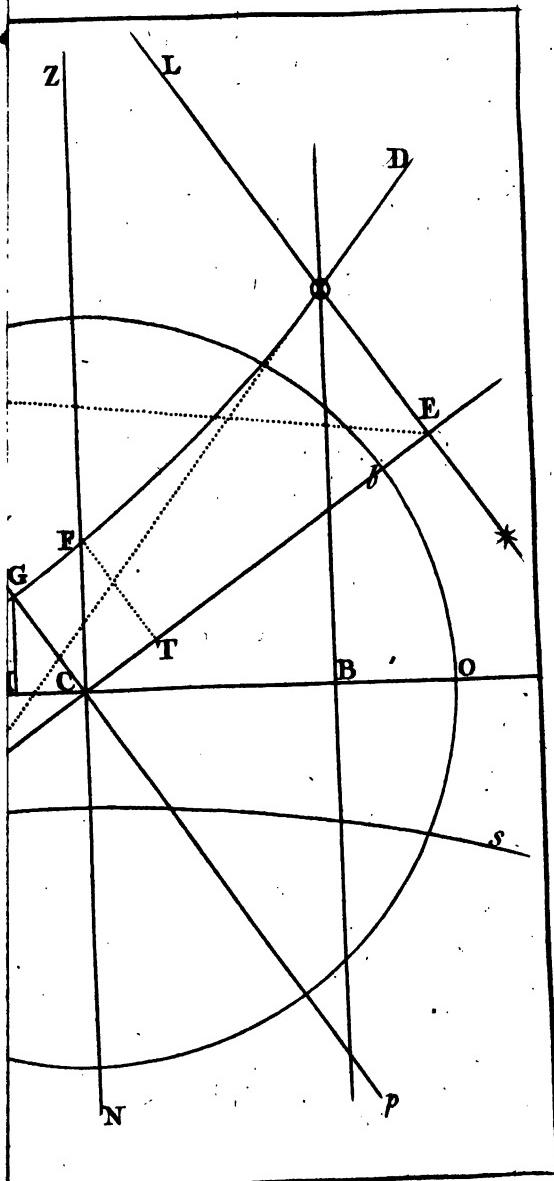
have the amplitude 32° 19', CI the comp. of his Fig.
azimuth at fix, 10° 44'. IG by Prop. XII. his 54.
altitude at fix, 14° 38'. CF his altitude when
eaft, 22° 25'. And by Prop. XI. ⊙B=41° 53',
his altitude a quarter paft nine. CB the comple-
ment of his azimuth at that time 32° 40'..

And the place of the moon or a ftar being given,
its place in the projection may be determined as
before, and all the requifites found.

Ex. 2.

To project the fphere upon the plane of the folfti-
tial colure for latitude 54½N. May 23, 1767, at
10 o'clock in the morning.

Stereographically.

The projection of the weftern hemifphere, the 55.
firft point of Libra, the projecting point. Defcribe
the folftitial colure PEpQ, and the equinoctial co-
lure Pp perpendicular to it; and through C draw the
equinoctial EQ perpendicular to Pp. Set 23° 28'
from E to ♋, and from Q to ♑, and draw the,
ecliptic ♋ ♑. Set the fun's longitude 61° 42'
from C to ⊙, and through ⊙ draw PⵙKp for the
10 o'clock meridian. Make KA (two hours or)
30°, and draw PAp for the meridian of the place.
Set the latitude of the place 54½ from A to Z,
and Z is the zenith. About the pole Z defcribe
the great circle BHS for the horizon of the place.
Through Z and ⊙ draw an azimuth circle Z⊙B.

Then you have ⊙K the fun's declination 20° 33'.
CK his right afcenfion 59° 35'. ⊙B his altitude
at 10 o'clok 49° 10'; the ∠AZ⊙ or PZ⊙ his
azimuth at 10=HB, 45° 44'. H the fouth point
of the horizon. I the point of the ecliptic that
is in the meridian. T the point of the ecliptic
that is fetting in the horizon.

Example 3.

To project the fphere on the plane of the horizon,
Lat. 35½, N. July 31, 1767, at 10 o'clock.

,56. , To project the upper hemisphere on a plane
parallel to the horizon. With the radius of pro-
jection and center C, describe the primitive circle
ADB. Through C draw the meridian PE, and AS
perpendicular to it for the prime vertical. Set
off CP 35½ the latitude and P is the N. Pole, and
perpendicular to CP draw P*p* the 6 o'clock meri-
, dian. Set the complement of the latitude from C
to E; and draw EQ perpendicular to CE for the
equinoctial. Make EB 30° (or 2 hours) and draw
the 10 o'clock meridian PB. Set the sun's decli-
nation 18° 27′ from B to. ☉. And ☉ is the place of
the sun at 10 o'clock. Through ☉ draw the azi-
muth circle CQ; likewise through ☉, a parallel to
the equinoctial EQ may easily be described by
Prop. XV. for the sun's parallel that day.

Then C☉ measured by Prop. XI. is 31° 30′ the
complement of the altitude. And the angle EC☉
measured by Cor. Prop. XII. is his azimuth, 65° 10′.

SCHOLIUM.

After this manner may any Problems of the
Sphere be solved by any of these projections, or
upon any planes, but upon some more commodi-
ously than upon others. And if in a spherical tri-
angle any sides or angles be required, they may be
projected from what is given therein, according to
any of these kinds of projection before delivered;
and it will be most easily done, when you chuse such
a plane to project on, that some given side may
be in the primitive, or a given angle at the center;
and then you need draw no more lines or circles
than what are immediately concerned in that Pro-
blem. But always chuse such a plane to project on,
where the lines and circles are most easily drawn,
and so that none of them run out of the scheme.

55.
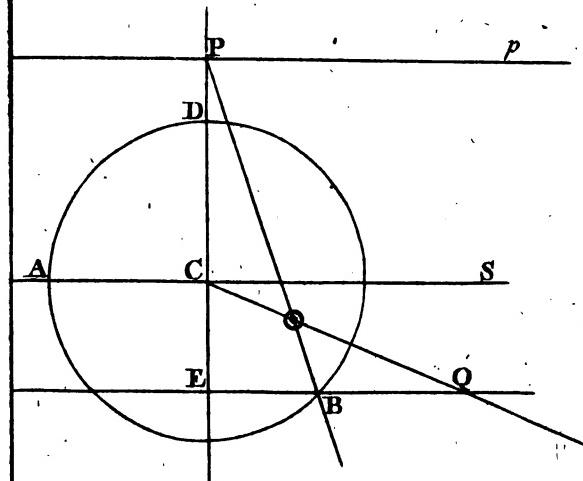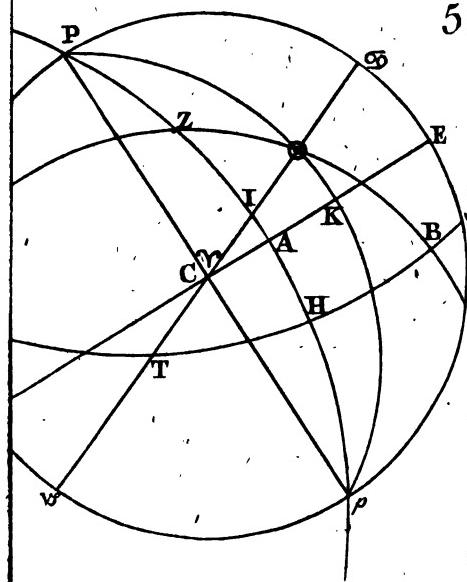
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L . A W S

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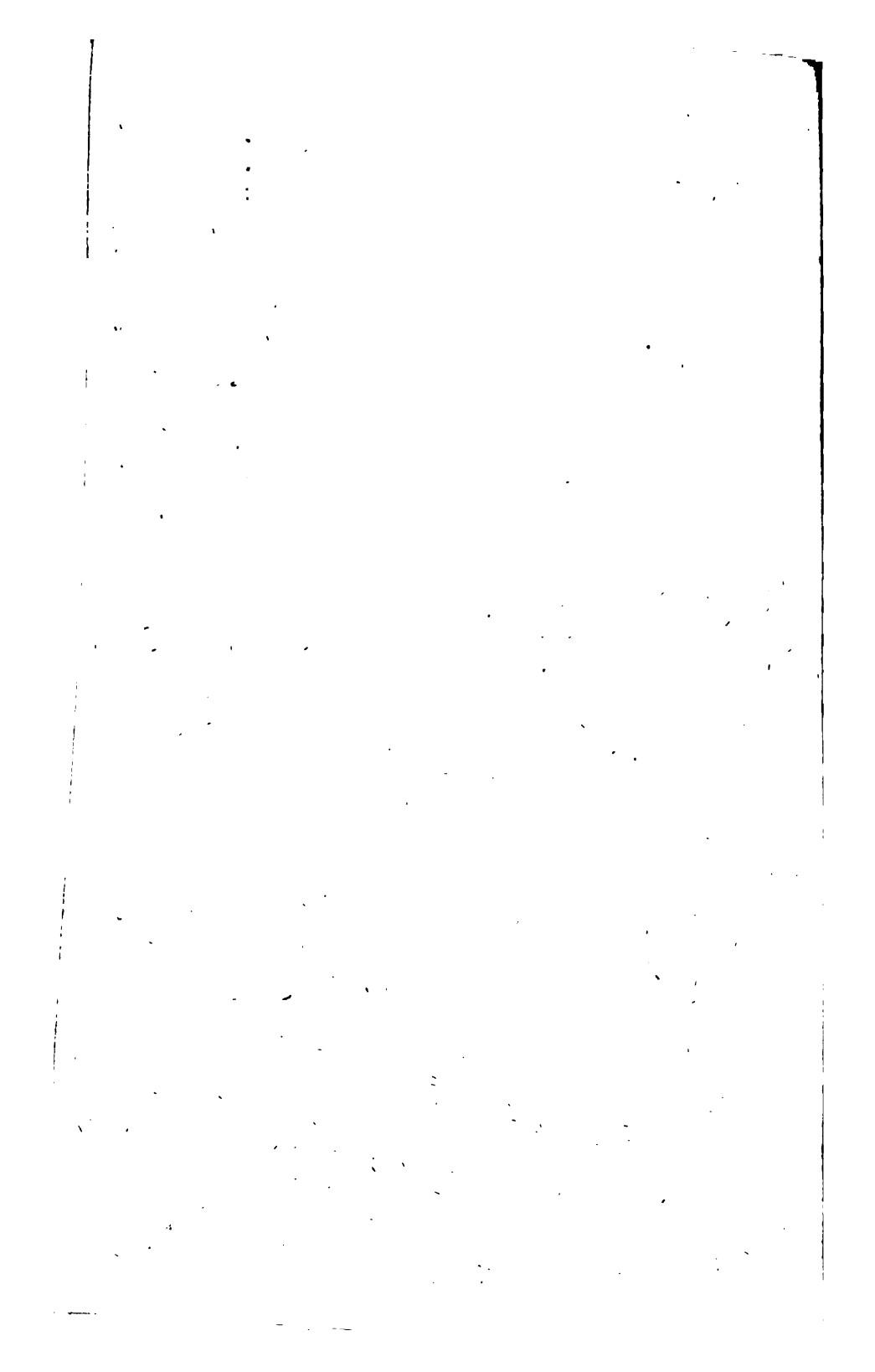
Centripetal and Centrifugal Force.

S H E W I N G

The MOTION of BODIES in Circular ORBITS,
and in the Conic Sections, and other Curves.

And explaining the perturbating Force of a third
Body. With many other things of the like
Nature.

Being a Work preparatory to ASTRONOMY, and
the very bafis thereof. And abfolutely neceffary
to be known by all fuch as defire to be Profi-
cients in that SCIENCE.

Solis uti varios curfus, lunæque meatus
Nofcere poffemus, quæ vis, & caufa cieret.

LUCRET. Lib. V.
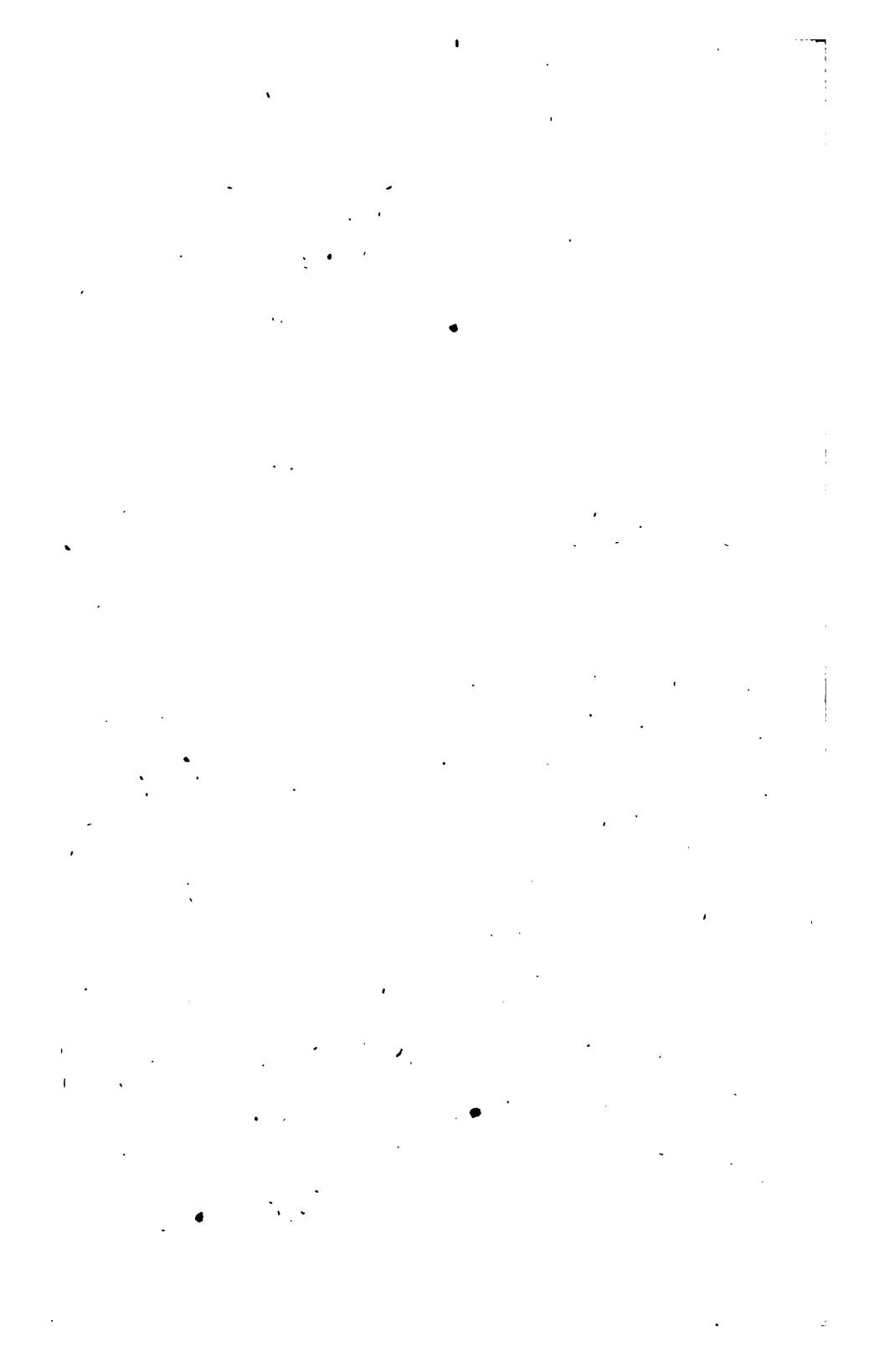
THE

PREFACE.

*I*N the following *Treatife*, *I have explained and demon-
*ftrated the Laws of Centripetal Forces ; a doctrine upon
which all Aftronomy is grounded; and without the knowledge
of which, no rational account can be given of the motions
of any of the celeftial bodies, as the Comets, the Planets, and
their Satellites. From thefe laws are derived the caufes of
the various feeming irregularities obferved in their motions ;
fuch as their accelerations and retardations, their approach-
ing to, and receding from the center of force ; irregularities
only in appearance ; but in reality, thefe motions are truly
regular and conformable to the eftablifhed laws of Nature.
From this foundation we trace the way or path of all the
planets, and difcover the origin and fpring of all the celeftial
motions, and clearly underftand and account for all the
phænomena thence arifing.*

*In the firft fection, you have the Centripetal Forces of
bodies revolving in circles ; their velocities, periodic times,
and diftances compared together ; their relations and propor-
tions to each other ; and that when they either revolve about
the fame center, or about different ones. The different
motions caufed by different forces, or by different central*
*attracting bodies, are here fhewn. We have given likewife
the periodic time of a fimple pendulum revolving with a
conical motion ; and alfo the center of Turbination, and the
periodic time of a compound pendulum, or a fyftem of bodies,
revolving with a conical motion ; as properly belonging to
the doctrine of Centripetal Forces.*

*In the fecond fection we have fhewn the motion of bodies
in the Ellipfis, Hyperbola, and Parabola ; and in other
Curves. The proportion of the Centripetal Forces, and
velocities in different parts of the fame Curve. The law of
Centripetal Force to defcribe a given Curve, and the velocity*

in

PREFACE.

in any point of it; and more particularly with refpect to that law of Centripetal Force that is reciprocally as the fquare of the diftance; which is the grand law of Nature in regard to the action of bodies upon one another at a diftance; and according to this law, is fhewn the motion of bodies round one another, and round the common center of gravity, and the orbits they will defcribe.

In the third fection we have given the difturbing or perturbating force of a third body, acting upon two others that revolve round one another. From thefe principles are deduced the errors caufed in the motion of a Satellite, moving round its primary planet. · Towards the end, are feveral propofitions, by means whereof, the motion of the Nodes, and variation of inclination of a Satellite's orbit, and fuch like things may be computed. As thefe things are all laid down for the fake of underftanding our own Syftem, I have inferted fome few things, by way of illuftration of the rules, in regard to the Moon and Jupiter. But as to the Moon, there are fome things fo very intricate, and require fuch long and tedious calculations, as would require a volume of themfelves; fo that the fmall room I am confined to cannot admit of them; and few would trouble themfelves to read them, if they were there. This laft fection concludes with a few things of another kind, but depending on the principles of Centripetal Forces.

Several of thofe things about Centripetal Forces are calculated by the method of Fluxions; and cannot eafily be done any other way; and moft of them taken from my book of Fluxions. And feveral other things relating to Centripetal Forces, you will alfo find in that book; being forry to trouble the reader too much with repeating what I have written and publifhed elfewhere.

W. Emerfon.

THE

L A W S

O F

Centripetal and Centrifugal Force.

DEFINITIONS.

DEF. I.

*T*HE *center of attraction*, is the point towards which any body is attracted or impelled.

DEF. II.

Centripetal force, is that force by which a body is drawn or impelled to a certain point, as a center. Here all the particles of the body are equally acted on by the force.

DEF. III.

Centrifugal force, is the refiftance a moving body makes to prevent its being turned out of its direct courfe. This is oppofite and equal to the centripetal force; for action and re-action are equal and contrary.

DEF. IV.

Angular velocity, is the quantity of the angle a body defcribes in a given time, about a certain point, as a center. *Apparent velocity* is the fame thing.

DEF. V.

Periodical time, is the time of revolution of a body round a center.

SECT. I.

The Motion of Bodies in Circular ORBITS.

PROP. I.

Fig. *THE centripetal forces, whereby equal bodies at equal diftances from the centers of force, are drawn towards thefe centers; are as the quantities of matter in the central bodies.*

For firice all attraction is made towards bodies, every part of the attracting body muft contribute its fhare in that effect. Therefore a body twice as great will attract the fame body twice as much; and one thrice as great, thrice as much, and fo on. Therefore the attraction of the central body; that is, the centripetal force, is as the quantity of matter in the attracting or central body.

Cor 1. *Any body whether great or little, placed at the fame diftance, is attracted through equal fpaces in the fame time, by the central body.*
For though a body twice or thrice as great as ano-ther, is drawn with twice or thrice the force; yet it will acquire no greater velocity, nor pafs through a greater fpace. For (Prop. V. Cor. 2. Mechan.) the velocity generated in a given time, is as the force directly, and quantity of matter reciprocally; and the force, which is the weight of the body, being as the quantity of matter; therefore the velocity generated is as the quantity of matter directly, and quantity of matter reciprocally, and therefore is a given quantity.

Cor.

Cor 2. *Therefore the centripetal force, or force* Fig. *towards the center, is not to be measured by the quantity of the falling body; but by the space it falls through in a given time.   And therefore it is sometimes called an accelerative force.*

PROP. II.

If a body revolves in a circle, and is retained in it, 1. *by a centripetal force, tending to the center of it; put* R = *radius of the circle or orbit described,* AC.

F = *absolute force, at the distance* R.

s = *the space a falling body could descend through, by the force at* A, *and*

t = *time of the descent.*

π = 3.1416.

Then its periodic time, or the time of one revolution, will be $\pi t \sqrt{\dfrac{2R}{s}}$.

And the velocity, or space it describes in the time t, will be $\sqrt{2Rs}$.

For let AB be a tangent to the circle at A; take AF an infinitely small arch, and draw FB perp. to AB, and FD perp. to the radius AC.   Let the body descend through the infinitely small height AD or BF, by the centripetal force in the time 1. Now that the body may be kept in the circular orbit AFE, it ought to describe the arch AF in the same time 1. The circumference of the circle AE is 2πR, and the arch AF = $\sqrt{2R \times AD}$.

By the laws of falling bodies $\sqrt{s} : t : : \sqrt{AD} :$

$t \sqrt{\dfrac{AD}{s}}$ = time of moving through AD or AF. And by uniform motion, as AF, to the time of its description : : circumference AFEA, to the time of

one

Fig.
1.

one revolution; that is, $\sqrt{2R \times AD} : t\sqrt{\dfrac{AD}{s}} ::$

$2\pi R$: periodic time $= \dfrac{2t\pi R}{\sqrt{2Rs}} = \pi t\sqrt{\dfrac{2R}{s}}$.

Alſo by the laws of uniform motion, $t\sqrt{\dfrac{AD}{s}}$

or time of deſcribing AF : AF or $\sqrt{2R \times AD}$: :
$t : \sqrt{2Rs} =$ the velocity of the body, or ſpace
deſcribed in time t.

Cor. 1. *The velocity of the revolving body, is equal
to that which a falling body acquires in deſcending
through half the radius AC, by the force at A uniform-
ly continued.*
For $\sqrt{s}$ (height) : $2s$ (the velocity) : : $\sqrt{\frac{1}{2}R}$ (the
height) : $\sqrt{2Rs}$, the velocity acquired by falling
through $\frac{1}{2}R$.

Cor. 2. *Hence, if a body revolves uniformly in a
circle, by means of a given centripetal force; the arch
which it deſcribes in any time, is a mean proportional
between the diameter of the circle, and the ſpace which
the body would deſcend through in the ſame time, and
with the ſame given force.*
For $2R$ (diameter) : $\sqrt{2Rs}$: : $\sqrt{2Rs}$: s; where
$\sqrt{2Rs}$ is the arch deſcribed, and s the ſpace def-
cended through, in the time t.

2.   Cor. 3. *If a body revolves in any curve AFQ,
about the center of force s; and if AC or R be the
radius of curvature in any point A; s = ſpace deſcend-
ed by the force directed to C. Then the velocity in A
will be $\sqrt{2Rs}$.*
For this is the velocity in the circle; and there-
fore in the curve, which coincides with it.

PROP.

PROP. III.

*If several bodies revolve in circles round the same 1.
or different centers; the periodic times will be as the
square roots of the radii directly, and the square roots
of the centripetal forces reciprocally.*

Let F = centripetal force at A tending to the
center C of the circle.
V = velocity of the body.
R = radius AC of the circle.
P = periodic time.

Then (Prop. II.) $P = \pi t \sqrt{\dfrac{2R}{s}}$. But s is as the

force F that generates it; whence $P = \pi t \sqrt{\dfrac{2R}{F}}$,

and since 2, π and t are given quantities, therefore

$P \propto \sqrt{\dfrac{R}{F}}$.

Cor. 1. *The periodic times are as the radii directly,
and the velocities reciprocally.*

For (Prop. II.) $V = \sqrt{2Rs} = \sqrt{2RF}$, and V^2

$= 2RF$, and $P = \pi t \sqrt{\dfrac{2R}{F}}$, and $PP = \pi^2 t^2 \times$

$\dfrac{2R}{F}$, therefore $P^2 V^2 = \pi^2 t^2 \times 4R^2$, and $P^2 =$

$\dfrac{\pi^2 t^2 \times 4R^2}{V^2}$, and $P = \dfrac{\pi t \times 2R}{V} \propto \dfrac{R}{V}$.

Cor. 2. *The periodic times are as the velocities di-
rectly, and the centripetal forces reciprocally.*

For $V^2 = 2Rs = 2RF$; and $R = \dfrac{VV}{2F}$, and $\dfrac{R}{V}$

$= \dfrac{V}{2F} \propto \dfrac{V}{F}$. But (Cor. 1.) $P \propto \dfrac{R}{V} \propto \dfrac{V}{F}$.

Fig.
1. Cor. 3. *If the periodic times are equal; the velo-
cities, and also the centripetal forces, will be as the
radii.*

For if P be given; then $\frac{R}{F}$, and $\frac{R}{V}$, and $\frac{V}{F}$
are all given ratios.

Cor. 4. *If the periodic times are as the square
roots of the radii; the velocities will be as the square
roots of the radii, and the centripetal forces equal.*

For (Prop. III. and Cor. 1.) putting $\sqrt{R}$ for P,
we have $\sqrt{R} \propto \sqrt{\frac{R}{F}} \propto \frac{R}{V}$. Therefore $1 \propto \frac{1}{\sqrt{F}}$

$\propto \frac{\sqrt{R}}{V}$, and $\sqrt{R} \propto V$, and $\sqrt{F}$ is a given quantity.

Cor. 5. *If the periodic times are as the radii; the
velocities will be equal, and the centripetal forces re-
ciprocally as the radii.*

For putting R for P, we have $R \propto \sqrt{\frac{R}{F}}$

$\propto \frac{R}{V}$; whence $\sqrt{R} \propto \frac{1}{\sqrt{F}}$, and $1 \propto \frac{1}{V}$; that

is, $R \propto \frac{1}{F}$, or the centripetal force is reciprocally

as the radius; and V is a given quantity.

Cor. 6. *If the periodic times are in the sesquiplicate
ratio of the radii; the velocities will be reciprocally as
the square roots of the radii, and the centripetal forces
reciprocally as the squares of the radii.*

Put R^3 for P, then $R^{\frac{3}{2}} \propto \sqrt{\frac{R}{F}} \propto \frac{R}{V}$; and

$R \propto \frac{1}{\sqrt{F}}$ or $RR \propto \frac{1}{F}$, and $\sqrt{R} \propto \frac{1}{V}$.

Cor. 7. *If the periodic times be as the nth power
of the radius; then the velocities will be reciprocally as
the n—1th power of the radii, and the centripetal
forces*

forces reciprocally as the $2n-1^{th}$ *power of the* Fig.
radii.   1.

Put R^n for P, then $R^n \propto \sqrt{\dfrac{R}{F}} \propto \dfrac{R}{V}$. Whence

$R^{2n} \propto \dfrac{R}{F}$, and $R^{2n-1} \propto \dfrac{1}{F}$. Alſo $R^{n-1} \propto \dfrac{1}{V}$.

PROP. IV.

If ſeveral bodies revolve in circles round the ſame 1.
or different centers; the velocities are as the radii di-
rectly, and periodic times reciprocally.

For putting the ſame letters as in Prop. III. we
have (by Prop. II.) $V = \sqrt{2Rs} = \sqrt{2RF}$; and P
$\propto \dfrac{V}{F}$ (by Cor. 2. Pr. III.) and PF $\propto$ V, and F $\propto$
$\dfrac{V}{P}$.   Whence $V = \sqrt{2RF} = \sqrt{2R \times \dfrac{V}{P}}$, and V^2
$= \dfrac{2RV}{P}$, and $V = \dfrac{2R}{P} \propto \dfrac{R}{P}$.

Cor. 1. *The velocities are as the periodical times,*
and the centripetal forces.
   For we had PF $\propto$ V.

Cor. 2. *The ſquares of the velocities are as the ra-*
dii and the centripetal forces.
   For $V = \sqrt{2RF}$.

Cor. 3. *If the velocities are equal; the periodic*
times are as the radii, and the radii reciprocally as
the centripetal forces.

For if V be given, its equal $\dfrac{R}{P}$ is a given ratio;

and $\sqrt{RF}$ is given, whence R $\propto \dfrac{1}{F}$.

Cor.

Fig.   Cor. 4. *If the velocities be as the radii, the perio-*
1.   *dic times will be the same; and the centripetal forces*
as the radii.

For then V or R $\propto \dfrac{R}{P}$, and 1 $\propto \dfrac{1}{P}$. Alſo R=

$\sqrt{2RF}$, whence R $\propto$ F.

Cor. 5. *If the velocities be reciprocally as the radii,*
the centripetal forces are reciprocrlly as the cubes of
the radii; and the periodic times as the ſquares of the
radii.

For put $\dfrac{1}{R}$ for V, then (Cor. 2.) $\dfrac{1}{R}=\sqrt{2RF}$,

$\dfrac{1}{RR}=2RF$, whence F $\propto \dfrac{1}{R^3}$.   Alſo $\dfrac{1}{R} \propto \dfrac{R}{P}$,
and P $\propto$ RR.

PROP. V.

1.   *If ſeveral bodies revolve in circles about the ſame*
or different centers; the centripetal forces are as the
radii directly, and the ſquares of the periodic times
reciprocally.

Put the ſame letters as in Prop. III. Then (Prop.
II.) $P = \pi t \sqrt{\dfrac{2R}{s}} = \pi t \sqrt{\dfrac{2R}{F}}$, and $PP = \pi\pi tt$

$\times \dfrac{2R}{F}$, and $PPF = 2\pi\pi ttR$; whence $F = \dfrac{2\pi\pi ttR}{PP}$

$\propto \dfrac{R}{PP}$.

Cor. 1. *The centripetal forces, are as the velocities*
directly, and the periodic times reciprocally.

For (Prop. IV.) V $\propto \dfrac{R}{P}$, and F $\propto \dfrac{R}{PP} \propto \dfrac{V}{P}$.

Cor. 2. *The centripetal forces, are as the ſquares*
of the velocities directly, and the radii reciprocally.

For

Fig.
1.

For (Cor. 1.) $F \propto \frac{V}{P}$, and $FP \propto V$. But (Prop.

III. Cor. 1.) $P \propto \frac{R}{V}$, therefore $FP \propto \frac{FR}{V}$,

therefore $\frac{FR}{V} \propto V$, and $F \propto \frac{VV}{R}$.

Cor. 3. *If the centripetal forces are equal; the velocities are as the periodic times; and the radii as the squares of the periodic times, or as the squares of the velocities.*

Cor. 4. *If the centripetal forces be as the radii, the periodic times will be equal.*

For $F \propto \frac{R}{PP}$, and $\frac{F}{R} \propto \frac{I}{PP}$, and if $\frac{F}{R}$ be a given ratio, $\frac{I}{PP}$ will be given, as also P.

Cor 5. *If the centripetal forces be reciprocally as the squares of the distances; the squares of the periodical times will be as the cubes of the distances; and the velocities reciprocally as the square roots of the distances.*

For writing $\frac{I}{RR}$ for F, then $\frac{I}{RR} \propto \frac{R}{PP}$, and $\frac{R^3}{P^2}$ a given quantity. And $\frac{I}{RR} \propto \frac{VV}{R}$, and $\frac{I}{R} \propto VV$, or $\sqrt{\frac{I}{R}} \propto V$.

PROP. VI.

If several bodies revolve in circles, about the same or different centers; the radii are directly as the centripetal forces, and the squares of the periodic times.

For (Prop. II.) putting the same letters as before, $P = \pi t \sqrt{\frac{2R}{s}} = \pi t \sqrt{\frac{2R}{F}}$, and $PP = \pi\pi tt \times \frac{2R}{F}$, and $PPF = 2\pi\pi ttR \propto R$.

Fig.   Cor. 1. *The radii are directly as the velocities and*
1. *periodic times.*

For (Prop. IV. Cor. 1.) PF $\propto$ V, but PPF ∞
R ; therefore PV $\propto$ R.

Cor. 2. *The radii are as the squares of the veloci-*
ties directly, and the centripetal forces reciprocally.

For (Prop. III. Cor. 2.) P $\propto \dfrac{V}{F}$, but (Cor. 1.)

R $\propto$ PV ; therefore R $\propto \dfrac{VV}{F}$.

Cor. 3. *If the radii are equal; the centripetal forces*
are as the squares of the velocities, and reciprocally
as the squares of the periodic times. And the veloci-
ties reciprocally as the periodic times.

For if R be given, $\dfrac{VV}{F}$, and PPF, and PV, are given

quantities, and F $\propto$ VV, or F $\propto \dfrac{1}{PP}$, and V $\propto \dfrac{1}{P}$

SCHOLIUM.

The converse of all these propositions and corol-
laries are equally true. And what is demonstrated
of centripetal forces, is equally true of centrifugal
forces, they being equal and contrary.

PROP. VII.

1.   *The quantities of matter in all attracting bodies,*
having others revolving about them in circles; are as
the cubes of the distances directly, and the squares of
the periodical times reciprocally.

Let M be the quantity of matter in any central
attracting body. Then since it appears, from all
astronomical observations, that the squares of the
periodical times are as the cubes of the distances,
of the planets, and satellites from their respective
<div align="right">centers</div>

centers. Therefore (Cor. 6. Prop. III.) the centri-
petal forces will be reciprocally as the fquares of the
diftances; that is, $F \propto \dfrac{1}{RR}$.　And (Prop. I.)
the attractive force at a given diftance, is as the
body M, therefore the abfolute force of the body
M is as $\dfrac{M}{RR}$. And (Prop. V.) fince $F \propto \dfrac{R}{PP}$, put
$\dfrac{M}{RR}$ inftead of F, and we have $\dfrac{M}{RR} \propto \dfrac{R}{PP}$, and
$M \propto \dfrac{R^3}{P^2}$.

Fig.
1.

Cor. 1. *Hence inftead of* F *in any of the foregoing
propofitions and their corollaries, one may fubftitute*
$\dfrac{M}{RR}$, *which is the force that the attracting body in* C,
exerts at A.

Cor. 2. *The attractive force of any body, is as
the quantity of matter directly, and the fquare of the
diftance reciprocally.*

PROP.　VIII.

*If the centripetal force be as the diftance from the
center* C. *A body let fall from any point* A, *will fall
to the center in the fame time, that a body revolving
in the circular orbit* ALEA, *at the diftance* CA,
would defcribe the quadrant AGL.

3.

The truth of this is very readily fhewn by fluxions;
thus, put $AC = r$, $AH = x$, $t =$ time of defcrib-
ing AH, $v =$ the velocity at H. $F =$ force at H,
which is as CH or $\overline{r - x}$. Then (Mechan. Cor. 2.
Prop. V.) the velocity generated is as the force and
time; that is, $\dot{v} \propto Ft$　Alfo (Mechan. Prop. III.
Cor. 1.) the time is as the fpace divided by the veloci-

ty;

ty; that is, $\dot{i} \propto \dfrac{\dot{x}}{v}$; therefore $\dot{v} \propto \dfrac{F\dot{x}}{v} \propto \dfrac{\overline{r-x} \times \dot{x}}{v}$,

and $v\dot{v} \propto \overline{r-x} \times \dot{x}$, and the fluent is $\dfrac{vv}{2} \propto rx -$

$\dfrac{xx}{2}$, or $vv \propto 2rx - xx$, and $v \propto \sqrt{2rx - xx}$ or
HG; that is, the velocity at H is as the ordinate
HG of the circle.

Now it is evident, that in the time the revolving body defcribes the infinitely fmall arch AF, the falling body will defcend through the verfed fine AD, and would defcribe twice AD in the fame time, with the velocity in D.   Therefore we fhall have,

velocity at F : velocity at D : : AF or FD : 2AD,
and velocity at D : velocity at H :: AF or FD : GH,

therefore,

velocity at F or G : velocity at H : : AF^2 : 2AD $\times$
GH : : $\dfrac{AF^2}{2AD}$: GH : : CA or CG : GH.   But
drawing an ordinate infinitely near GH; by the nature of the circle, it will be, as GC : GH : : fo the increment of the curve AG : to the increment of the axis AH.   And Therefore, vel. at G : vel. at H : : as the increment of AG : to the increment of AH.   Therefore fince the velocities are as the fpaces defcribed, the times of defcription will be equal; and the feveral parts of the arch AGL are defcribed in the fame times as the correfpondent parts of the radius AHC.   And by compofition, the arch AG and abfciffa AH, as alfo the quadrant AL and radius AC, are defcribed in equal times.

Cor. 1. *The velocity of the defcending body at any place* H, *is as the fine* GH.

Cor. 2. *And the time of defcending through any verfed fine* AH, *is as the correfpondent arch* AG.

Cor.

Cor. 3. *All the times of falling from any altitudes* Fig.
whatever, to the center C, *will be equal.* 3.

For thefe times are $\frac{1}{4}$ the periodic times; and
(Prop. V. Cor. 4.) thefe periodic times are all equal.

Cor. 4. *In the time of one revolution, the falling
body will have moved from* C *through* E, *and back
again through* C *to* A, *meeting the revolving body
again at* A.

Cor. 5. *The velocity of the falling body at the cen-
ter* C, *is equal to the velocity of the revolving body.*

For the velocities are as the lines GH and GC;
and thefe are equal, when G comes to L.*

PROP. IX.

If a pendulum AB *be fufpended at* A, *and be made* 4.
to revolve by a conical motion, and defcribe the circle
BEDH *parallel to the horizon.*

Put $\pi = 3.1416$; $p = 16.\frac{1}{2}$ *feet, the fpace def-
cended by gravity in the time t.*

Then the periodical time of B *will be* $\pi t \sqrt{\dfrac{2AC}{p}}$.

For (Mechan. Prop. VIII.) if the axis AC re-
prefents the weight of the body, AB will be the
force ftretching the ftring, and BC the force tend-
ing to the center C. Alfo (Mechan. Prop. VI.) if
the time is given, the fpace defcribed will be as the
force; whence $AC : BC : : p : \dfrac{BC}{AC} p =$ the fpace
defcended towards C, by the force BC, in the time
t. This is the fpace s in Prop. II. Therefore in-
ftead of s put its value in the periodical time, and
(by Prop. II.) we fhall have the periodical time of the

pendulum $= \pi t \sqrt{\dfrac{2R}{s}} = \pi t \sqrt{2BC \times \dfrac{AC}{BC \times p}}$

$= \pi t \sqrt{\dfrac{2AC}{p}}$.

Cor.

Fig.
4.

Cor. 1. *In all pendulums, the periodic times are as the square roots of the heights of the cones,* AC.

For π, t, and p are given quantities.

Cor. 2. *If the heights of the cones be the same, the periodic times will be the same, whatever be the radius of the base* BC.

Cor. 3. *The semiperiodic time of revolution, is equal to the time of oscillation of a pendulum, whose length is* AC, *the height of the cone.*

For by the laws of falling bodies, $t\sqrt{\dfrac{AC}{2p}}=$ time of falling through $\frac{1}{2}$AC; and therefore (Mechan. Prop. XXIV.) $1 : \pi :: t\sqrt{\dfrac{AC}{2p}} : \pi t\sqrt{\dfrac{AC}{2p}}$

$= \frac{1}{2}\pi t\sqrt{\dfrac{2AC}{p}}$, the time of vibration, which is half the periodical time.

Cor. 4. *The space descended by a falling body in the time of one revolution, will be* $\pi\pi \times 2$AC.

For tt (time) : p (height) :: $\pi\pi tt \times \dfrac{2AC}{p}$ (per. time) : $\pi\pi \times 2$AC $=$ height descended in that time.

Cor. 5. *The periodic time, or time of one revolution, is equal to* $\pi\sqrt{2} \times$ *time of falling through* AC.

For the time of falling through AC is $t\sqrt{\dfrac{AC}{p}}$.

Cor. 6. *The weight of the pendulum is to the centrifugal force; as the height of the cone* AC, *to the radius of the base* CB. *And therefore when the height* CA *is equal to the radius* CB; *the centripetal or centrifugal force is equal to the gravity.*

PROP.

PROP. X.

Suppose a system of bodies A, B, C, to revolve 5. *with a conical motion about the axis* TR *perp. to the horizon, so as to keep the same side always towards the axis of revolution, and the same position among themselves.*

To find the periodical time of the whole system.

1. Let A, B, C, be all situated in one plane passing through TR. From A, B, C let fall the perpendiculars Aa, Bb, Cc, upon the axis TR. And let A, B, C represent the quantities of matter in the bodies A, B, C. Also put $h = 16\frac{1}{12}$ feet the height a body falls in the time t by gravity; $\varpi = 3.1416$; P = the periodic time of the system.

By the resolution of forces, Ta (gravity) : Aa (force in direction Aa) $:: h : \dfrac{Aa}{Ta} h =$ space descended by A towards a in the time 1, which is as the velocity generated by the force Aa. Therefore $\dfrac{Aa}{Ta} hA =$ motion generated in A in direction Aa. And the force in direction Aa to move the system towards TR, by the power of the lever TA, is $\dfrac{Aa}{Ta} hA \times Ta$ or $Aa \times hA$. This is the centripetal force of the system, arising from the gravity of A. In like manner the centripetal forces arising from B and C, will be $Bb \times hB$ and $Cc \times hC$.

By the laws of uniform motion, $P : 2\varpi \times Aa ::$ $t : \dfrac{2\varpi t \times Aa}{P} =$ arch described by A in the time t. And $\dfrac{4\varpi\varpi tt \times Aa^2}{PP \times 2Aa}$ or $\dfrac{2\varpi\varpi tt \times Aa}{PP} =$ distance it is drawn from the tangent in that time, or as the velocity generated; and therefore $\dfrac{2\varpi\varpi tt \times Aa}{PP} \times A$ = motion

Fig.
5.

= motion of A tending from the center a, by the revolution of the fyftem. And the force in direction aA, to move the fyftem from TR, by the power of the lever TA, will be $\dfrac{2\varpi\varpi tt \times Aa}{PP}$ A $\times$ Ta. And this is the centrifugal force of the fyftem arifing from the revolution of A. And in like manner the centrifugal forces arifing from B and C, will be $\dfrac{2\varpi\varpi tt \times Bb}{PP}$ B $\times$ Tb, and $\dfrac{2\varpi\varpi tt \times Cc}{PP}$ C $\times$ Tc.

But becaufe the whole fyftem always keeps at the fame diftance from the axis TF, in its revolution; therefore the fum of all the centripetal forces muft be equal to the fum of all the centrifugal forces. Whence A$a \times b$A + B$b \times b$B + C$c \times b$C = $\dfrac{2\varpi\varpi tt}{PP} \times$ into A$a \times$ T$a \times$ A + B$b \times$ T$b \times$ B + Cc $\times$ T$c \times$ C. And confequently P = $\varpi t \sqrt{\dfrac{2}{b}} \times$ $\dfrac{\text{A}a \times \text{T}a \times \text{A} + \text{B}b \times \text{T}b \times \text{B} + \text{C}c \times \text{T}c \times \text{C}}{\text{A}a \times \text{A} + \text{B}b \times \text{B} + \text{C}c \times \text{C}}$.

2. If the bodies are not all in one plane, let N be the center of gravity of the bodies A, B, C. And through N draw the plane TNR; and from all the bodies, let fall perpendiculars upon that plane. Then the periodic time will be the fame as if all the bodies were placed in thefe points where the perpendiculars cut the plane. For if m be one of the bodies, and mC perp. to the plane. Then the centripetal and centrifugal forces of m in direction cm, will be $cm \times bm$ and $\dfrac{2\varpi\varpi tt \times Tc}{PP}$ $m \times mc$. But the force cm is divided into the two forces cC, Cm. And all the forces Cm deftroy one another, becaufe the plane, TNc, paffes through their center of gravity. Therefore the plane is only acted on by

the

the remaining force cC. So that the centripetal and centrifugal forces will be the same as before, when the body was placed in C; and the periodic time is the same.

Cor. 1. *If* Nn *be drawn from the center of gravtiy perp. to* TF; *then the periodic time of the syftem,* $P = \pi t \sqrt{\dfrac{2}{b}} \times$

$$\frac{Ta \times Aa \times A + Tb \times Bb \times B + Tc \times Cc \times C}{Nn \times \overline{A + B + C}}.$$

For (Mechan. Prop. XXXV.) $Aa \times A + Bb \times B + Cc \times C = Nn \times \overline{A + B + C}$.

Cor. 2. *The length of a simple pendulum, making two vibrations, or an exceeding small conical motion, in the same periodic time, will be*.

$$\frac{Ta \times Aa \times A + Tb \times Bb \times B + Tc \times Cc \times C}{Nn \times \overline{A + B + C}}.$$

For let TO be the height of the cone defcribed by the pendulum; then (Prop. IX) $PP = \dfrac{2\pi\pi tt}{b} \times$ TO; therefore TO $=$

$$\frac{Ta \times Aa \times A + Tb \times Bb \times B + Tc \times Cc \times C}{Nn \times \overline{A + B + C}}.$$

Cor. 3. *If* TO *be the length of an ifocronal perdulum, then* O *is the center of gravity of all the peripheries defcribed by* A, B, C; *each multiplied by the body; whether* A, B, C *be the places of the bodies, or the points of projection upon the plane* TNR.

For if $Aa \times A$, $Bb \times B$, $Cc \times C$ be taken for bodies, their center of gravity will be diftant from T, the length

$$\frac{Ta \times Aa \times A + Tb \times Bb \times B + Tc \times Cc \times C}{Aa \times A + Bb \times B + Cc \times C}$$ (Me-

chan.

Fig.
5.
chan. Prop. XXXV.) which is equal to TO by
Cor. 2. and the peripheries are as the radii, A*a*,
B*b*, C*c*.

Cor 4. *If any of the bodies be on the contrary side*
of the axis TR, *or above the point of suspension* T;
that distance must be negative.

Cor. 5. *If any line or plane figure be placed in the*
plane TNR; *then the point* O, *which gives the length*
of the pendulum, will be the center of gravity, of
the surface or solid, described in its revolution.

SCHOLIUM.

The point O, which gives the length of the iso-
chronal pendulum, is called the *center of turbina-*
tion or revolution.   And the plane TNR passing
through the center of gravity, the *turbinating plane.*

SECT.

SECT. II.

The Motion of Bodies in all Sorts of CURVE LINES.

━━━━━━━━━

PROP. XI.

THE areas, which a revolving body defcribes by Fig.
radii drawn to a fixed center of force, are propor- 6.
*tional to the times of defcription; and are all in the
fame immovable plane.*

Let S be the center of force; and let the time
be divided into very fmall equal parts. In the firft
part of time let the body defcribe the line AB;
then if nothing hindered, it would defcribe BK =
AB, in the fecond part of time; and then the area
ASB=BSK. But in the point B let the centri-
petal force act by a fingle but ftrong impulfe, and
caufe the body to defcribe the line BC. Draw KC
parallel to SB, and compleat the parallelogram
BKC*r*, then the triangle SBC = SBK, being be-
tween the fame parallels; therefore SBC = SBA,
and in the fame plane. Alfo the body moving uni-
formly, would in another part of time defcribe C*m*
= CB; but at C, at the end of the fecond part of
time, let it be acted on by another impulfe, and
carried along the line CD; draw *m*D parallel to
CS, and D will be the place of the body after the
third part of time; and the triangle SCD = SC*m*
= SCB, and all in the fame plane. After the
fame manner let the force act fucceffively at D,
E, F, &c. And making D*n* = DC, and E*o* =
ED, &c. and compleating the parallelograms as
before;

Fig.
6. before; the triangle $CSm = CSD = DSn = DSE$ $= ESo = ESF$, &c. and all in the fame immovable plane. Therefore in equal times equal areas are defcribed; and by compounding, the fum of all the areas is as the time of defcription. Now let the number of triangles be increafed, and their breadth diminifhed *ad infinitum*; and the centripetal force will act continually, and the figure ABCDEF, &c. will become a curve; and the areas will be proportional to the times of defcription.

Cor. 1. *If a body defcribes areas proportional to the times, about any point; it is urged towards that point by the centripetal force.*
For a body cannot defcribe areas proportional to the times, about two different points or centers, in the fame plane.

Cor. 2. *The velocity of a body revolving in a curve, is reciprocally as the perpendicular to the tangent, in that point of the curve.*
For the area of any of thefe little triangles being given; the bafe (which reprefents the velocity) is reciprocally as the perpendicular.

7. *Cor.* 3. *The angular velocity at the center of force, is reciprocally as the fquare of its diftance from that center.*
For if the fmall triangles CSD and SBA be equal, they are defcribed in equal times. The area $CSD = \dfrac{SC \times CQ}{2}$, and area $SBA = \dfrac{SB \times BP}{2}$, therefore $SC \times CQ = SB \times BP$. But the angle CSD : angle ASB : : CQ : cq : : $SC \times CQ$:$SC \times cq$: : $SB \times BP$: $SC \times cq$: : area SBA : area Scq : : $SB^2 : Sc^2$ or SC^2.

PROP.

PROP. XII.

If a body revolving in any curve VIL, *be urged by a centripetal force tending towards the center* S; *the centripetal force in any point* I *of the curve will be as*

$\dfrac{p}{p^3 d}$; *where* p = *perpendicular* SP *on the tangent at* I, *and* d = *the diſtance* SI.

For take the point K infinitely near I, and draw the lines SI, SK; and the tangents IP, Kf; and the perpendiculars SP, Sf. Alſo draw Km, Kn parallel to SP, SI, and KN perp. to SI.

The triangles ISP, IKN, nKm, are ſimilar; as alſo IKm, IPq. Therefore Iq or IP : IK : : qP : Km. And PS : IP : : Km : mn. And IN : IK : : mn : nK. And multiplying the terms of theſe three proportions, IP × PS × IN : IK × IP × IK : : qP × Km × mn : Km × mn × nK. That is, PS × IN : IK² : : qP : nK = $\dfrac{Pq \times IK^2}{PS \times IN}$. But (Mechan. Prop. VI.) the ſpace nK, through which the body is drawn from the tangent, is as the force and ſquare of the time; that is (Prop. XI.) as the force and ſquare of the area ISK, or as the force × SI² × KN², or becauſe SI × KN = twice the triangle ISK = IK × SP; therefore nK is as the force × IK² × PS². Therefore the force at I is as

$$\dfrac{n\mathrm{K}}{\mathrm{IK}^2 \times \mathrm{PS}^2} = \dfrac{Pq \times \mathrm{IK}^2}{\mathrm{PS} \times \mathrm{IN} \times \mathrm{IK}^2 \times \mathrm{PS}^2} = \dfrac{Pq}{\mathrm{PS}^3 \times \mathrm{IN}} = \dfrac{p}{p^3 d}.$$

Cor. 1. *The centripetal force at* I *is as* $\dfrac{n\mathrm{K}}{\mathrm{SI}^2 \times \mathrm{KN}^2}$, *or as* $\dfrac{n\mathrm{K}}{\mathrm{SP}^2 \times \mathrm{IK}^2}.$

Q   Cor.

Fig.
8.

Cor. 2. *Hence the radius of curvature in* I, *is* $=$ $\dfrac{SI \times IN}{Pq}$.

For that radius $= \dfrac{IK^2}{Km} =$ (by the fimilar trian-

gles IKm, IqP) $\dfrac{IK \times IP}{Pq} =$ (by the fimilar trian-

gles IPS, INK) $\dfrac{SI \times IN}{Pq}$.

Cor. 3. *If* R *be the radius of curvature in* I, *the centripetal force at* I *will be as* $\dfrac{d}{p^3 R}$ *or* $\dfrac{SI}{R \times SP}$.

For $R = \dfrac{SI \times IN}{Pq}$, and $\dfrac{Pq}{IN} = \dfrac{SI}{R}$; whence the

force $\dfrac{Pq}{PS^3 \times IN} = \dfrac{SI}{R \times PS^3}$.

PROP. XIII. *Prob.*

To find the law of the centripetal force, requifite to make a body move in a given curve line.

Let the diftance SI $= d$, the perpendicular SP (upon the tangent at I) $= p$; then from the na-ture of the curve, find the value of p in terms of d, and fubftitute it and its fluxion, in the quantity $\dfrac{\dot{p}}{p^3 d}$.

Or find the value of $\dfrac{nK}{SI^2 \times KN^2}$ or $\dfrac{nK}{SP^2 \times IK^2}$

Any of thefe will give the law of centripetal force by the laft Prop.

Ex. 1.

9.

If a body revolves in the circumference of a cir-cle; to find the force directed to a given point S.

Draw

Draw SI to the body at I, SP perp. to the tan- Fig.
gent PI, SG perp. to the radius CI. Then SP = 9.
GI; becaufe SGIP is a parallelogram. Put SI =
d, SP $= p$, SC $= a$, CI $= r$, CD $= x$, ID be-
ing perp. to SD. Then in the obtufe angle SCI,
$SI^2 = SC^2 + CI^2 + 2SCD$, or $dd = aa + rr + 2ax$;

whence $x = \dfrac{dd - aa - rr}{2a}$. The triangles

SCG and CID are fimilar, whence CI (r) : CD

(x) : : SC (a) : CG $= \dfrac{ax}{r} = \dfrac{dd - aa - rr}{2r}$; and

$p = r + \dfrac{dd - aa - rr}{2r} = \dfrac{dd + rr - aa}{2r}$; and

$\dot{p} = \dfrac{d\dot{d}}{r}$. Therefore the force $\left(\dfrac{\dot{p}}{p^3 d}\right)$ is as $\dfrac{d\dot{d}}{rp^3 d} =$

$\dfrac{d}{rp^3} = \dfrac{d \times 8r^3}{r \times \overline{dd + rr - aa}\,]^3}$; that is, the force is as

$\dfrac{d}{\overline{dd + rr - aa}\,]^3}$.

And if $a = r$, the force is as $\dfrac{1}{d^5}$.

Ex. 2.

If a body revolves in an ellipfis; to find the force 10.
tending to the center C.

Let $\frac{1}{2}$ tranfverfe CV$=r$, $\frac{1}{2}$ conjugate CD$=c$,
draw CI $= d$, and its femiconjugate CR $= b$.
Then by the properties of the ellipfis (Con.
Sect. B. I. Prop. XXXIV.) $bb + dd = rr + cc$,
whence $b = \sqrt{rr + cc - dd}$; and (ib. Prop.
XXXVII.) b or $\sqrt{rr + cc - dd} : c :: r : p =$

$\dfrac{cr}{\sqrt{rr + cc - dd}}$; and $\dot{p} = \dfrac{crd\dot{d}}{\overline{rr + cc - dd}\,]^{\frac{3}{2}}}$. There-

Q 2   fore

Fig.
10. fore $\dfrac{\dot{p}}{p^3\dot{d}} = \dfrac{crd\dot{d}}{\overline{rr+cc-dd}|^{\frac{1}{2}}} \times \dfrac{\overline{rr+cc-dd}|^{\frac{1}{2}}}{c^3r^3\dot{d}} = \dfrac{d}{ccrr}$.

Therefore the force is directly as the diftance CI.

After the fame manner, the force tending to the

center of an hyperbola, will be found $\dfrac{-d}{ccrr}$, which

is a centrifugal force, directly as the diftance.

Ex. 3.

11. *If a body revolves in an ellipfis, to find the law of
centripetal force, tending to the focus S.*

Let the femitranfverfe OV $= r$, the femiconju-
gate OD $= c$, draw SI $= d$; and OI, and its con-
jugate OK $= b$.

Then (Con. Sect. B. I. Prop. XXXV.) $2rd - dd$
$= bb$; and (ib. Prop. XXXVI.) b or $\sqrt{2rd - dd}$:

$c :: d : p = \dfrac{cd}{\sqrt{2rd - dd}}$; and $\dot{p} =$

$$\dfrac{cd\sqrt{2rd - dd} - cd \times \overline{2rd - dd}|^{-\frac{1}{2}} \times \overline{rd - dd}}{2rd - dd} =$$

$$\dfrac{cd \times \overline{2rd - dd} - cd \times \overline{rd - dd}}{\overline{2rd - dd}|^{\frac{3}{2}}} = \dfrac{crd\dot{d}}{\overline{2rd - dd}|^{\frac{3}{2}}}.$$

Therefore $\dfrac{\dot{p}}{p^3\dot{d}} = \dfrac{crd\dot{d} \times \overline{2dr - dd}|^{\frac{3}{2}}}{c^3d^3\dot{d} \times \overline{2dr - dd}|^{\frac{3}{2}}} = \dfrac{crd}{c^3d^3} = \dfrac{r}{ccdd}$.

Therefore the centripetal force is as $\dfrac{1}{dd}$, or reci-

procally as the fquare of the diftance.

Ex. 4.

12. *If a body revolves in the hyperbola* VI; *to find the
law of centripetal force, tending to the focus* S.

Draw SI, and the tangent IT, and SP perp.
upon it. And let the femitranfverfe SO $= r$, fe-
miconjugate

miconjugate $= c$, SI $= d$, SP $= p$, and $b =$ femi-
conjugate to IO.

Then (Con. Sect. B. II. Prop. XXXI.) $2rd +$
$dd = bb$, and $b = \sqrt{2rd + dd}$. And (ib. Prop.
XXXII.) b or $\sqrt{2rd + dd} : c : : d : p = $
$\dfrac{cd}{\sqrt{2rd+dd}}$; whence $\dot{p} =$

$$\frac{c\dot{d} \times \sqrt{2rd + dd} - cd \times \overline{2rd + dd}^{-\frac{1}{2}} \times \overline{r\dot{d} + d\dot{d}}}{2rd + dd}$$

$$= \frac{c\dot{d} \times \overline{2rd + dd} - cd \times \overline{r\dot{d} + d\dot{d}}}{\overline{2rd + dd}^{\frac{3}{2}}} = \frac{crd\dot{d}}{\overline{2rd + dd}^{\frac{3}{2}}}.$$

Therefore $\dfrac{\dot{p}}{p^3 d} = \dfrac{crd\dot{d} \times \overline{2rd + dd}^{\frac{3}{2}}}{\overline{2rd+dd}^{\frac{3}{2}}\dot{d} \times c^3 d^3} = \dfrac{crd}{c^3 d^3} =$

$\dfrac{r}{ccdd}$. Therefore the centripetal force is as $\dfrac{r}{ccdd}$ or

$\dfrac{1}{dd}$; that is, reciprocally as the square of the dif-
tance.

And in like manner the force towards the other

focus F, is $\dfrac{-r}{ccdd}$, or as $\dfrac{-1}{dd}$, which is a centrifugal

force reciprocally as the square of the diftance.

Ex. 5.

If a body revolves in the parabola VI; *to find the*
force tending to the focus S.

Draw IS, and the tangent IT, and SP perp. to
it. And put SI $= d$, SP $= p$, *latus rectum $= r$.*
Then (Con. Sect. B. III. Prop. II. and Cor. 3.
Prop. XII.) $pp = \frac{1}{4}rd$, and $2p\dot{p} = \frac{1}{4}r\dot{d}$; and $\dot{p}$

$= \dfrac{r\dot{d}}{8p} = \dfrac{r\dot{d}}{4\sqrt{rd}}.$ And $\dfrac{\dot{p}}{p^3 d} = \dfrac{r\dot{d}}{4\sqrt{rd} \times \frac{1}{4}rd\sqrt{rd} \times d}$

$=$

Fig.
13. $= \dfrac{8r}{4rrdd} = \dfrac{2}{rdd}$. Therefore the centripetal force is reciprocally as the fquare of the diftance CI.

Hence, in all the Conic Sections the centripetal force tending to the focus, is reciprocally as the fquare of the diftance from the focus.

Ex. 6.

14. *Let* VI *be the logarithmic fpiral, to find the force tending to the center* S.

Draw the tangent IP, and SP perp. to it, let SI $= d$, SP $= p$; then the ratio of d to p is always given, fuppofe -as m to n. Then $p = \dfrac{n}{m}d$, and $\dot{p} = \dfrac{n}{m}\dot{d}$. Confequently $\dfrac{\dot{p}}{p^3 \dot{d}} = \dfrac{n\dot{d}}{m} \times \dfrac{m^3}{n^3 d^3 \dot{d}} = \dfrac{mm}{nnd^3}$; and the centripetal force is as $\dfrac{1}{d^3}$, or reciprocally as the cube of the diftance.

PROP. XIV.

15. *The velocity of a body moving in any curve* QAO, *in any point* A; *is to the velocity of a body moving in a circle at the fame diftance; as* $\sqrt{\dot{pd}}$ *to* $\sqrt{\dot{dp}}$. Putting $d=$diftance SA, and $p=$SP *the perpendicular on the tangent at* A.

Let AR be the radius of curvature; from the point a in the curve infinitely near A, draw am, an parallel to AS, AR. Let C=velocity in the curve; $c=$velocity in the circle. By fimilar triangle SP (p) : SA (d) : : an : am : : centripetal force tending to R : centripetal force tending to S : ; (Prop. V. Cor. 2.) $\dfrac{CC}{AR} : \dfrac{cc}{AS}$. But (Prop. XII. Cor. 2.) AR$=\dfrac{d\dot{d}}{\dot{p}}$; whence $p : d :: \dfrac{CC\dot{p}}{d\dot{d}} :$

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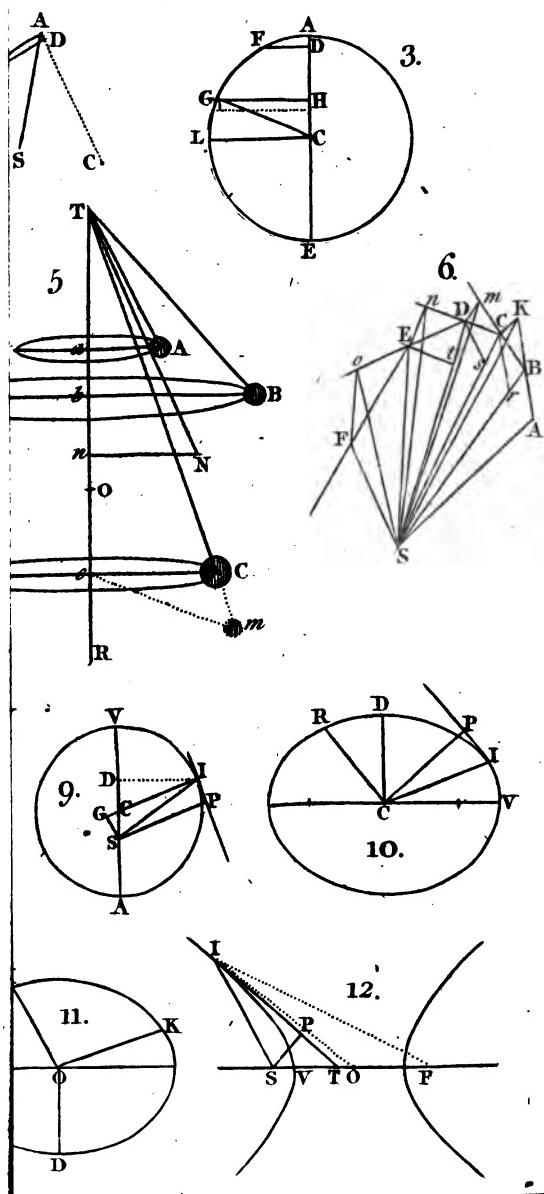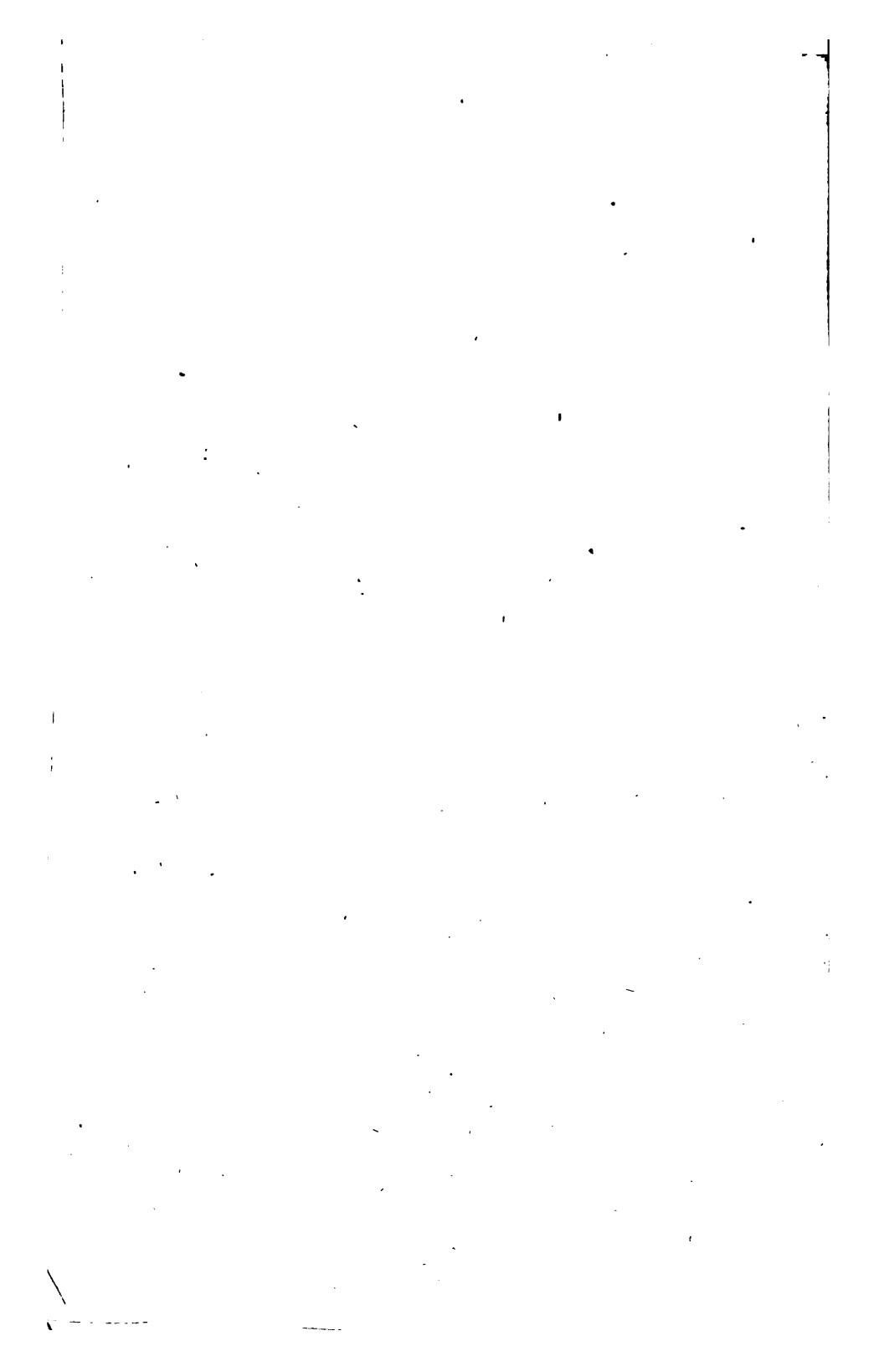
$\frac{cc}{d} :: CC\dot{p} : cc\dot{d}.$ And $p\dot{d} : d\dot{p} :: CC : cc.$

Cor. 1. *If* r=*half the tranverse axis of an el-
lipfis ; then the velocity of a body revolving round the
focus is to that in a circle at the fame diftance.; as*
$\sqrt{2r-d} : \sqrt{r}.$

For $\dot{p} = \dfrac{crd\dot{d}}{\overline{2rd-dd}|^{\frac{3}{2}}}$ (See Ex. 3. Prop. XIII.),

and $p = \dfrac{cd}{\sqrt{2rd-dd}}.$ And the fquares of the

velocities in the curve, and in the circle, are as

$\dfrac{cd\dot{d}}{\sqrt{2rd-dd}}$ and $\dfrac{crdd\dot{d}}{\overline{2rd-dd}|^{\frac{3}{2}}},$ or as 1 and $\dfrac{rd}{2rd-dd},$

or as $2r-d$ to r.

Cor. 2. *Suppofe as before, the velocity of a body
revolving round the center of an ellipfis, is to the ve-
locity in a circle at the fame diftance ; as half the con-
jugate diameter to that diftance, is to the diftance.*

For $p = \dfrac{cr}{\sqrt{rr+cc-dd}},$ and $\dot{p} =$

$\dfrac{crd\dot{d}}{\overline{rr+cc-dd}|^{\frac{3}{2}}}.$ Whence, the fquares of thefe ve-

locities are as $\dfrac{cr\dot{d}}{\sqrt{rr+cc-dd}}$ and $\dfrac{crdd\dot{d}}{\overline{rr+cc-dd}|^{\frac{3}{2}}}$

or as 1 to $\dfrac{dd}{rr+cc-dd}$ or as $rr+cc-dd$ to $dd,$

or as bb to dd.   See Ex. 2. Prop. XIII.

Cor. 3. *The velocity in a parabola round the focus,
is to the velocity in a circle at the fame diftance; as*
$\sqrt{2}$ *to* 1.

For $p = \frac{1}{2}\sqrt{rd},$ and $\dot{p} = \dfrac{rd\dot{d}}{4\sqrt{rd}}$ (See Ex. 5.

Q 4   Prop.

Fig. Prop. XIII.)   Whence the fquares of thefe veloci-
15.
ties are as $\frac{1}{2}\dot{d}\sqrt{rd}$ and $\dfrac{rd\ddot{d}}{4\sqrt{rd}}$, or as $\frac{1}{2}rd$ to $\frac{1}{4}rd$;
that is as 2 to 1.

Cor. 4. *The velocity of a body in the logarithmic
fpiral in any point, is the fame as the velocity of a
body at the fame diftance in a circle.*

For $p=\dfrac{n}{m}d$, and $\dot{p}=\dfrac{n}{m}\dot{d}$, (Ex. 6. Prop. XIII.)

And the fquares of the velocities are as $\dfrac{n}{m}\dot{d}\dot{d}$ and

$\dfrac{n}{m}\dot{d}\dot{d}$, that is, equal.

PROP. XV. *Prob.*

16.   *To find the force which, acting in direction of the
ordinate* MP, *fhall caufe the body to move in that curve.*

Draw *mp* parallel and infinitely near MP, and
M*s* parallel to AP.   Then univerfally the force is
always as the fpace *mr*, through which it is drawn
from the tangent, in a given time.   But *ms* or *rs*
is the fluxion and *mr* the fecond fluxion of the ordi-
nate PM.   Therefore making the fluxion of the
time conftant; or which is the fame thing, making
the fluxion of the axis conftant; find the fecond
fluxion of the ordinate, which will be as the force.

Ex. 1.

Let the curve be an ellipfis whofe equation is $y=\dfrac{c}{r}\sqrt{2rx-xx}$.   Putting AP $=x$, PM $=y$, $r=$
femitranfverfe, $c=$ femiconjugate.   Then $\dot{y}=\dfrac{c}{r}\times\dfrac{rx-xx}{\sqrt{2rx-xx}}$,   and $\ddot{y}=\dfrac{c}{r}\times\dfrac{-x\sqrt{2rx-xx}}{2rx-xx}$

$$- \frac{c}{r} \times \overline{r - \dot{x}} \times \overline{r\dot{x} - \dot{x}\dot{x}} \times \overline{2r\dot{x} - \dot{x}\dot{x}}^{-\frac{1}{2}}$$
$$\overline{2r\dot{x} - \dot{x}\dot{x}} = \frac{c}{r} \times$$

$$\frac{- 2r\dot{x} + \dot{x}\dot{x} - \overline{r - \dot{x}}^{2}}{\overline{2r\dot{x} - \dot{x}\dot{x}}^{\frac{3}{2}}} = \frac{c}{r} \times \frac{-rr}{\overline{2r\dot{x} - \dot{x}\dot{x}}^{\frac{3}{2}}} = \frac{-cr}{y^3}.$$

That is, the force is as $\dfrac{-1}{y^3}$, or reciprocally as the cube of the ordinate. The fame is true of the circle, which is one fort of ellipfis.

<p align="center">Ex. 2.</p>

Let the curve be a parabola, AP $= x$, PM $-$ y, and $r\dot{x} = y\dot{y}$; then $r\ddot{x} = 2\dot{y}\dot{y}$, and $2\dot{y}\dot{y} + 2y\ddot{y}$ $= 0$; therefore $y\ddot{y} = - \dot{y}\dot{y}$, and $\ddot{y} = -\dfrac{\dot{y}\dot{y}}{y} = -$

$\dfrac{rr\dot{x}\dot{x}}{4\dot{y}\dot{y} \times y} = \dfrac{-rr}{4y^3}$, and the force as $\dfrac{-1}{y^3}$, or reciprocally as the cube of the ordinate.

<p align="center">PROP.   XVI.</p>

If the law of centripetal force be reciprocally as the fquare of the diftance; the velocities of bodies revolving in different ellipfes about one common center, are directly as the fquare roots of the parameters, and reciprocally as the perpendiculars to the tangents at thefe points of their orbits.

Let d, D be the diftances in two ellipfes; r, c, l, p; and R, C, L, P, the femitranfverfe, femiconjugate, *latus rectum*, and perpendicular in the two ellipfes. Then the fquares of the velocities in two circles whofe radii are d, D, (by Prop. IV. Cor. 2.) will be as $d \times$ force in d, and D $\times$ force in D; that is, as $\dfrac{d}{dd}$ and $\dfrac{D}{DD}$, or as $\dfrac{1}{d}$ and $\dfrac{1}{D}$.

Then (Prop. XIV. Cor. 1.),

<p align="right">velocity</p>

Fig. velocity in the ellipſis d : vel. in the circle d : : $\sqrt{2r-d}$: $\sqrt{r}$.

and vel. in the circle d : vel. in the circle D : : $\sqrt{\dfrac{1}{d}}$: $\sqrt{\dfrac{1}{D}}$.

And (Prop. XIV. Cor. 1.)

vel. in the circle D : vel. in the ellipſis D : : $\sqrt{R}$: $\sqrt{2R-D}$.

Therefore vel. in the ellipſis d : vel. in the ellipſis

$$D : : \sqrt{\frac{2r-d \times R}{d}} : \sqrt{\frac{2R-D \times r}{D}} : : \sqrt{\frac{2r-d}{dr}}$$

$$: \sqrt{\frac{2R-D}{DR}}.$$

But (Con. Sect. B. I. Prop. 36.) $p = c\sqrt{\dfrac{d}{2r-d}}$,

and $p\sqrt{\dfrac{2r-d}{d}} = c$, and $\sqrt{\dfrac{2r-d}{d}} = \dfrac{c}{p}$, and

$\sqrt{\dfrac{2r-d}{dr}} = \dfrac{c}{p\sqrt{r}} = \dfrac{\sqrt{\frac{1}{2}l}}{p}$ (becauſe $\dfrac{cc}{r} = \frac{1}{2}l$ by

the Conic Sections.) In like manner $\sqrt{\dfrac{2R-D}{DR}} =$

$\dfrac{\sqrt{\frac{1}{2}L}}{P}$. Whence, vel. in the ellipſis d : vel. in

the ellipſis D : : $\dfrac{\sqrt{l}}{p}$: $\dfrac{\sqrt{L}}{P}$.

Cor. 1. *Hence the velocities in the two ellipſis, are*

as $\sqrt{\dfrac{2r-d}{dr}}$, *and* $\sqrt{\dfrac{2R-D}{DR}}$.

Cor. 2. *Alſo the ſquares of the areas deſcribed in the ſame time, are as the parameters.*

For the areas are as the arches × perpendiculars, or as the velocities × perpendiculars; that is, as $\dfrac{\sqrt{l}}{p} \times p$ and $\dfrac{\sqrt{L}}{P} \times P$, or as $\sqrt{l}$ and $\sqrt{L}$.

Cor. 3. *The velocity of a body in different parts of its orbit is reciprocally as the perpendicular upon the*

tangent

tangent at that point; and therefore is as $\sqrt{\dfrac{2r-d}{d}}$. Fig.

For the parameter is given.

Cor. 4. *The velocity in a conic section at its great-est or least distance, is to the velocity in a circle at the same distance; as the square root of the parameter, to the square root of twice that distance.*

For here $d = D = p = P$, and $L = 2D$. There-fore the velocity in the ellipsis, to the velocity in the circle; as $\dfrac{\sqrt{l}}{P} : \dfrac{\sqrt{2D}}{P} :: \sqrt{l} : \sqrt{2D}$.

Cor. 5. *The velocity in an ellipsis at its mean dis-tance, is the same as in a circle at the same distance.*

For if d be the mean distance, then $p = c$. And if D be the radius of the circle, then $L = 2D$, and $P = D$. Whence, vel. in the ellipsis : to the vel. in the circle :: $\dfrac{\sqrt{l}}{c} : \dfrac{\sqrt{2D}}{D} ::$ (because $cc = \frac{1}{2}lr$)

$\dfrac{1}{\sqrt{\frac{1}{2}r}} : \dfrac{1}{\sqrt{\frac{1}{2}D}} :: \sqrt{D} : \sqrt{r}$. But $D = r$, there-fore the velocites are equal.

Cor. 6. *Both the real and apparent velocity round* 17. *the focus F, is greatest at A, the nearest vertex; and least at B, the remote vertex.*

For the real velocity is reciprocally as the per-pendicular, which is least at A and greatest at B. And the apparent velocity at F is reciprocally as the square of the distance from F, which distance is least at A, and greatest at B, (Cor. 2. and 3, Prop. XI.)

Cor. 7. *The same things supposed, and PC, CK* 23. *being semiconjugates; the velocity in the curve, is to the velocity towards the focus F; as CK to* $\sqrt{CK^2 - CD^2}$.

For

Fig.　For vel. in the curve : vel. towards $F : : Pp :$

23. $pn : : FP : NP : : d : \sqrt{dd - pp}.$　But $pp =$ $\dfrac{ccd}{2r - d}$, and $dd - pp = \dfrac{2rd - dd - cc}{2r - d}d.$　Whence vel. in the curve : vel. towards $F : : d :$

$$\sqrt{\dfrac{2rd - dd - cc}{2r - d}}d : : \sqrt{2rd - dd} : \sqrt{2rd - dd - cc}$$

: : (Con. Sect. B. I. Prop. XXXV.) CK :
$\sqrt{CK^2 - CD^2}.$

Cor. 8. *The afcending or defcending velocity is the greateft when* FP *is half the* latus rectum, *or when* FP *is perp. to* AB.

For $\sqrt{2rd - dd} : \sqrt{2rd - dd - cc} : :$ vel. in

the curve $\left(\dfrac{\sqrt{2r - d}}{d}\right)$: vel. towards $F =$

$\sqrt{\dfrac{2rd - dd - cc}{dd}}$; and making the fquare of this

velocity a maximum, then $\dfrac{2rd - dd - cc}{dd} = m,$ and

$\overline{2r\dot{d} - 2d\dot{d}} \times dd - 2d\dot{d} \times \overline{2rd - dd - cc} = 0;$ and $rd - dd - 2rd + dd + cc = 0,$ and $- rd + cc = 0.$　Whence $d = \dfrac{cc}{r} =$ half the *latus rectum.*

Cor. 9. *If* FR, *the diftance from the focus to the curve be* $= \sqrt{CA \times CD}$; *then* R *is the place where the angular motion about the focus* F, *is equal to the mean motion.*

For the area of a circle whofe radius FR is $=$ $\sqrt{CA \times CD}$ is equal to the area of the ellipfis; and if we fuppofe them both defcribed in equal times, then the fmall equal parts at R will be defcribed in equal times; and therefore the angular velocities at F will be equal; and both equal to the mean motion.　The angular motion in the el-
lipfis

lipfis from B to R will be flower ; and from R to Fig.
A fwifter, than the mean motion.

PROP. XVII.

*If the centripetal forces be reciprocally as the 17.
fquares of the diftances; the periodic times in ellipfes,
will be in the fefquiplicate ratio of the tranfverfe
axes AB; or the fquares of the periodic times, will
be as the cubes of the mean diftances FD, from the
common center.*

Put the fimbols as in the laft, and t, T, for the
periodical times. Then by the nature of the el-
lipfis $cc = \frac{1}{2} lr$, and $c = \sqrt{\frac{1}{2} lr}$, and $rc = r\sqrt{\frac{1}{2} lr}$.
And for the fame reafon $RC = R\sqrt{\frac{1}{2} LR}$. Alfo
(Prop. XVI. Cor. 2.) the areas defcribed in the
fame time are as the fquare roots of the parame-
ters ; and therefore the whole areas of the ellipfes,
are as the periodical times multiplied by the fquare
roots of the parameters. But the whole areas are
alfo as the rectangles of the axes; therefore the
rectangles of the axes are as the periodical times
multiplied by the fquare roots of the parameters;
that is, rc or $r\sqrt{\frac{1}{2} lr}$: RC or $R\sqrt{\frac{1}{2} LR}$: : $t\sqrt{l}$:
$T\sqrt{L}$. And fquaring, $\frac{1}{2} lr^3$: $\frac{1}{2} LR^3$: : ttl : TTL.
That is, r^3 : R^3 : : tt : TT. And t : T : : $r^{\frac{3}{2}}$: $R^{\frac{3}{2}}$: :
$2r^{\frac{3}{2}}$: $2R^{\frac{3}{2}}$.

Cor. 1. *The areas of the ellipfes are as the periodic
times multiplied by the fquare roots of the parameters.*

Cor. 2. *The periodic time in an ellipfis, is the fame
as in a circle, whofe diameter is equal to the tranfverfe
axis AB; or the radius equal to the mean diftance ED.*

▪ Cor. 3. *The quantities of matter in central attract-
ing bodies, that have others revolving about them in
ellipfes; are as the cubes of the mean diftances, divi-
ded by the fquares of the periodical times.*

For·

Fig.
17.
For (Cor. 2.) the periodic times are the same when the mean diftances are equal to the radii; and the reft follows from Prop. VII.

Cor. 4. *Whatever has been demonftrated in Sett. I. concerning the forces, velocities and periodic times of bodies moving in circles; hold equally true in ellipfes, taking the mean diftances inftead of the radii.*
This will appear by Cor. 2. or by comparing this Prop. with Cor. 5. Prop. V. for the periodic times. And by Cor. 5. Prop. XVI. for the velocities; the forces being reciprocally as the fquares of the diftances, in both cafes.

PROP. XVIII.

18. *If the centripetal forces be directly as the diftances; the periodic times of bodies moving in ellipfes round the fame center, will be all equal to one another.*

Let AEL be an ellipfis, AGL a circle on the fame axis AL, C the center of both. Draw the tangent A·D, and *np*F parallel to it, and D*n*, B*p* parallel to AC : AF being very fmall. Then D*n* equal to B*p* will be as the centripetal force; and therefore AD and AB, or A*n* and A*p* will be defcribed in the fame time, in the circle and ellipfis. Confequently the areas defcribed in thefe equal times will be A*n*C and A*p*C. But thefe areas are to one another as *n*F to PF, or as GC to EC; that is, as the area of the circle AGL to the area of the ellipfis AEL. Therefore fince parts proportional to the wholes are defcribed in equal times; the wholes will be defcribed in equal times. And therefore the periodic times, in the circle and ellipfis, are equal.

But (Prop. V. Cor. 4.) the periodic times in all circles are equal, in this law of centripetal force; and therefore the periodic times in all ellipfes are equal.

Cor.

Cor. *The velocity at any point* I *of an ellipsis, is* Fig. *as the rectangle of the two axes* AC, CE; *divided* 18. *by the perpendicular* CH, *upon the tangent at* I.

For the arch I × CH is as the area defcribed in a fmall given part of time, and that is as the whole area (becaufe the periodic times are equal) or as AC × CE. And therefore the arch I or the velocity, is as $\dfrac{AC \times CE}{CH}$.

PROP.   XIX.

The denfities of central attracting bodies, are reciprocally as the cubes of the parallaxes of the bodies revolving about them (as feen from thefe central bodies) and reciprocally as the fquares of the periodic times.

For the denfity multiplied by the cube of the diameter, is as the quantity of matter; that is (by Prop. XVII. Cor. 3.) as the cube of the mean diftance divided by the fquare of the periodical time of the revolving body. And therefore the denfity is as the cube of the diftance, divided by the cube of the diameter, and by the fquare of the periodic time. But the diameter divided by the diftance is as the angle of the parallax; therefore the denfity is as 1 divided by the cube of the parallax, and the fquare of the periodic time.

Cor. *The denfities of central attracting bodies (having others revolving about them) are reciprocally as the cubes of the apparent diameters of thefe central bodies (feen from the revolving bodies), and reciprocally as the fquares of the periodic times.*

PROP.   XX.

If two bodies A, B, *revolve about each other;* 19. *they will both of them revolve about their center of gravity.*

Let

Fig. 19.   Let C be the center of gravity of the bodies A, B, acting upon one another by any centripetal forces. And let AZ be the direction of A's motion; draw BM parallel to AZ, for the direction of B. And let AZ, BH be described in a very small part of time, so that AZ may be to BH, as AC to BC; and then C will be the center of gravity of Z and H, because the triangles ACZ and BCH are similar. Whence AC : CB : : ZC : CH. But as the bodies A and B attract one another, the spaces A*a* and B*b* they are drawn through will be reciprocally as the bodies, or directly as the distances from the center of gravity; that is A*a* : B*b* : : AC : BC. Compleat the parallelograms A*c* and B*d*; and the bodies, instead of being at Z and H, will be at *c* and *d*. But since AC : BC : : A*a* : B*b*. By division AC : BC : : *a*C : *b*C. But AC : BC : : AZ : BH : : *ac* : *bd*. Whence *a*C : *b*C : : *ac* : *bd*. Therefore the triangles *c*C*a*, and *d*C*b* are similar, whence C*c* : C*d* : : *ac* : *bd* : : AC : BC : : B : A. Therefore C is still the center of gravity of the bodies at *c* and *d*.

In like manner, producing B*d* and A*c*, till *dg* be equal to B*d*, and *cq* to A*c*; and if *cf*, *dh*, be the spaces drawn through by their mutual attractions; and if the parallelograms *ce*, *di*, be compleated. Then it will be proved by the same way of reasoning, that C is the center of gravity of the bodies at *q* and *g*, and also at *e* and *i*, where A describes the diagonals A*c*, *ce*, &c. and B the diagonals B*d*, *di*, and so on *ad infinitum*.

If one of the bodies B is at rest whilst the other moves along the line AL. Then the center of gravity C will move uniformly along the line CO parallel to AL. Therefore if the space the bodies move in, be supposed to move in direction CO, with the velocity of the center of gravity; then the center of gravity will be at rest in that space,

and

and the body B will move in direction BH parallel Fig.
to CO or AZ; and then this cafe comes to the 19.
fame as the former. Therefore the bodies will
always move round the center of gravity, which is
either at reft, or moves uniformly in a right line.

If the bodies repel one another; by a like
reafoning it may be proved that they will con-
ftantly move round their center of gravity.

If the lines CA, Cc, Ce, &c. be equal; and
CB, Cd, Ci, &c. alfo equal. Then it is the cafe
of two bodies joined by a rod or ftring; or of
one body compofed of two parts. This body or
bodies will always move round their common cen-
ter of gravity.

Cor. 1. *The directions of the bodies in oppofite points
of the orbits, are always parallel to one another.*
For fince AZ : Zc : : BH : Hd; and AZ, Zc
parallel to BH, Hd; therefore the ∠ ZAc = ∠
HBd, and Bd parallel to Ac. And for the fame
reafon di is parallel to ce, &c.

Cor. 2. *Two bodies, acting upon one another by
any forces; defcribe fimilar figures about their common
center of gravity.*
For the particles Ac, Bd of the curves are pa-
rallel to one another, and every where proportional
to the diftances of the bodies AC, BC.

Cor. 3. *If the forces be directly as the diftances;
the bodies will defcribe concentrical ellipfes round the
center of gravity.*

Cor. 4. *If the forces be reciprocally as the fquares
of the diftances; the bodies will defcribe fimilar el-
lipfes or fome conic fections, about each other, whofe
center of gravity is in the focus of both.*

R   PROP.

PROP. XXI.

20.   *If two bodies* S, P, *attract each other with any forces, and at the same time revolve about their center of gravity* C. *Then if either body* P, *with the same force, describes a similar curve about the other body* S *at rest; its periodical time, will be to the periodical time of either about the center of gravity; as the square root of the sum of the bodies* ($\sqrt{S+P}$), *to the square root of the fixed or central body* ($\sqrt{S}$.)

Let PV be the orbit described about C, and P*v* that described about S. Draw the tangent P*r*, take the arch PQ extremely small, and draw CQR, also draw S*qr* parallel to CR, and then PQ and P*q* will be similar parts of the curves PV and P*v*.

Now the times that the bodies are drawn from the tangent through the spaces QR, *qr*, with the same force, will be as the square roots of the spaces QR, *qr*; that is (because of the similar figures CPRQ and SP*rq*) as $\sqrt{CP}$ to $\sqrt{SP}$; that is, (by the nature of the center of gravity) as $\sqrt{S}$ to $\sqrt{S+P}$. But the times wherein the bodies are drawn from the tangent through RQ, *rq*, are the times wherein the similar arches PQ, P*q* are described; and these times are as the whole periodic times. Therefore the periodic time in PV, is to the periodic time in P*v*; as $\sqrt{S}$ to $\sqrt{S+P}$.

Cor. 1. *The velocity in the orbit* PV *about* C, *is to the velocity in the orbit* P*v* *about* S; *as* $\sqrt{S}$ *to* $\sqrt{S+P}$.

For the velocities are as the spaces divided by the times; therefore, vel. in PV : vel. in P*v* : :

$$\frac{PQ}{\sqrt{S}} : \frac{Pq}{\sqrt{S+P}} :: \frac{CP}{\sqrt{S}} : \frac{SP}{\sqrt{S+P}} :: \frac{S}{\sqrt{S}} : \frac{S+P}{\sqrt{S+P}}$$

$$:: \sqrt{S} : \sqrt{S+P}.$$

Cor. ·

Cor. 2. *Bodies revolving round their common cen-* Fig. *ter of gravity, defcribe areas proportional to the times.* 20.

PROP. XXII.

If the forces be reciprocally as the fquares of the 20. *diftances; and if a body revolves about the center* L *in the fame periodical time, that the bodies* S, P, *re-volve about the center of gravity* C. *Then will* SP : LP :* : $\sqrt[3]{S+P} : \sqrt[3]{S}$.

Let PN be the orbit defcribed about L. Then (Prop. XXI.) per. time in PQ : per. time in Pq : : $\sqrt{S} : \sqrt{S+P} :: \sqrt{CP} : \sqrt{SP}$. And (Prop. XVII.) per. time in Pq : per. time in PN : : SP$^{\frac{1}{2}}$: LP$^{\frac{1}{2}}$; fuppofing PQ, PN, fimilar arches, and the body S placed in L. Therefore per. time in PQ : per. time in PN : : $\sqrt{CP} \times$ SP$^{\frac{1}{2}}$: $\sqrt{SP} \times$ LP$^{\frac{1}{2}}$: : $\sqrt{CP \times SP^2}$: $\sqrt{LP^3}$. · But the periodic times are equal; therefore $\sqrt{CP \times SP^2} = \sqrt{LP^3}$, and LP^3 = CP $\times$ SP^2, and LP = $\sqrt[3]{CP \times SP^2}$. But LP : SP : : $\sqrt[3]{CP \times SP^2}$: SP or $\sqrt[3]{SP^3}$: : $\sqrt[3]{CP}$: $\sqrt[3]{SP}$: : $\sqrt[3]{S}$: $\sqrt[3]{S+P}$.

Cor. 1. *If the forces be reciprocally as the fquare of the diftances; the mean diftance* (SP) *of the bodies moving about their center of gravity* C; *is to their mean diftance when either body* P *moves about the other body* S *at reft, in the fame periodic time; as the cube root of the fum of the bodies* S + P, *to the cube root of the fixt or central body* S.

For in fig. 20. make SP = LP, the mean dif-tance; then SP : Sp : : $\sqrt[3]{S+P}$: $\sqrt[3]{S}$.

Cor. 2. *If the forces be reciprocally as the fquares of the diftances; the tranfverfe axis of the ellipfis*

R 2   *defcribed*

Fig. *defcribed by either body* P *about the center of gravity*
20. C, *is to the tranfverfe axis defcribed by* P *about the other body* S *at reft, in the fame periodical time; as the cube root of the fquare of the central body* $(\sqrt[3]{SS})$ *to the cube root of the fquare of the fum of the bodies* $(\sqrt[3]{\overline{S+P^2}})$

For Sp : SP : : $\sqrt[3]{CP}$: $\sqrt[3]{SP}$.

SP : CP : : $\sqrt[3]{SP^3}$: $\sqrt[3]{CP^3}$, therefore

it is Sp : CP : : $\sqrt[3]{CP \times SP^3}$: $\sqrt[3]{CP^3 \times SP}$: : $\sqrt[3]{SP^2}$

: $\sqrt[3]{CP^2}$: : $\sqrt[3]{\overline{S+P^2}}$: $\sqrt[3]{SS}$.

Or thus.

PL : PC : : PS^2 : PL^2 : : $\overline{S+P^{\frac{2}{3}}}$: S$^{\frac{2}{3}}$.

Cor. 3. *If two bodies attracting each other move about their center of gravity. Their motions will be the fame as if they did not attract one another, but were both attracted with the fame forces, by another body placed in the center of gravity.*

PROP. XXIII. *Prob.*

21.   *Suppofe the centripetal force to be directly as the diftance. To determine the orbit which a body will defcribe; that is projected from a given place* P, *with a given velocity, in a given direction* PT.

By Ex. 2. Prop. XIII. the body will move in an ellipfis, whofe center is C the center of force; and the line of direction PT will be a tangent at the point P. Draw CR perp. to PT. And let the diftance CP $=d$. CR $=p$, femitranfverfe axis CA $=$ R, femiconjugate axis CB $=$ C. CG (the femiconjugate to CP) $=$ B. $f =$ fpace a body would defcend at P, in a fecond, by the centripetal force. $v =$ the velocity at P, the body is projected with, or the fpace it defcribes in a fecond.
                                         Then

Then $\sqrt{2df}$ = velocity of a body revolving in a circle at the diftance CP.

Then (Prop. XIV. Cor. 2.) $v : \sqrt{2df} :: B : d$, and $B\sqrt{2df} = dv$, and $2BBdf = ddvv$, whence $BB = \dfrac{dvv}{2f}$, and $B = v\sqrt{\dfrac{d}{2f}}$. But (Con. Sect. B. I. Prop. XXXIV.) $RR + CC = BB + dd = \dfrac{vvd}{2f} + dd$. And (ib. Prop. XXXVII.) $CR = Bp = pv\sqrt{\dfrac{d}{2f}}$. Therefore $RR + CC + 2RC = \dfrac{vvd}{2f} + dd + 2pv\sqrt{\dfrac{d}{2f}}$, and $R + C = \sqrt{\dfrac{vvd}{2f} + dd + 2pv\sqrt{\dfrac{d}{2f}}} = m$. Alfo $RR + CC - 2RC = \dfrac{vvd}{2f} + dd - 2pv\sqrt{\dfrac{d}{2f}}$, and $R - C = \sqrt{\dfrac{vvd}{2f} + dd - 2pv\sqrt{\dfrac{d}{2f}}} = n$. Therefore $R = \dfrac{m + n}{2}$, and $C = \dfrac{m - n}{2}$.

Then to find the pofition of the tranfverfe axis AD. Let F, S be the foci. Then (by Con. Sect. B. I. Prop. II. Cor.) we fhall have SC or $CF = \sqrt{RR - CC}$. Put $FP = x$; then $SP = 2R - x$, and (ib. Prop. XXXV.) SP × PF or $2Rx - xx = BB$, and $RR - 2Rx + xx = RR - BB$, and $R - x = \pm\sqrt{RR - BB}$; whence $x = R \pm \sqrt{RR - BB}$; that is, the greater part $FP = R + \sqrt{RR - BB}$, and the leffer part $SP = R - \sqrt{RR - BB}$. Then in the triangle PCF or PCS, all the fides are given, to find the angle PCF or PCA.

Cor.

Fig.
21.
Cor. *The periodical time in seconds, is* 3.1416 $\sqrt{\dfrac{2d}{f}}$.

For arch $\sqrt{2df}$: time $1''$ :: circumference 3.1416

$\times 2d : 3.1416 \sqrt{\dfrac{2d}{f}}$ the periodical time in a circle

whose radius is d. And by Prop. XVIII. the periodical time is the same in all circles and ellipses.

PROP. XXIV. *Prob.*

22. *Suppofing the centripetal force reciprocally as the square of the diftance; to determine the orbit which a body will defcribe; that is projeEted from a given place* P, *with a given velocity, in a given direEtion* PT.

By Prop. XIII. the body will move in a conic fection, whofe focus is S the center of force. And the line of direction PT will be a tangent at the point P. Let the diftance SP $= d$, tranfverfe axis AD $= z$, $f =$ fpace a body will defcend at P in a fecond, by the centripetal force, $v =$ the velocity the body is projected with from P, or the fpace it defcribes in a fecond. Then $\sqrt{2df}$ is the velocity of a body revolving in a circle at the diftance SP.

Then (Prop. XIV. Cor. 1.) $v : \sqrt{2df} :: \sqrt{z - d}$: $\sqrt{\tfrac{1}{2}z}$. Whence $v\sqrt{\tfrac{1}{2}z} = \sqrt{2dfz - 2ddf}$, and $vvz = 4dfz - 4ddf$; and $4dfz - vvz = 4ddf$, whence $z = \dfrac{4ddf}{4df - vv} = $ AD. And PH $= z -$

$d = \dfrac{dvv}{4df - vv}$. Therefore if $4df$ is greater than vv, z is affirmative, and the orbit is an ellipfis. But if leffer, z is negative, and the curve is a hyperbola, and if equal, it is a parabola.

Draw SR perp. to PT, and let SR $= p$. Alfo draw from the other focus H, HF perp. to PT. Then

Then (Con. Sect. B. I. Prop. X.) the angle SPR Fig.
= angle HPF, whence the triangles SPR, HPF 22.
are fimilar; therefore SP (d) : SR (p) : : HP $(z -$

$d)$: HF $= \dfrac{z-d}{d} p$; and (ib. Prop. XXI.) SR ×

HF or $\dfrac{z-d}{d} pp$ = rectangle DHA or CB^2, the

fquare of half the conjugate axis; therefore CB =

$p\sqrt{\dfrac{z-d}{d}}$.

In the triangle SPH; the angle SPH and the
fides SP, PH are given, to find the angle PSH,
the pofition of the tranfverfe axis.

Cor. 1. *The periodical time in the ellipfis* APDB

$= 3.1416 \times \dfrac{4ddf}{\overline{4df - vv}|^{\frac{1}{2}}}$.

For $3.1416 \sqrt{\dfrac{2d}{f}}$ = periodic time in the circle

whofe radius is d. And (Prop. XVII.) $\overline{2d^{\frac{3}{2}}}$: 3.1416

$\sqrt{\dfrac{2d}{f}}$: : $z^{\frac{3}{2}}$: period. time in the ellipfis = 3.1416

$\sqrt{\dfrac{2d}{f}} \times \dfrac{\overline{z}|^{\frac{3}{2}}}{2d} = 3.1416 \times \dfrac{4ddf}{\overline{4df - vv}|^{\frac{1}{2}}}$.

Cor. 2. *The* latus rectum *of the axis* AD *is* =
$\dfrac{ppvv}{ddf}$.

Cor. 3. *Hence the tranfverfe axis and the periodic
time will remain the fame, whatever be the angle of
direction* SPT.
Fqr no quantities but $d, f,$ and v are concerned;
all which are given.

Some people have dreamed that there may be a
fyftem of a fun and planets revolving about it,
within any fmall particle of matter; or a world in
miniature.   But this cannot be; for though mat-
ter is infinitely divifible; yet the law of attraction
of the fmall particles of matter, not being as the
fquares of the diftances reciprocally, but nearer the
cubes; therefore the revolution of one particle of
matter about another, cannot be performed in an
ellipfis, but in fome other curve; where it will con-
tinually approach to or recede from the center;
and fo at laft will lofe its motion. Such motions as
thefe can be nothing like that of a fun and planets.

PROP. XXV.

24.   *If a body revolves in the circumference of a circle*
ZPA, in a refifting medium, whofe denfity is given.
To find the force at any place P, tending to the cen-
ter C; as alfo the time, velocity, and refiftance. Sup-
pofing the refiftance as the fquare of the velocity.

Draw PC, and dp parallel and infinitely near it,
cutting the tangent Pd in d.   And put $CZ = r$,
$ZP = z$, time of defcribing $ZP = t$, velocity at
$P = v$, refiftance $= R$, $f =$ force at P, $g =$ force
of gravity at Z, $c =$ velocity in Z.   And let a
body moving uniformly with the velocity 1, through
the fpace 1, in the time 1, meet the refiftance 1
in the medium. And let a body defcend through the
fpace a, by the force g at Z, in the fame time 1.

1. By the laws of uniform motion, the fpace is
as the time × velocity.   Whence 1 (fpace) : 1 × 1

(time × vel.) : : $\dot{z}$: $vt = \dot{z}$, whence $\dot{t} = \dfrac{\dot{z}}{v}$.

2. By the nature of the circle, $dp = \dfrac{z^2}{2r} = \dfrac{vv\dot{t}t}{2r}$.

3. By

3. By accelerated motion, the fpace is as the Fig. force × fquare of the time; whence $g \times 1^2$ (force 24. × time^2) : a (fpace) :: $\ddot{f}\ddot{t}$: dp cr $\dfrac{vv\ddot{t}t}{2r}$ :: $2rf : vv$

$= \dfrac{2arf}{g}$. And $cc = 2ar$.

4. The velocity generated (or deftroyed) is as the force × time; therefore, $g \times 1$ (force × time) : $2a$ (velocity) :: $R\dot{t}$: $-\dot{v} = \dfrac{2aR\dot{t}}{g} = \dfrac{2aR\dot{z}}{gv}$,

and $-\dot{v}v = \dfrac{2aR\dot{z}}{g}$.

5. The refiftance is as the fquare of the velocity, whence 1^2 (vel.^{2}) : 1 (refiftance) :: vv : R $= vv$.

Therefore $-\dot{v}v = \dfrac{2aR\dot{z}}{g} = \dfrac{2avv\dot{z}}{g}$.   And $-$

$\dfrac{\dot{v}}{v} = \dfrac{2a\dot{z}}{g}$, whence $-$ log. $v = \dfrac{2az}{g}$, and corrected,

log. $\dfrac{c}{v} = \dfrac{2az}{g}$.

Again, fince $\dfrac{2arf}{g} = vv$, $\dfrac{arf}{g} = v\dot{v} = -$

$\dfrac{2avv\dot{z}}{g}$, and $\dot{f} = -\dfrac{2vv\dot{z}}{r} = -\dfrac{4arf\dot{z}}{gr}$, and $-\dfrac{\dot{f}}{f}$

$= \dfrac{4a\dot{z}}{g}$, and $-$ log. $f = \dfrac{4az}{g}$, and corrected, log.

$\dfrac{g}{f} = \dfrac{4az}{g}$.

Alfo $\dot{t} = \dfrac{\dot{z}}{v} = \dfrac{-g\dot{v}}{2avv}$, and $t = \dfrac{g}{2av}$, and cor-

rected $t = \dfrac{g}{2av} - \dfrac{g}{2ac}$.

Cor.

Fig.   Cor. 1. *Hence $v =$ number belonging to the lo-*
24.
garithm log. $c - \dfrac{2az}{g}$. *And $f =$ number belonging to*

the logarithm log. $g - \dfrac{4az}{g}$.

Cor. 2. *Therefore the logarithms of v and f, each
of them severally decreases equally, in describing equal
spaces, ad infinitum. And therefore at every revolu-
tion, the log. of v is equally diminished, and likewise
that of f. But the body will revolve for ever, for
when v is 0, t will be infinite.*

Cor. 3. *Hence if the central body at* C, *was so
diminished that its log. may decrease equally in descri-
bing equal spaces, or in each revolution, after the man-
ner as before mentioned; then the body will perpetu-
ally revolve in a circle, in a medium of uniform den-
sity.*

SECT.
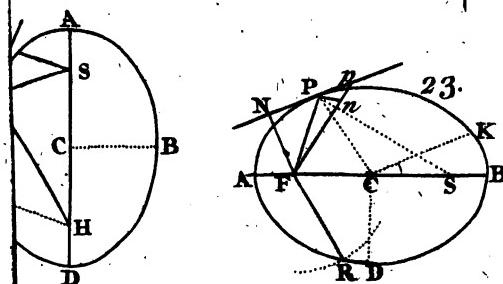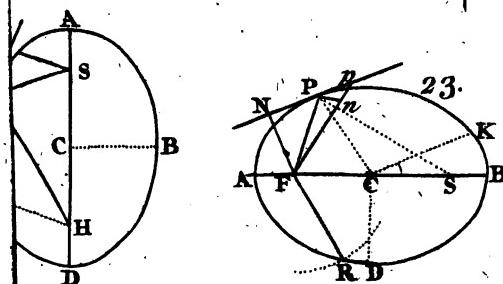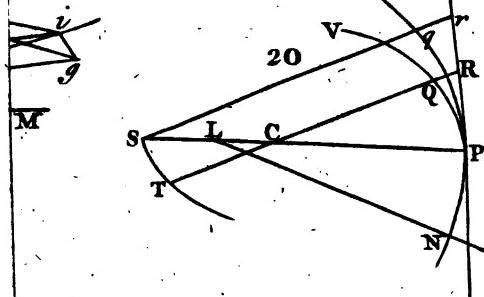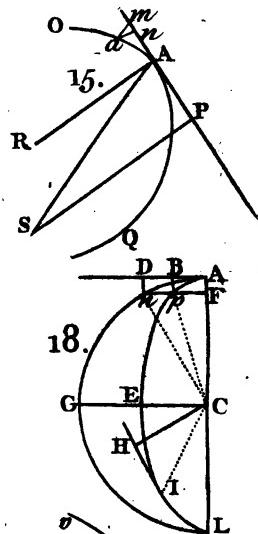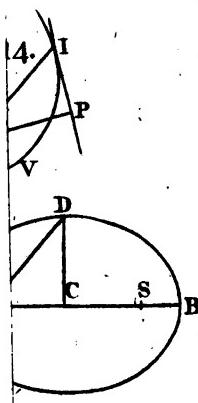
SECT. III.

The Motion of three Bodies acting upon one another; the perturbating Forces of a third Body. The Motion of Bodies round an Axis at rest, or having a progressive Motion, and other Things of the same Nature.

PROP. XXVI.

IF a body be projected from A, in a given direction AD, and be attracted to two fixed centers S, T, not in the same plane with AD; the revolving triangle SAT, drawn through the moving body, shall describe equal solids in equal times, about the line ST.

Divide the time into infinitely small equal parts; it is plain that equal right lines AB, BC, CD, &c. would be described in these equal times; and consequently that all the solid pyramids STAB, STBC, STCD, &c. are equal, which would be described in the same equal times; if the moving body was not acted on by the forces S and T.

But let the forces at S and T, act at the end of the several intervals of time; as suppose the force T to act at B in direction BT; so that the body, instead of being at C, is drawn from the line BC, in the direction CF, parallel to BT. And in like manner it is drawn from the line BC, by the force S, in direction CE parallel to SB. And therefore, by the joint forces, the body at the end of the time, must be somewhere in the plane ECF parallel to SBT,

Fig. SBT, as at I. But (Geom. VI. 17.) the folid
25 pyramids STBI and STBC, are equal; being con-
tained between the parallel planes ECF and SBT,
and therefore have equal heights; whence STBI =
pyramid STAB.

In like manner continue BI, making IK = BI;
and in the next part of time, the body would ar-
rive at K, defcribing the pyramid STIR equal to
STBI, But being drawn from tHe line IK, by the
forces S, T, in the directions KL, KN, parallel
to IS, IT; the body will be found at the end of
the time, fomewhere in the plane LKN parallel to
SIT, as fuppofe at O, and then it will have de-
fcribed the folid STIO = STIK = STBI = py-
ramid STAB.

And in the fame manner producing IO to P, till
OP = OI. Then the body, attracted from O, by
the forces S, T, will defcribe another equal pyra-
mid. And fo it will continue to defcribe equal py-
ramids in equal times; and confequently the whole
folids defcribed are proportional to the times of de-
fcription.

Cor. 1. *When the number of* lineolæ AB, BI, IO,
&c. is increafed, and their magnitude diminifhed, ad
infinitum; *the orbit* ABIO, *becomes a curve.*

Cor. 2. *Any line* AB *is a tangent at* A, BI *at* B,
&c. A, B, *&c. being any points in the orbit.*

Cor. 3. *But the orbit* ABIO *is not contained in
one plane, except in fome particular cafes.*
For that the orbit may not deviate from a plane;
the forces on both fides thereof, ought to be alike.

PROP.

PROP. XXVII.

If the body T *revolves in the orbit* TH, *about the* 26.
body S *at a great diftance,* whilft *a leffer body* P *re-*
volves about T *very near ;   and if* C *be the centripe-*
tal force of S *acting upon* T.   *Then the difturbing*
force of S *upon* P *is* $= \frac{3PK}{ST}C.$ *Suppofing* PK *pa-*

rallel, and KT *perp. to* ST.   *And* $\frac{PT}{ST}C =$ *the in-*

creafe of centripetal force from P *towards* T.

Let ST $= r$, PT $= a$, PK $= y$, $g =$ force of
gravity, $b =$ fpace defcended thereby in time 1,
$s =$ the fpace defcended in the time 1, by the force
C, $p =$ periodic time of T about S, and $t =$ per.
time of P about T, $\gamma =$ centripetal force of T at
P, $\pi = 3.1416$.
Since attraction is reciprocally as the fquare of
the diftance, then force of S acting at T : force of

S acting at P : $: \frac{1}{ST^2} : \frac{1}{SP^2} : : \frac{1}{rr} : \frac{1}{\overline{r-y}^2} : : r :$

$r + 2y$, nearly: And force of S acting at T : to
difference of the forces : : $r : 2y$; that is, $r : 2y : :$

C $: \frac{2y}{r}$C = difference of the forces ; and this is

the fingle force by which P is drawn from the or-
bit QAZ in direction KP or PS.
But fince the motion will be the fame, whether
the fingle force PS act in the direction PS; or the
two forces PT, TS act in the directions PT, TS;
fubftitute thefe two for that fingle one ; therefore
proceeding as before, the force of S acting at T :

force of S acting at P : $: \frac{1}{rr} : \frac{1}{\overline{r-y}^2}.$ And force

of

Fig. of S acting at P in direction PS : force of S act-
26. ing on P in direction of TS : : PS : TS : : $r-y$:

r : : $\dfrac{1}{r}$: $\dfrac{1}{r-y}$. Therefore *ex æquo*, force of S acting

at T : force of S acting on P in direction TS : :

$\dfrac{1}{r^3}$: $\dfrac{1}{\overline{r-y}^3}$: : $\dfrac{1}{r^3}$: $\dfrac{1}{r^3-3r^2y}$: : $\dfrac{1}{r}$: $\dfrac{1}{r-3y}$: : r :

$r+3y$, nearly. And the force at T : difference

of the forces : : r : $3y$; or r : $3y$: : C : $\dfrac{3y}{r}$ C = dif-

turbing force of P, acting in direction parallel to
TS. And PK (y) : PT (a) : : increase of the dif-

turbing force in direction PK $\left(\dfrac{y}{r}C\right)$: $\dfrac{a}{r}$C, the

addition of the centripetal force in direction PT.

For when the disturbing force was $\dfrac{2y}{r}$C, there was

no addition of centripetal force at T, but a dimi-
nution thereof; as appears by the following Corol.

Cor. 1. *The simple disturbing force, whereby* P *is*

drawn towards S, *is* = $\dfrac{2y}{r}$C. *And the diminution of*

centripetal force of P *towards* T, *is* = $\dfrac{v}{r}$C. *And the*

accelerating force at P *in the arch* PA, *is* = $\dfrac{z}{r}$ C.

Putting z = *sine of* 2PQ, v = *versed sine of* 2PQ.
For let x = PK, and draw KI perp. to PT;
then by similar triangles, PT (a) : PK (y) : :

PK : PI : : force PK $\left(\dfrac{2y}{r}C\right)$: force in direction

IP or TP = $\dfrac{2yy}{ar}$C = $\dfrac{v}{r}$C.

Also PT (a) : TK (x) · : PK : KI : : force PK
(2y

$\left(\frac{2y}{r}C\right)$: force in direction KI or PA $=\frac{2xy}{ar}C=$ Fig. 26.

$\frac{z}{r}C$. By Trigon. B. I. Prop. II. Schol.

Cor. 2. *The disturbing force at* P *is* $=\frac{q}{59\frac{1}{2}}\gamma$, q *being the sine of the distance from the quadrature,* P *the moon,* S *the sun.*

For (Prop. V.) $C=\frac{ttr}{ppa}\gamma$, and $\frac{3y}{r}C=\frac{3tty}{ppa}\gamma=$

$\frac{3q\gamma}{178\frac{1}{4}}$ (because $\frac{y}{a}=\frac{q}{1}$) $=\frac{q}{59\frac{1}{2}}\gamma$ nearly.

Cor. 3. *If* S *be the sun,* P *a body in the equinoctial of the earth; the disturbing force at* P *is* $=$ $\frac{qg}{12852000}$.

For when P is at the moon's orbit, the force is $\frac{q}{59\frac{1}{2}}\gamma$; but $g=60\times60\gamma$, or $\gamma=\frac{1}{3600}g$, therefore

the force becomes $\frac{qg}{59\frac{1}{2}\times3600}$, and at the earth is

$\frac{qg}{59\frac{1}{2}\times60^3}$.

Cor. 4. *If* S *be the moon,* P *a body on the equinoctial of the earth. The disturbing force at* P *is* $=$ $\frac{qg}{2880000}$.

For the general perturbating force was $\frac{3y}{r}C$, and here C must be the centripetal force at the moon. Now the centripetal force of the earth, at the distance of the moon is $\frac{1}{60^2}g$. And the moon being 40 times less than the earth, the centripetal force

of

of the moon, at the fame diftance, is $\dfrac{1}{40 \times 60^2}g$:

put this for C, then the force of the moon upon

the equinoctial, is $\dfrac{3y}{r} \times \dfrac{g}{40 \times 60^2} = \dfrac{3yg}{60a \times 40 \times 60^2}$

$= \dfrac{yg}{20 \times 40 \times 60^2}$

Cor. 5. *The difturbing force of the fun, to that of the moon, upon the equinoctial; is as* 1 *to* 4.46.

For thefe forces are as $\dfrac{1}{12852000}$ and $\dfrac{1}{2880000}$

or as 288 to 1285, or as 1 to 4.46.

Cor. 6. *If f be the apparent diameter, and d the denfity of the perturbating body. Then the difturbing force will always be as* df^3y.

For that force is $\dfrac{3y}{r}C$ or as $\dfrac{yC}{r}$. Let its diame-

ter $= b$, M $=$ its quantity of matter. Then C

is as $\dfrac{M}{rr}$; that is, as $\dfrac{db^3}{rr}$. Therefore the difturbing

force is as $\dfrac{db^3y}{r^3}$, or as $dy \times f^3$.

Cor. 7. *If* P *be a point in the equator of the earth,* S *the fun.*
The centrifugal force of P :
is to the perturbating force PT : :
As the fquare of the earth's periodical time about the fun pp :
to the fquare of the earth's periodical time about its axis tt.

Let $t =$ time of revolution of the earth round

its axis; then $t : 2\pi a$ (circumference) : : $1'' :$

$\dfrac{2\pi a}{t} =$ arch defcribed in one fecond ; and the verf-

ed

Fig. 26.

ed fine $= \frac{4\pi a a}{tt \times 2a} = \frac{2\pi\pi a}{tt} =$ afcent or defcent by the earth's centrifugal force.   But forces are as their effects, whence $h : g : : \frac{2\pi\pi a}{tt}$ (afcent) $: \frac{2\pi\pi ag}{ttb}$ the centrifugal force itfelf.   But the perturbating force is $\frac{a}{r}C = \frac{asg}{rb}$.   Whence the centrif. force :

perturbating force $: : \frac{2\pi\pi ag}{ttb} : \frac{asg}{rb} : : \frac{2\pi\pi}{tt} : \frac{s}{r} : :$

$2\pi\pi r : tts : : \frac{2\pi\pi r}{s} : tt.$ But $\frac{2\pi\pi r}{s} = pp.$ For $\sqrt{2rs} :$

$1'' : : 2\pi r : p = \frac{2\pi r}{\sqrt{2rs}},$ and $pp = \frac{4\pi\pi rr}{2rs} = \frac{2\pi\pi r}{s}$

Cor. 8. *Hence the body* P *is accelerated from the quadratures* Q, Z, *to the fiziges* A, B; *and retarded 'from the fiziges to the quadratures. And moves fafter, and defcribes a greater area, in the fiziges than in the quadratures.*

PROP. XXVIII.

The fame things fuppofed as in the laft Prop. the linear error generated in P *in any time, is as the difturbing force and fquare of the time. And the angular error, feen from* T, *will be as the force and fquare of the time directly, and the diftance* TP *reciprocally.*

For the motion generated in a given part of time, by any force, will be as that force; and in any other time as the force and the fquare of the time. The motion fo generated is the linear error of P, as it is carried out of its proper orbit, by the force $\frac{3y}{r}C.$ And that error, as feen from T, is as the

S   angle

Fig. angle it is feen under; and therefore is as that linear
26. error, divided by the diftance TP ; and therefore
is as the force and fquare of the time, divided by
the diftance.

Cor. 1. *The linear error generated in one revolu-
tion of* P, *is as the difturbing force and fquare of
the periodical time* $\frac{3ca}{r}tt$. *And the angular error in
one revolution is as the force and fquare of the perio-
dic time divided by the diftance.*

Cor. 2. *The mean linear error of* P *in any given
time, will be as the force and periodical time,* $\frac{a}{r}Ct$.
*And the mean angular one, as the force and periodical
time, divided by the diftance.*
For let the given time be 1 ; then t (time) :
$\frac{aC}{r}tt$ (whole error) : : 1 : $\frac{aCt}{r}$, the error in the
given time.

Cor. 3. *The mean lineal error in any given time, is
as* TP *and the periodical time of* P *directly, and the
fquare of the periodical time of* T *reciprocally. And
the mean angular error, as the periodical time of* P
directly, and the fquare of the periodical time of T
reciprocally.

For (Prop. V.) C is as $\frac{r}{pp}$, and $\frac{at}{r}$C is as $\frac{at}{r}$ ×
$\frac{r}{pp}$ or $\frac{at}{pp}$. And the angular error as $\frac{t}{pp}$.

Cor. 4. *In any given time, the lineal error is as*
TP *and the periodical time of* P *directly, and the
cube of* ST *reciprocally. And the angular error as
the periodical time of* P *directly and the cube of* ST
reciprocally.

For

For (Prop. XVII.) pp is as r^3, therefore $\dfrac{at}{pp}$ is as $\dfrac{at}{r^3}$. Fig. 26.

Cor. 5. *The linear error in a given time is as* $\dfrac{a^{\frac{5}{2}}}{r}C$, *and the angular error as* $\dfrac{a^{\frac{3}{2}}}{r}C$.

For t is as a^3, and $\dfrac{at}{r}C$ as $\dfrac{a^{\frac{5}{2}}}{r}C$.

Cor. 6. *And universally, the angular errors in the whole revolution of any satellites; are as the squares of the periodic times of the satellites directly, and the squares of the periodic times of their primary planets reciprocally. And the mean angular errors are as the periodical times of the satellites, divided by the squares of the periodic times of their primary planets.*

For by Cor. 1. the angular error is as the force and square of the time divided by the distance; that is, as $\dfrac{Ca}{r} \times \dfrac{tt}{a}$; that is, (because C is as $\dfrac{r}{pp}$) as $\dfrac{tt}{pp}$. The rest is proved in Cor 3.

PROP. XXIX.

If a spheriod **AB** *revolves about an axis* **ST** *in free space, which axis is in an oblique situation to the spheriod; the spheriod will, by the centrifugal force, be moved by degrees into a right position* ab; *and afterwards by its libration, into the oblique position* αβ *And then will return back into the positions* ab, **AB**; *and so vibrate for ever.*

Let C be the center of the spheroid; D the center of gravity of the end ICLB; E that of the end ICLA; Dd, Ee perp. to ST. Then the centrifugal force of the end CB, supposing it to act

wholly

Fig.
27.
wholly at D, in direction dD, having nothing to
oppose it, will move the end CB from B towards
b, with a force which is as Cd. And at the same
time, the centrifugal force of the end CA, acting
in direction eE, will move the end CA from A to-
wards a, conspiring with the motion of the end
CB ; by which means it will by degrees come into
the position ab. And then by the motion acquir-
ed, it will come into the position $\alpha\beta$, making
the angle SCα = SCB. And the motion being
then destroyed, it will return back, by the like
centrifugal forces, acting the contrary way ; and
be brought again into the positions ab, and AB ;
and continue to vibrate thus perpetually.

Cor. *Hence if the axis of the earth is not precise-
ly the same as the axis of its diurnal rotation; the
earth will have such a libration as is here described,
but exceeding small. This is supposing it a solid bo-
dy; but if it was a fluid, it would by the centrifu-
gal force, form itself into an oblate spheroid.*

PROP. XXX. *Prob.*

28. *If a globe* APBQ *in free space, revolve about
the axis* SCT, *in direction* ADB; *and if any force
applied at* V, *the end of the radius* CV, *acts by a
single impulse in direction* VG *perp. to* CV, *in the
plane* VCD. *To find the axis about which the globe
shall afterwards revolve.*

Suppose the great circle VBQA perp. to the
line of direction VG ; and if VH, VI be 90 de-
grees ; it is plain, if the first motion was to cease;
the globe by the impulse at V, would revolve round
the axis IH, which by the first motion was round
the axis PQ. Therefore by both motions toge-
ther it will move round neither of them. Now
, since

Fig.
28.

since a point of the surface moving with the great-
est velocity about ST, will move along the great
circle ADB ; and a point having the greatest velo-
city about IH, moves along the great circle VDE.
Therefore a point that will have the greatest velo-
city, by the compound motion, will also be in a
great circle passing through D. Therefore in the
great circles DB, DE take two very small lines
Dr, Do, in the same ratio as the velocities in AD,
and VD ; and compleat the parallelogram D*qpr*.
Then through D and *p* draw a great circle KD*p*L ;
and a point having the greatest motion, arising from
a composition of the other two motions, will move
along KD*p*L.   Therefore finding F, R, the poles
of the circle KD*p*L, FR will be the new axis of
revolution, or the axis sought.   And the velocity
about the axis FR will be proportional to Dp ;
VBQA being always supposed perp. to GV, or to
the plane DVC.

Note, if you suppose an equal force applied at
E, in direction contrary to GV, it will by that
means keep the center C of the globe unmoved,
and will likewise generate twice the motion in the
globe.

Cor. 1. *The greater the force is that is applied to*
V, *the greater the distance* PF *is, to which the pole
is removed.   And if several impulses be made succes-
sively at* V, *when* V *is in the circle* APB, *the pole* F
will be moved further and further towards H, *in the
circle* APB.

For several small forces or impulses have the
same effect as a single one equal to them all.

Cor. 2. *If the force act at* P, *in direction perp. to*
the plane CPB ; *and* Dr, Do *be as the velocities along*
DB *and* DQ.   *The great circle* KDL (*passing through*
D *and p*) *is the path of the point* D ; *and its pole* F,
or axis of revolution RF ; *the pole being translated*

Fig. *from* P *to* F. *And if the impulfe be exceeding fmall,*
29. PF *will alfo be exceeding fmall.*

Cor. 3. *If the force at* P *always aƐts in parallel*
direƐtions, whilft the globe turns round. The pole
will make a revolution in a fmall circle upon the fur-
face of the globe, in the time of the globe's rotation,
and the contrary way to the globe's motion.

For let a fingle impulfe at P tranflate the pole to
F; and afterwards when the globe has made half a
revolution, and the point P is come to *p*; then if a
new impulfe be made at F, the pole will be tranf-
lated to *p* which is now P; that is, it will be mov-
ed back to its firft place on the globe. So that
in any two oppofite points of rotation, the place of
the pole is moved contrary ways, and fo is carried
back again the fame diftance. And fince the globe
revolves uniformly, if the force aƐt uniformly, it
will move the pole all manner of ways, or in all
manner of direƐtions upon its furface; that is, it
will defcribe a circle, which will end where it be-
gun. And in defcribing this polar circle, the mo-
tion will be contrary to the motion of the globe;
for fuppofe PFB an immovable plane. If the globe
ftood ftill, the pole would move in a great circle,
in the plane PFB. But fince all the points of the
globe which come fucceffively to the plane PB, are
not yet arrived at it, but are fo much further fhort
of it, as PF is greater; it is evident all thefe points
will lie on this fide the plane PB. And as any
fixed point will defcribe a circle on the moving
globe, contrary to the motion of the globe; fo will
a point that is not fixed, but moving in the plane
PFB, likewife defcribe a circle (or fome curve)
contrary to the motion of the globe. Or fhorter
thus, fuppofe the globe to ftand ftill, and the di-
reƐtion of the force to move backward; then the
relative motion will be the fame as before; and
then

Fig.
29.
then the pole F will move backward too, as it will
follow the force, being at right angles to it.

Cor. 4. *Since the pole by one impulse is translated
to* F; *the new pole* F *is therefore another point of
the material globe, distinct from* P. *And the particle*
P *that was before at rest, will now revolve about the
particle* F *at rest.*

For the new pole F is that particle of the globe
which happened to be revolving in F, when the
impulse was made at P.   The matter of the great
circle ADB does not come into the circle KDL,
but only the point D of it.   For when the force is
impressed, the other particles M, N, by the com-
pound motion, will be made to revolve in the di-
rections M*m*, N*n*, parallel to D*p*; and therefore
will describe lesser circles about F; whilst only D
describes the great circle KDL.

Cor. 5. *What is demostrated of a sphere is true
also of an oblate spheroid, whose axis is* PQ; *and
the force impressed at* P, *acting in direction perp. to*
CPB, *or parallel to* CD *the radius.*

Cor. 6. *But if the force at* P *act in direction con-
trary to the foregoing (as in case of an oblong sphe-
roid); the pole of rotation will be moved from* P *to-
wards* A, *contrary to the way of the other motion.*

Cor. 7. *And in general the pole* P *will always be
moved in a direction perp. to that of the power; and
towards the same way as the spheroid revolves.*

Cor. 8. *Hence after every half rotation of the
globe round its axis, the places upon the globe change
their latitude a little; which, after an entire rotation
et urn to the same quantity. But this variation is so
rifling, as to come under no observation.*

This is evident, because the pole is-altered; and

Fig.
29.

PROP. XXXI. *Prob.*

30.   *Let* AB *be an oblate ſpheroid, whoſe axis is* PC; *and let it revolve round that axis, in the order* ADB, *which is its equinoctial; and if any force act at* P, *in direction* PG *perp. to* PC, *and in the plane* P⊙C, *which moves ſlowly about, in the order* ADB. *To find the motion generated in the ſpheroid.*

Let EL be an immovable plane like the ecliptic, in which the center C of the ſpheroid always remains. ON another plane parallel to it. Erect CM perp. to theſe planes ; and make the angle MCN = the angle B≏C, in which the equinoctial B≏ cuts the ecliptic CL. Suppoſe the ſpherical ſurface OVN to be drawn, whoſe pole is M ; and produce CP to cut it at R, in the circle ORN. Then PCD and RCM are in one plane, and both of them perp. to P≏C and P♈C. Now to find the motion of the axis of the ſpheroid. Here OVN is the upper ſide of the ſurface.

This Prop. differs from the laſt in ſeveral reſpects. The laſt Prop. regards only the motion of the pole upon the ſurface of the globe, and that is cauſed by a motion which is generated in the globe itſelf. But in this Prop. we conſider the motion of the axis CR in the fixed ſpherical ſurface OVN ; which always proceeds in one direction, as long as the moving force keeps its poſition. In the laſt Prop. the motion of the globe round its axis is performed in a very ſhort time ; but here the revolution of the force, in the moving plane P⊙C, is a long time in its period.

Now by the laſt Prop. Cor. 7. the force PG will always move the axis of rotation CPR in a direc-
tion
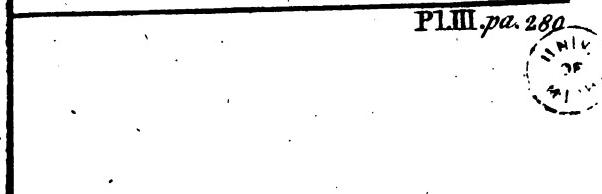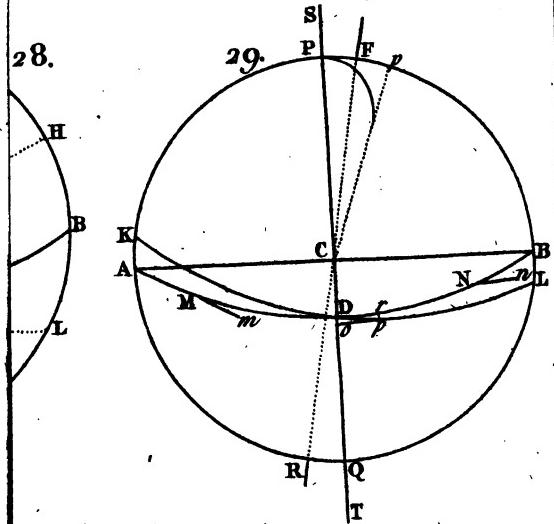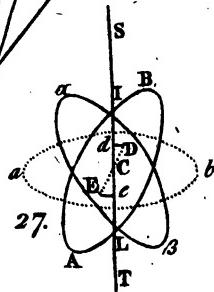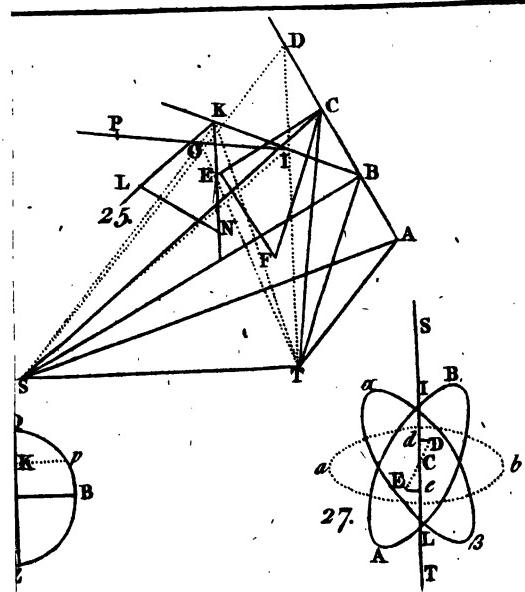
tion perp. to PG. Therefore fuppofe the plane Fig.
P⊙C to revolve flowly round PC, and we fhall 30.
find that in the beginning of the motion, when ⊙
or F is at ♎, then the plane (P⊙C or) P♎C will
be perp. to the plane CRM, and at that time the
motion of R will be directed from R towards M.
And when that plane comes to the pofition PF⊙C,
the motion of R will be directed to fome place be-
tween N and V. And when it is got to the tropic
D, then R's motion is directed along the circum-
ference RV; for then P⊙C coincides with CRM,
But when P⊙C arrives at f, the motion will be
directed from R to fome point t without the circle.
And laftly, when P⊙C is at the other interfection
♈, beyond B; the motion of R will be directed to
m oppofite to M. The refult of all which is, that
the pole R will defcribe fuch a curve as R1234;
and then the fame force begins again at ♎; which
being repeated, another fimilar figure 456 is de-
fcribed by R; and fo on for more. The fame
force I fay is repeated, for when the plane P⊙C
comes on the other fide of the globe, the force
acts the contrary way, and therefore it is all one as
if it acted on the firft fide of the globe.

It muft be obferved, that as R moves through 1234,
the interfection ♎ gradually moves towards E. And
as to the force PG, it may be fuppofed variable,
at different pofitions of the plane P⊙C. And
according to the quantity of force in the feveral
places, different curves (1234) will be defcribed.

Cor. 1. *Hence it is evident, that the inclination of
the planes* ADB *and* ECL, *is greateft when* P⊙C
paffes through ♎ *and* ♈. *And leaft when it paffes
through the tropic* D. *And that the inclination de-
creafes from the node* ♎ *to the tropic* D, *and increafes
from the tropic to the node.*

For

Fig.   For when P⊙C is in ♎, R is fartheſt from M;
30   but when it is in D, R is at 2, and its direction
is parallel to R4, and then 2M is leaſt.

Cor. 2.   *After a revolution of the plane* PF⊙C
*(in which the force always acts), the inclination comes
to the very ſame it was at firſt.*

For at any two points F, *f*, equidiſtant on each
ſide from the tropic; the force is directed contrary
ways, from and to the circle RV; and therefore
the motion, in the curve R1234, being alſo equal
and contrary, from and towards RV, they mutu-
ally deſtroy one another; and therefore after a re-
volution, or rather half a revolution, the pole R
is brought back to the circle RV, and then the
angle RCM is the ſame as it was at firſt.

Cor. 3.   *The motion of the pole* R, *reckoned in the
circle* OVN, *is always from* R *towards* V, *then through*
N, O, *and* R.

For though the motion of R towards and from M,
in the line M*m*, in one revolution, is equal both
ways; and ſo R is always brought to the circle
again; yet-the motion conſidered along the circle
is always in the order RVN. Thus it goes through
the curve 123 to 4, ſo that after half a revolution
of P⊙C, it is advanced forward in the circle RV,
the length R4.

Cor. 4.   *The motion along the circle is ſometimes
faſter and ſometimes ſlower.   At* 2 *it moves faſteſt
of all; at* R *and* 4, *it moves ſloweſt,* or rather *is
ſtationary for a moment.*

Cor. 5.   *The pole* R, *and the nodes move the con-
trary way about, to what* AB *revolves.*

Cor. 6.   *If the force* PG *ſtands ſtill, the pole* R
will ſtill move backwwards as before; and that in a
right

right line, or rather, a great circle. And if PG moves
backward through BDFA; *the pole* R *will still go back-*
ward; but then the curve R24, *will be concave to-*
wards M, *like* 87R, *being contrary to the other*
where the force moved forward.

Cor. 7. *But in an oblong spheroid, where the force*
acts in direction GP, *quite contrary to the other;*
R *will describe the curve* R78, *without the circle*
ORV; *every particle of it in a contrary direction to*
these of R24. *And therefore the pole* R, *and the*
nodes ♈ *and* ♎ *will move the same way about as* ADB
revolves, and contrary to what they do in an oblate
spheroid.
  For the force being directed the contrary way;
of consequence the motion must be so too.

Cor. 8. *And in an oblong spheroid, if the force* GP
move the contrary way about; yet the pole R *will*
still move forward.   And the curve described by R,
will have its convexity the contrary way.

Cor. 9. *Hence if the quantities and proportions of*
these forces, in different places be known; it will not
be difficult to delineate the curve R1234, *upon the*
spherical surface OVNM.

PROP. XXXII. *Prob.*

If a planet (or the moon) move in the orbit ATE*t.*
round an immovable center C, *whose plane is inclined*
to the plane of the ecliptic AQE; *and a force acts*
upon it in lines perp. to GZ, *and parallel to the eclip-*
tic, directed always from the plane GZ *to either side.*
To find the motion of the nodes A, E; *and the varia-*
tion of the orbit's inclination PAO.

  Let ATZE be half the orbit raised above the
ecliptic AQ, AE the line of the nodes; T, *t*,
                                                    the

Fig. the tropics. Draw CM perp. to the ecliptic AQE,
31. and CR perp. to the orbit ATE. Round M as
a pole defcribe a fpherical furface RVNX; then
RC will be the axis of the orbit, and MC of the
ecliptic, or circle AQE. Through the points T, C,
M, draw the plane RMC; and through A, C, M,
another plane cutting the circle RNF in X and H;
then RF is perp. XH, and the circle RNX paral-
lel to GEQ. Note, RNX is the upper fide of the
furface.

Let the planet be at P, and let P2 be the fpace
it defcribes in any fmall time, and the line P1 the
fpace it would be drawn through in the fame time, by
the force acting from the plane MGZ. Compleat the
parallelogram P123, and P3 will be its direction
by the compound force. Now as the line P1 is
parallel to the ecliptic, it is plain the point 3 will be
below the plane of the orbit; and the plane CP2
will be moved into the pofition CP3, revolving
about CP; confequently the axis RC will be mov-
ed in a direction perp. to CP. And the pole R
will be moved to fome point between F and H.
This being duly attended to, the motion of the
pole R will be known for all the places of P in
the orbit GATZ. For about G the motion of R
is directed perp. to Mn; at A it moves perp. to
MX, or in direction RM. At T it moves parallel
to MH, or in the curve RV. Approaching to Z,
it moves perp. from Mn. So that in the paffage
of the planet P, from G to Z, through GAPTZ,
the pole R of its orbit, moves through the curve
R1234. But in the other half of the orbit ZEtG,
as the force is directed the contrary way from the
plane GZMN, the pole R will return back at 4,
and defcribe a fimilar curve 45678. So that when
the planet P has made one revolution, the pole of
its orbit R will be found at 8. But in this pofition
of the nodes, the point 8 will be within the fphe-

rical

rical furface RHNX, not reaching the periphery Fig.
RV. ·For the feveral parts of the curve be- 31.
ing defcribed in all directions in refpect of the
line *n*N; the points R, 4, will be equidiftant from
*n*N; and likewife 4, 8, for the fame reafon.

Now fuppofe'the force and the plane GZN*n* to
revolve about the axis CM in the order GATE.
Then after it is come to fuch a pofition, that the
afcending node A is as far on the other fide of G,
fuppofe at *a*; then the pole R will be as far on the
other fide of V, fuppofe at *r*; and being alfo as far
from N*n*, on the fame fide, the curves (12468)
will approach VH there, by the fame degrees as
they receded from it at RV. And therefore the
pole R will by degrees be brought to the circle
again. Thus in every two correfpondent points on
each fide V, the forces and their effects balance
one another, and R will be at the fame diftance
from the circle RVH And therefore after half a
revolution of the plane GZ to the nodes, the angle
RCM, and confequently the inclination of the or-
bit, comes to the fame as at firft. And likewife as
the pole R moves forward or backward in the
circle RVH, the motion of the nodes A, E, will
be forward or backward.

Cor. 1. *In this pofition of the nodes at* A *and* E,
the inclination of the orbit ATE *will be diminifhed
every revolution of* P. *But on the oppofite fide at* a,
the inclination increafes every revolution of P.

For the points 4, 8, come nearer and nearer to
M, and the contrary at *r*.

Cor. 2. *When the nodes are at* A, E; *the inclina-
tion decreafes; when the planet is in* GT *or* Z*t*; *and
increafes in* TZ *and* *t*G.

For R moves to 3, whilft P moves through GT.
At 3 it is at its neareft diftance to M ; from 3 to

Fig. 4, R recedes from M, whilft P moves through TZ.
31. And the like on the other half of the orbit.

Cor. 3. *When the planet is in* GA *and* ZE, *the
nodes go forward.* But *in* AZ *and* EG, *they go
backward.*

For whilft R paffes through R1, its motion is
forward, viz. from R toward *n*, and at 1 where it
moves parallel to RM, it is ftationary; that is,
when P is in A. Through 1234, R moves back-
ward, or towards V; and then P is in AZ.

Cor. 4. *In general, the nodes are always regreffive,
except when P is between a node, and its neareft qua-
drature; and then they are progreffive, wherever the
nodes are fituated.*

Cor. 5. *The nodes go fafteft back when the planet,
is in* T *and* t.
For then R is at 3 and 7.

Cor. 6. *The inclination varies moft, when* P *is at*
A *and* E.
For then R is at 1 and 5.

Cor. 7. *And from the various fituations of the
nodes, and the place of* P, *it may eafily be determined,
when the inclination increafes or decreafes, in any cafe.*

Cor. 8. *Hence if the quantities of thefe forces were
known, it would not be difficult to delineate the motion
of the pole* R, *upon the fpherical furface* RXFH;
*and at any time to find the inclination, and place of
the node.*

Cor. 9. *And to find the nature of the curve*
R1234 *defcribed by the pole* R. *Snppofing the force
directed always to the fun; and to be as the diftance
of* P *from the plane* GZ.

32. Ler RDB be the curve, and let the tangent *t*DT
revolve about the curve RDB, begining at R, fo
as

as the end T may move uniformly through all the Fig.
points of the compass, in the same manner as P 32.
moves through its orbit ATZE*t*. It is plain this
is one property of the curve RDT.

Now since the sun's rays fall at the same obli-
quity upon all parts of the plane ATQ, therefore
the force to draw P in a direction parallel to these
rays, being the same at equal distances from the
plane GZ, and always as the distance; therefore by
the resolution of motion, the distance that P is
drawn perpendicularly from the plane of its orbit,
will also be as that distance; and that is as the va-
riation of the orbit's inclination. Therefore if P,
instead of moving to 2 move to 3, then the force
at P or PB (fig. 31.) will be as the angle 2P3: suppo-
sing the sun's distance from the node to remain the 33.
same, during one revolution of P.

But when the sun or the force alters its position, 34.
it will be greater or less on that account, in pro-
portion to the sine of OL (where OL is perp. to
AL), and that is as the sine of AO, the distance
from the node, the angle A being given. From
hence it follows that universally, the force acting
on P will be always as BP × S.AO; that is, as
S.GP × S.AO (fig. 31.); that is, as the sine of
the distance of P from the quadratures, and the
sine of the distance of the sun at O from the node.

Now let us find the nature of the curve R1234,
supposing AO to remain the sane for one revolu- 32.
tion of P. Put RA = *x*, AD = *y*, RD = *z*.
Since by the generation of the curve, the angle
AD*t* = arch GP, and the force is as the sine of

GP or of AD*t*, and $\frac{x}{z}$ = S.AD*t*. Also it is plain,

the increment of the curve at D is as that force;

therefore $\frac{\dot{x}}{z}$ is as $\dot{z}$. And since in passing through the

particle

Fig. particle of the curve z, the line Dt is fuppofed to
32. change its direction uniformly, therefore the angle

of contact is given; whence $\dot{z}$ or $\dfrac{\dot{x}}{z}$ is as the ra-

dius of curvature, or as $\dfrac{\ddot{z}\dot{y}}{\dot{x}}$; that is, $\dfrac{\ddot{x}}{\dot{z}}$ is as $\dfrac{\ddot{z}\dot{y}}{\dot{x}}$, or

$a\dot{x}\ddot{x} = \dot{z}^2\dot{y}$ ($\dot{z}$ being given), and the fluent is

$\dfrac{a\dot{x}^2}{2} = \dot{y}\dot{z}^2$, and $\dot{x} : \dot{z} :: \sqrt{y} : \sqrt{\frac{1}{4}a} ::$ as the fub-

tangent : to the tangent; which is the property of
the cycloid, $\frac{1}{4}a$ being $=$ CB, the diameter of the
generating circle.

Now at different diftances of O from the node,
the cycloid defcribed wtll be greater in proportion
to the fine of AO (fig. 31.); and even in the fame
cycloid, the latter part will be greater than the
former part, as AO grows greater; all the parts
of it increafing as the fine of AO increafes; and
the greateft cycloid will be when A is in the qua-
dratures; and the leaft when in the fyziges, where
it is reduced to nothing.

<div align="center">SCHOLIUM.</div>

From the foregoing folution, thefe obfervations
may be made.

1. Though the curve R24 has been determined
to be a cycloid, yet it is nearer an epicycloid. For
at R it fets off nearly in a direction perp. to GZ,
and during its generation (that is, whilft P per-
forms a femirevolution) the point A moves to-
wards G; and fuppofing the force at O to be fix-
ed, the laft particle of the curve at 4 would be
parallel to that at R. But as O really moves for-
ward, fome number of degrees, fuppofe 14, and
continues to do fo, all the femirevolution; there-
fore every particle of the curve will have other di-
rections

rections in its defcription, being more curve than Fig.
before; and at laft the tangents at R and 4, will 34.
make an angle of 14 deg. which is the fame as if
an epicycloid was defcribed on the convex fide of
a circle, going through an arch of it equal to 14
degrees.

2. Thus the curve defcribed by R would be
nearly an epicycloid, when the force at O is every
where of the fame quantity; yet as O moves about,
the force will increafe and decreafe in proportion to
the fine of AO; therefore, if you will fuppofe
fuch an epicycloid defcribed as above mentioned,
and moreover imagine the radius of the generating
circle to fwell or increafe, in the fame ratio as the
S.AO increafes; then fuch an epicycloid will near-
ly reprefent the curve defcribed by R. For then
every part of it will be greater or lefs, in propor-
tion to the force that generates it. But enough of
this. All that I fhall add on this head is the folu-
tion of the two following problems, upon account
of their curiofity, as depending on the foregoing
principles.

PROP. XXXIII. *Prob.*

*To find the difturbing force of Jupiter or Saturn,
upon the earth in its orbit; having that of the fun
upon the moon given.*

Let the matter in the fun and Jupiter be as m to 26.
1. E, I, L, the periodic times of the earth, Ju-
piter and the moon. A, B, the diftances of the
earth and Jupiter from the fun. D the moon's dif-
tance from the earth. C, c, the centripetal forces
of the fun and Jupiter.

Then (by Prop. XXVII.) the difturbing force of

S upon P, is $\frac{3PK}{ST}C$ or as $\frac{PT}{ST}C$. Therefore if S be
the fun, and P the moon, the difturbing force

<div align="center">T</div> is

Fig. 26. is $\frac{D}{A}C$; but if S be Jupiter, P the earth, T the

fun; then the force is $\frac{A}{B}c$. That is, the fun's dif-

turbing force upon the moon is to Jupiter's dif-

turbing force upon the earth; as $\frac{D}{A}C$ to $\frac{A}{B}c$; or

as DBC to A^2c. But (Cor. 2. VII.) $C=\frac{m}{AA}$, and

$c=\frac{1}{BB}$. Therefore the fun's force upon the moon,

is to Jupiter's upon the earth; as $\frac{DBm}{AA}$ to $\frac{AA}{BB}$, or

as DB^3m to A^4; that is (Prop. XVII.), as DI^2m :

AE^2. That is, the fun's difturbing force upon the

moon, is to Jupiter's difturbing force upon the

earth; as $D \times II \times m$, to $A \times EE$. But that of

the moon is known, and confequently that of Ju-

piter. And if for I and m, we put Saturn's peri-

odic time, and quantity of matter; Saturn's dif-

turbing force will be known.

Cor. 1. *The angular errors generated in the moon*
by the fun, are to the errors generated in the earth
by Jupiter in the fame time, : : *as* IIL $\times m$, *to* E^3.

For (Prop. XXVIII. Cor. 2.) thefe errors are as

the forces and periodic times, divided by the dif-

tances. Therefore the fun's effect to Jupiter's, is as

$\frac{D \times II \times m \times L}{D}$ to $\frac{A \times E^2 \times E}{A}$; or as IIL$m$ to E^3.

Cor. 2. *Hence the error generated in the moon by*
the fun, is to the error generated in the earth by Ju-
piter; as 11230 *to* 1, *and to that generated by Sa-*
turn, as 196076 *to* 1.

For put $I=4332\frac{1}{4}$ days, $L=27\frac{1}{3}$, $m=1067$,

$E=365\frac{1}{4}$; then $\frac{I^2Lm}{E^3}=11230$. And putting

$$I=$$

31.

32.

34.

Pl. IV. *pa 290*
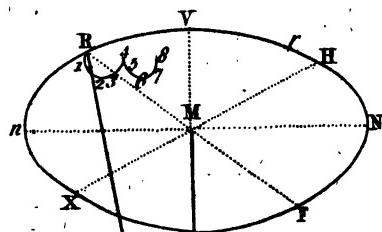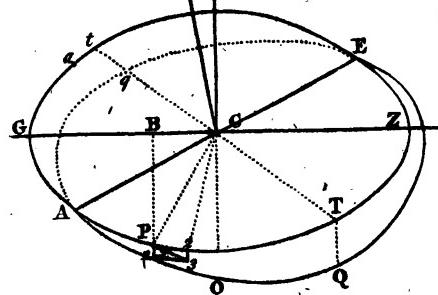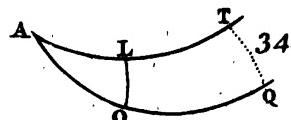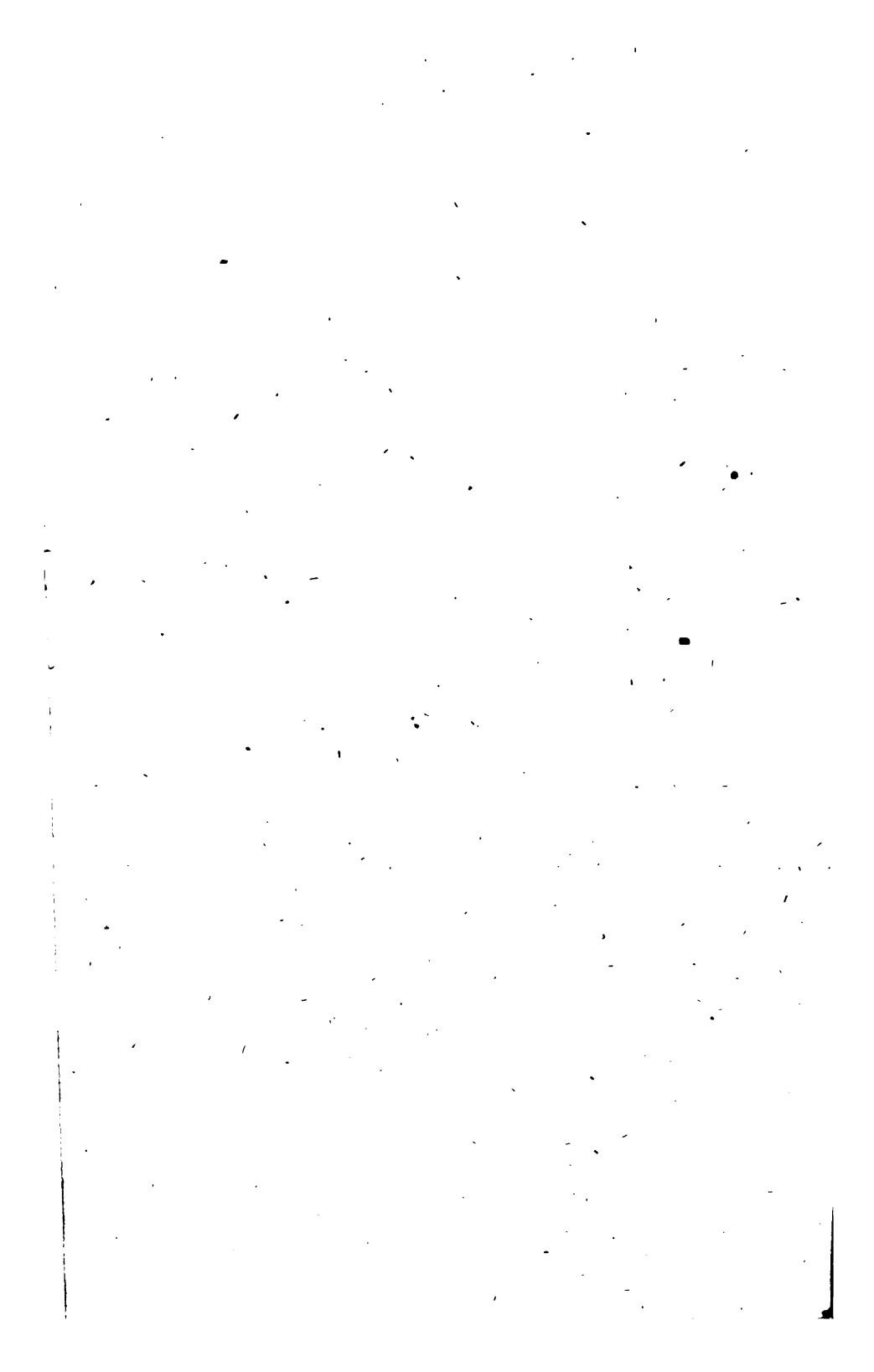
$I = 10759\frac{1}{4}$, and $m = 3021$, for Saturn; then Fig.
$\dfrac{I^2 L m}{E^3} = 196076$.   26.

Cor. 3. *The force of Saturn to the force of Jupiter to disturb the earth, is as* 1 *to* $17\frac{1}{2}$.

Cor. 4. *The motion of the nodes of the earth's orbit by Jupiter's action, in* 100 *years, is* $10' \; 20''\frac{1}{2}$. *And by Saturn's,* $35''\frac{2}{3}$.
For the motion of the moon's nodes in a year is $19° \; 20' \; 32''$, or $69632''$; this divided by 11230 gives $6''.2005$, multiplied by 100, is $620''.05$, which increased in the ratio of the cosine of inclination of Jupiter's orbit $(1° \; 19' \; 10'')$, to that of the moon's $(5° \; 8'\frac{1}{2})$, produces $10' 22''\frac{1}{4}$. Which diminished in the ratio of 1 to $17\frac{1}{2}$, gives $35''\frac{2}{3}$ for Saturn.

Cor. 5. *The motion of the earth's aphelion by the action of Jupiter, is* $21' \; 44''$ *in* 100 *years, in consequentia.   And by Saturn,* $1' \; 14''\frac{1}{2}$.
For the motion of the moon's apogee is $40° \; 40' \; 43''$, or $146443''$ in a year. This divided by 11230 gives $13.04''$; which multiplied by 100, gives $1304''$ or $21' 44''$. And divided by $17\frac{1}{2}$, gives $74''\frac{1}{2}$.

PROP. XXXIV. *Prob.*

To find the variation of inclination of the earth's 35. *orbit, by the action of Jupiter in* 100 *years; and the like for Saturn.*

Let ♈ ♋ ♎ ♑ be the ecliptic, or plane of the earth's orbit; GFH the orbit of Jupiter; G Jupiter's ascending node; E, I, Q, the poles of the ecliptic, Jupiter's orbit, and the equator, respectively; ECK a circle parallel to GF; and DmQ a circle parallel to the ecliptic. The pole Q here moves regularly along the circle Q/D, by the pre-

cession

Fig.
35.

ceffion of the equinoxes; which circle is no way affected or altered by Jupiter's action; becaufe Jupiter cannot be fuppofed to have any force to move the equinoctial points, or alter their regular motion. But he has a power of acting upon the whole body of the earth, and altering its orbit, and confequently the pole E of the ecliptic; which pole is therefore made to move along the circle ECK. Therefore we muft fuppofe the orbit of Jupiter fixed, and confequently the pole I, and circle ECK. And now we have to compute the motion of E along the circle ECK.

The preceffion of the equinoxes in
100 years is — — $1° \ 23' \ 20''$
And (by the laft prob.) the motion of
Jupiter's nodes in 100 years is $10' \ 22''\frac{1}{2}$ or $622''\frac{1}{2}$
Jupiter's afcending node G (1755,
angle QEG) -- —   ♋ $8° \ 20'$
Inclination of Jupiter's orbit — $1° \ 19' \ 10''$.
Therefore make the angle QEa = $1° \ 23'\frac{1}{3}$, and EIC = $10' \ 22''\frac{1}{2}$. Upon Ea let fall the perp. Co; then Eo is the decreafe of EQ or Ea, which (Prop. XXXII.) is the fame as the decreafe of the inclination of the planes of the ecliptic and equinoctial.

In the triangle EIC, by reafon of the very fmall angle EIC, we fhall have as rad : S.IC $(1° \ 19' \ 10'')$: : angle EIC $(622''\frac{1}{2})$: EC $=\dfrac{\text{EIC} \times \text{S.IC}}{\text{rad}}=$
$14''.3$. To the angle GEQ $(8° \ 20')$, add QEa ($1° \ 23'\frac{1}{3}$), then aEG or oEC = $9° \ 43'\frac{1}{3}$. Then in the very fmall right lined triangle ECo, rad: EC : : cof. oEC $(9° \ 43'\frac{1}{3})$: Eo $=\dfrac{\text{EC} \times \text{cof.}o\text{EC}}{\text{rad}}= 14''.1.$

Or Eo $=\dfrac{\text{EIC} \times \text{S.IC} \times \text{cof.}o\text{EC}}{\text{rad}^2}$, the decreafe of
inclination of the equinoctial in 100 years by the
action

Fig.
35.

action of Jupiter. And this decrease will amount
to a minute in 425 years.

If the same computation was applied to Saturn,
putting EIC = 35″ ⅔, IC = 2° 30′ 10″, oEC =
(21° 21′ 36″ + 1° 23′ ⅓) 22° 44′ 56″; the decrease
by Saturn will be 1″.44. Therefore the decrease
by both will be 15″.54; which will be a minute in
386 years.

Cor. 1. *The inclination will decrease till* E *and a
be at their nearest distance in the two circles, which
will be above* 6000 *years; and then it will increase
again. It has likewise been decreasing for above* 8000
years.

For the diameters of the circles EK and DQ,
being about as 1 to 17. And the angle IEQ being
81° 40′, and the difference of the motions of E
and Q being 1° 13′; it will decrease nearly as ma-
ny centuries as is the quotient of 81° 40′ divided
by 1° 13′, which is 67. Also the supplement 98° 20′
divided by 1° 13′, gives 81 centuries, it has been
decreasing.

Cor. 2. *But the increase or decrease for every cen-
tury is not* 15″.54, *as determined in this particular si-
tuation. For as it approaches to its maximum or mini-
mum, it varies very slow, and at these places is at a
stand for a long time.*

Cor. 3. *The inclination can never be less than* 20°
50′ 54″; *nor greater than* 26° 7′ 20″.

For the nearest and greatest distances of the two
circles EK, DQ amount but to these. And there
must be many revolutions, before they can light
upon these two points, if the world can be sup-
posed to exist so long.

SCHOLIUM.

The disturbing forces of Jupiter and Saturn here
made use of are derived from that of the sun up-

on

Fig.
35.

on the moon; but thefe forces are really more than are here determined. For in calculating the difturbing force of the fun (by Prop. XXVII.), the forces upon T and p are as rr to $rr + 2ry + yy$; but by reafon of the great diftance of the fun, the part yy is left out, as being extremely fmall. But in the cafe of Jupiter and Saturn it is otherwife, and therefore yy muft be taken in. Whence the difturbing force muft have an additional increafe, which is as $2ry$ to yy, or as $2r$ to y, which in Jupiter is as 10 to 1, and in Saturn as 19 to 1. Therefore Jupiter's difturbing force muft be increafed by $\frac{1}{10}$th, and Saturn's by $\frac{1}{19}$th; which being done, their effects will be proportional, and the decreafe of inclination of the ecliptic in 100 years, by Jupiter and Saturn will become 15″.45, and 1″.51; and by both 16″.96 or 17″ nearly; which will amount to a minute in 353 years.

But if the obfervations of the antients can be depended on, the obliquity decreafes fafter than this. For by the obfervations of *Ariftarchus*, *Eratofthenes*, *Hyparchus*, *Ptolemy*, and *Theon*, the obliquity was found to be 23° 51′. And none of them lived 300 years before Chrift, and two of them after. So that in little more than 2000 years, there is a difference of 22′; which is more than a minute in 100 years, and is more than three times as faft as we have here determined it.

I fhall now proceed to fome things of another kind, relating to centripetal forces; which as far as I can find, have not been meddled with by any body before.

PROP. XXXV. *Prob.*

36.   *If the circle* GDFE *be moved along the right line* AB, *whilft it turns round its axis; to find its motion upon a horizontal plane.*

Suppofe the circle inlined in any given angle to the horizon, the line of direction AB being at firft

in
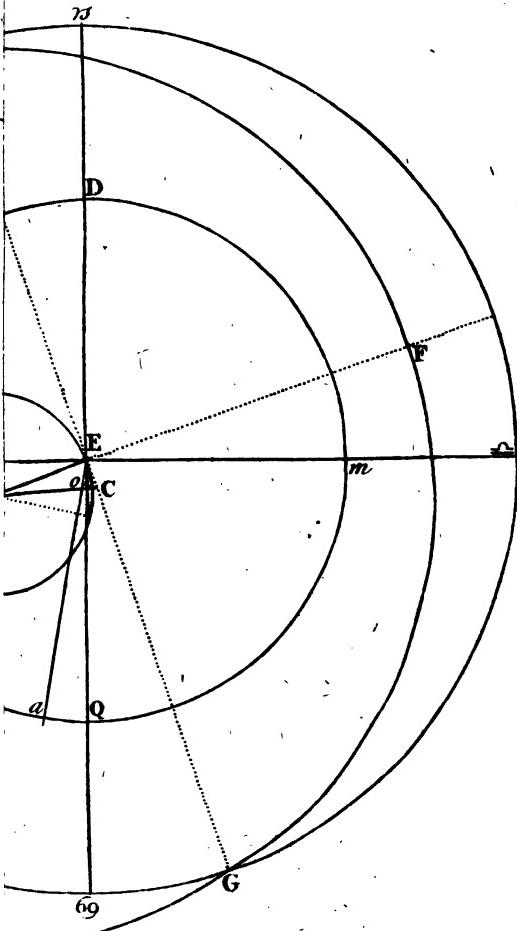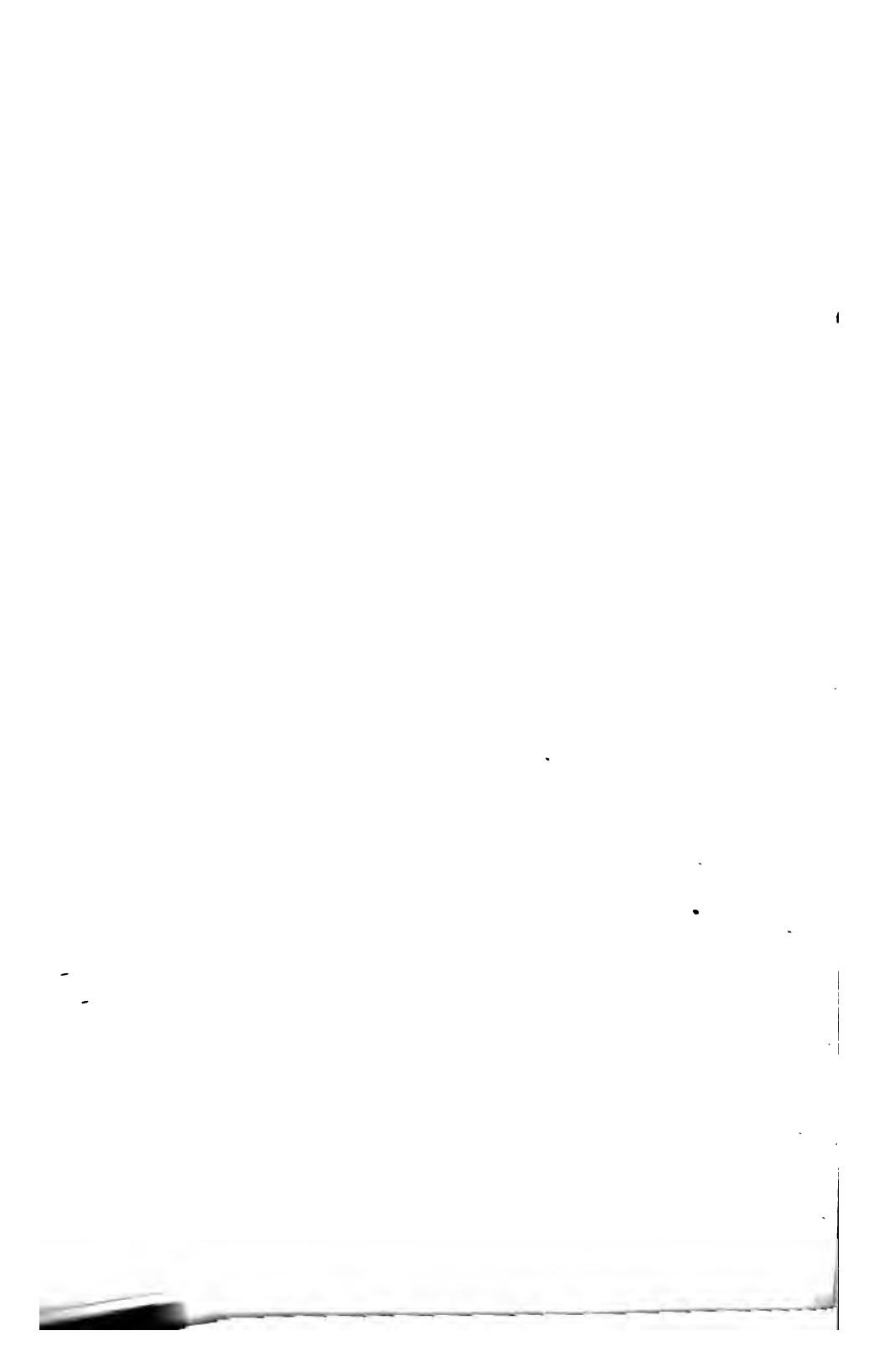
in its plane; and let it move round its axis CP, Fig.
which is perpendicular to its plane, with any velo- 36.
city. Let O be the center of ofcillation.

Now the circle will endeavour to defcend by its
gravity, in the fame manner, as the fingle point O
would do.   Therefore fuppofe the point F to def-
cend through F*f* in a moment of time, and that F
is transferred to I in the fame time. Then if the
parallelogram F*fn*I be compleated, the point F
by the compound motion, will move along F*n*.
By this means CP, the axis of revolution, will be
transferred to the pofition C*q*, inclining more to-
wards B.   But when the circle has made half a
revolution, and G is come to the place F; the
points in F, proceeding in the track F*n*, will move
the axis C*q* forwards, as before; that is (by the
turning of the circle) it will throw it into its former
place CP. So that during a revolution, the axis is
thrown contrary ways in all the oppofite points,
and fo is always reftored back to its firft place in
regard to the plane.   Therefore the circle always
revolves about the axis CP, whilft CP continually
inclines more and more forward; that is, the plane
of the circle continually alters its pofition; and
the variation of its pofition is known from the lines
FI, *n*I; and is equal to the angle *n*FI or PC*q*.
And fince the circle endeavours to move along a
line which is in the plane *n*FG, it will no longer
go along AB, but deviate from it into a new track,
which is now to be found.

It may be convenient to imagine the circle to
be a poligon with an infinite number of fides.
Then let KL be the horizon, M*g* the plane of the
circle, *g* being a point in the right line AB; let
the next point of the circle (or angle of the poli-
gon) defcend to the horizon, through the very fmall
fpace *rt*; then in the right angled triangle *trg*, S.
inclination (*tgr*) : *tr* or *n*I (which is as S.*n*FI) : :

T 4   rad

Fig.
36.
rad $(1) : tg = \dfrac{S.nFI}{S.tgr}$. Therefore if the circle be

moved through Gg in the fametime, then from g (in

the line AB) fetting off $gt = \dfrac{S.nFI}{S.\text{inclination}}$, then t

will be a point in the curve GtR through which the
circle will pafs. After the fame manner it will
again deviate from the laft direction tb; and de-
fcribe the curve GRVWX.

Generally the whirling motion, round its axis, is
equal to its progreffive motion; for the friction of
the plane foon reduces it to that. But take away
the friction, and thefe two different motions may
be what you will.

Cor. 1. *The curvature in any place, is reciprocally
as thefe three quantities, the velocity of rotation, the
progreffive velocity, and the tangent of inclination.*

For the curvature is as the angle tGg, that is, as

$\dfrac{tg}{Gg}$ or as $\dfrac{S.nFI}{Gg \times S.\text{inclination}}$; that is, as

$\dfrac{nI}{FI \times Gg \times S.\text{inclin.}}$; that is, as $\dfrac{\text{Cof. incl.}}{FI \times Gg \times S.\text{inclin.}}$

or as $\dfrac{1}{FI \times Gg \times \text{tan. inclin.}}$. For nI is as the co-

fine of inclination, being the fpace defcribed upon
the inclined plane nI.

Cor. 2. *Taking away all impediments, the circle
always keeps the fame inclination to the horizon.*

For the pofition of the plane FnI is fuch, that
the axes CP, Cq, are both parallel to it. If we
fuppofe gravity to act by a fingle impulfe at O,
then F will move to n, and P to q. And the plane
of the circle endeavouring to defcend a little at D,
and rife at E; a new point of the circle as t, ly-
ing beyond G will inftantly touch the plane; by
which means it leaves the line AG. And fince at
every

every point of contact as G, the pole P moves (at each impulse of gravity) in a line perp. to CG, and alfo parallel to the tangent arch at G; and the like at every new point of contact; it is plain CP is always alike inclined to the horizon. Confequently, when gravity is continual, the circle coming continually to new points of contact, the axis CP will always revolve round at the fame inclination, and therefore the plane of the circle will alfo have the fame inclination to the horizon.

This might alfo be proved after the manner of the XXXth Prop. not confidering the progreffive motion of the circle.

All this is fuppofing there is no refiftance, friction, or other irregularity. But fince in fact, the refiftance of the air continually leffens its motion, and the fmoothnefs of the plane it runs on, caufes the foot or bottom of the circle to flide outward, which continually leffens the inclination, and brings the axis more upright; and the more oblique the plane of the circle is, the fafter it flides out. Upon thefe accounts it can never defcribe a circle, but only a fort of fpiral line; and the plane of the circle defcending lower and lower, at laft falls flat upon the horizon.

Cor. 3. *Hence a circle moving without any refiftance, &c. upon a horizontal plane; will defcribe a circle upon that plane.*

For the velocity and inclination continuing the fame; the curvature of the track defcribed, will be every where alike.

Cor. 4. *And to find the diameter of the circle or orbit defcribed.*

Let t be a very fmall part of time wherein Gg is defcribed, v the velocity of projection per fecond, b the fpace defcended by gravity in time t, s and c the fine and cofine of the circles inclination, $f =$

Fig. $16\frac{1}{12}$ feet; then will $cb = $ fpace defcended along
36. the inclined plane Ff; and by the laws of motion,
$$1'' : v :: t'' : Gg = tv.$$

and $tt : b :: 1'' : f = \dfrac{b}{tt}.$

Then whilft O has moved through the length Gg,
t or I has defcended through the fpace cb on an
inclined plane parallel to Ff. But we proved $gt = $
$\dfrac{nI}{s} = \dfrac{tr}{s} = \dfrac{cb}{s}.$ And therefore the diameter of the

orbit $= \dfrac{\overline{Gg}^2}{tg} = \dfrac{\overline{Gg}^2 \times s}{cb} = \dfrac{ttvvs}{cb} = \dfrac{vvs}{cf} = \dfrac{vv}{f} \times $ tan.
inclination.

Cor. 5. *Alfo to find the periodic time, or time of
one revolution.*

Let $D = $ diameter of the orbit, then by Cor.

laft, $\dfrac{\overline{Gg}^2 \times s}{cb} = D$, and $Gg = \sqrt{\dfrac{cbD}{s}}$. The cir-

cumference of the orbit is $\dfrac{\pi \times \overline{Gg}^2 \times s}{cb}$ (putting π

$= 3.1416$); and by uniform motion, $Gg : t ::$

$\dfrac{\pi \times \overline{Gg}^2 \times s}{cb} : \dfrac{\pi ts \times Gg}{cb}$ the periodic time $= \dfrac{\pi st}{cb}$

$\sqrt{\dfrac{cbD}{s}} = \pi\sqrt{\dfrac{ttsD}{bc}} = \pi\sqrt{\dfrac{SD}{fc}}.$

Or thus,

$Gg\ (tv) : t ::$ circumference $\dfrac{\pi vvs}{fc}$: periodic time

$= \dfrac{\pi vs}{fc} = \dfrac{\pi v}{f} \times $ tan. inclination.

PROP. XXXVI. *Prob.* Fig.

If DEF *be the furface of a right cone, whofe* 37.
axis AE *is perpendicular to the horizon, and* DHFG
a circular plane parallel to the horizon; and if a cir-
cle ab revolves round in the periphery DHFG, *with*
its axis 1B2 *always parallel to the fide of the cone*
DE, *where it then is. To find the periodic time of*
ab, in the circle DHFG.

Draw DBA perp. to DE, and BC perp. to AC.
Let $\pi = 3.1416$, $h =$ fpace defcended by gravity
in the time t.

Then if the force of gravity be reprefented by
AC, the centrifugal force at B to keep the circle
ab in that pofition, through its whole revolution, will
be denoted by BC.   Then it will be AC (the gra-
vity) : BC (the cent. force) : : $h : \dfrac{BC}{AC} h =$ fpace
defcended towards C, by the force in direction
BC = the effect of the centrifugal force.   There-
fore (by Prop. II.) the periodic time is

$$\pi t \sqrt{\dfrac{2BC}{\dfrac{BC}{AC} \times h}} = \pi t \sqrt{\dfrac{2AC}{h}}.$$

Cor. 1. *Hence the periodic time* =

$$\pi t \sqrt{\dfrac{2BC}{h}} \times tan.\ inclination\ ABC.$$

For S.A : S.B : : BC : AC $= \dfrac{SB}{S.A} \times BC = \dfrac{S.B}{cof.B}$
$\times$ BC = tan. B $\times$ BC ; whence, the periodic time

$$= \pi t \sqrt{\dfrac{2BC}{h}} \times tan.\ ABC.$$

Cor. 2. *Draw* BI *parallel to* DE, *and* DL *pa-*
rallel to BC ; *then the periodic time* $= \dfrac{\pi t \times BC \times \sqrt{2}}{\sqrt{CI} \times h}.$

For AC $= \dfrac{BC^2}{CI}.$ Cor.

Fig.
37.

Cor. 3. *Hence the curve defcribed on the conic fur-face, is the fame as that defcribed on a horizontal plane, as explained in the laft Prop. All the difference is, that the conic furface hinders the circle ab from fliding outward, which the horizontal plane cannot do; except it be fuppofed to be fo rough, that the circle cannot flide on it.*

36. For in Cor. 4. of the laft Prop. v is put for the velocity of G along AB; but putting it for the velocity at C, you will have the diameter of the orbit paffing through C: that is, (fig. 37.) inftead of 2DL

37. we fhould find 2BC for the diameter D of the orbit. And by Cor. 5, the periodic time is $\pi t \sqrt{\dfrac{sD}{bc}}$. And in Cor. 1. of this Prop. the periodic time is

$$\pi t \sqrt{\dfrac{2BC}{b}} \times \text{tan. inclination;}$$ which is equal to the

former, becaufe $D = 2BC$, and $\dfrac{s}{c} = $ tan. of the inclination. So the curve is the fame in both cafes.

PROP. XXXVII. *Prob.*

38. *To explain the motion of a top, or fuch like whirling body.*

Let ABC be a top whirling about in the order AEC; FDG a circle defcribed by any point D in the furface, K its center of gravity, BKS the axis of the top.

If the top be upright upon the foot B; that is, if BS be perp. to the horizon, and moves fwiftly about; it will continue upright till the motion flacken. But when it is going to fall, it will lean to one fide; therefore fuppofe D to be the loweft point in the circle FG; then the top endeavours by its gravity to defcend towards D. Let the force of gravity alone move the point D through the fpace D*o*, in a very fmall time; during which, the rotary motion

motion would carry the point D to *r*.   Compleat Fig.
the parallelogram D*rpo*, and the point D will be 38.
carried through D*p*; that is, the circle FDG will come
into the pofition D*p*; and therefore the axis BKS
(perp, to the circle FG) will be moved in a direc-
tion towards H, perp. to DK ; and the poiht S
moved to *n*; the particle S*n* being parallel to D*p*.
After the fame manner by a new impulfe of gravi-
ty at *p*, the loweft point ; the circle FDG will be
moved into a new pofition, below D*p*, and the
point S carried from *n* to *t*.   And fo by every im-
pulfe of gravity, the point S will be moved gra-
dually forward, through the circle S*ntqlz* ; and thus
the top recovers itfelf from falling; the motion of
S being always parallel to that of D.   And there-
fore the motion of the axis BS will be the fame
way about, as the top's motion is.   And thus the
point S will continue to make feveral revolutions
by a flow motion, whilft the top makes its revolu-
tions about its axis, by a fwift motion.

Cor. 1. *This motion of the top and its axis, is fi-
milar to the motion of an oblong fpheroid, and its
nodes.*

For (Cor. 6. Prop. XXX.) the nodes move the
fame way about as the body revolves, and fo does
the axis of the top ; and therefore this motion may
be called the motion of the nodes of the top.

Cor. 2. *When the top's motion is very fwift, 'the
circle* S*ql* *is very fmall; and as it grows flower, that
circle will grow bigger and bigger, till the top falls.*

For when the top's motion is very fwift, D*r* will
be greater, and the angle *r*D*p* lefs; and the circle .
D*p* will deviate lefs from DG.   And gravity hav-
ing little power to difturb its motion, the circle S*ql*
will be extremely fmall, and the top will revolve
about the axis in appearance unmoved.   But as the
top's

Fig. top's motion by refiftance and friction grows lefs
38. and lefs, D*r* will be lefs; and the circle D*p* will de-
     viate more from DG; that is, gravity will have
     more and more power to difturb its motion; and the
     axis BS will defcribe a greater and greater circle
     with the point S, or rather a fpiral, till at laft the
     top falls down.

39.   Cor. 3. *As the top grows flow, and the motion
     weak, and the pole* S *defcribes greater and greater cir-
     cles, the foot* B *is thrown out to the oppofite fide, de-
     fcribing a circle* B*b*, *which is greater as* S*ql is greater;
     and goes the fame way about.*
        For the center of gravity K always endeavours
     to be at reft, whilft the body revolves about.
     Therefore when the top grows weak, and the pole
     S defcribes greater circles, the foot B is thrown fur-
     ther out to the oppofite fide ; and being always op-
     pofite, will defcribe a circle proportional to S*ql*;
     the foot B going the fame way about as S does.
     And thefe circles B*b* will continually grow greater
     and greater till the top falls down.   Till then the
     top rolls about and about from the pofition CAB
     to the oppofite pofition *cab*, till the motion end,
     and the top falls down.   And thefe are the princi-
     pal phœnomena of the motion of a top.

FINIS.

ERRATUM, in Plate, Fig. 23.

P*n* fhould be perp. to F*p*.

37.

39.

Pl. VI. the End